King School

Math Review Summer Assignment

For Students Entering MAT 701

2021-2022

Each section has an "In a Nutshell" review followed by practice problems.

- -Please read through the review first, then complete the circled problems.
- -All work should be done on a separate sheet of paper in order.
- -Each problem should be clearly labeled with the page number and problem number,
- -Every answer should be circled.
- -No final answers without supporting work will be accepted. ANY SUMMER ASSIGNMENT SUBMITTED WITHOUT SUFFICIENT ACCOMPANYING WORK WILL NOT BE ACCEPTED AND WILL REQUIRE YOU TO REDO THE ENTIRE ASSIGNMENT.

Please be sure to use notes from the previous year as well as educational resources on the Internet such as Kahn Academy to refresh your memory if need be.

- -Working with a tutor on these exact problems is not permitted.
- -This assignment will be handed in the first day of class and will count towards your first graded assignment.
- -You will submit all of the pages with your work, but you do not need to submit this packet.

IN A MUTST

Factors and Multiples

In $4 \times 3 = 12$,

- 4 and 3 are factors of 12.
- 12 is a multiple of both 4 and 3.
- The smallest common multiple of two or more numbers is called the least common multiple (LCM).
- The largest common factor of two or more numbers is called the **greatest common factor (GCF)**.

Exponents



$$5^3 = 5 \times 5 \times 5$$

Order of Operations for Evaluating Expressions

- In general, calculate from left to right.
- Evaluate the expression within the innermost pair of parentheses first if there is more than one pair of parentheses.
- Evaluate the exponents next.
- Do multiplication and division before addition and subtraction.

Properties of Multiplication

- Commutative Property of Multiplication $a \times b = b \times a$ For example: $5 \times 4 = 4 \times 5$.
- Identity Property of Multiplication $a \times 1 = a$
- Zero Property of Multiplication $a \times 0 = 0$
- Associative Property of Multiplication $(a \times b) \times c = a \times (b \times c)$ For example: $(2 \times 3) \times 4 = 2 \times (3 \times 4)$.
- Distributive Property of Multiplication $a \times (b+c) = a \times b + a \times c$ For example: $5 \times (3+2) = 5 \times 3 + 5 \times 2$, $(6-4) \times 8 = 6 \times 8 - 4 \times 8$.

Properties of Division

- Identity Property of Division $a \div 1 = a$
- Zero Property of Division $0 \div a = 0$, where $a \ne 0$
- We can divide the sum of, or the difference between, two numbers in parentheses by dividing each number in the parentheses.
 For example: (8 + 4) ÷ 2 = 8 ÷ 2 + 4 ÷ 2,
 (20 10) ÷ 5 = 20 ÷ 5 10 ÷ 5.
- When we multiply or divide both the dividend and the divisor by the same number, the quotient remains the same. For example: $24 \div 8 = (24 \div 4) \div (8 \div 4)$, $6 \div 2 = (6 \times 3) \div (2 \times 3)$.

Example 9

Evaluate the following expressions.

(a)
$$4^2 \div (12 - 8)$$

(b)
$$25 - (2 + 6 \times 3)$$

(c)
$$27 \div 3^2 + (20 - 2^3)$$

(d)
$$((44-2\times 2^2)\div 3^2+4)\div 2$$

Solution

(a)
$$4^2 \div (12 - 8) = 16 \div 4$$

= 4

(b)
$$25 - (2 + 6 \times 3) = 25 - (2 + 18)$$

= $25 - 20$
= 5

(c)
$$27 \div 3^2 + (20 - 2^3) = 27 \div 3^2 + (20 - 8)$$

= $27 \div 3^2 + 12$
= $27 \div 9 + 12$
= $3 + 12$
= 15

(d)
$$((44-2 \times 2^2) \div 3^2 + 4) \div 2 = ((44-8) \div 3^2 + 4) \div 2$$

= $(36 \div 3^2 + 4) \div 2$
= $(36 \div 9 + 4) \div 2$
= $(4+4) \div 2$
= $8 \div 2$
= 4

REMARK

When there are multiple operations or exponents inside the parentheses, make sure you follow the correct order of operations inside the parentheses as well.

Try It! 9

Evaluate the following expressions.

(a)
$$17 + (21 + 3^2) \div (5 \times 2)$$

(b)
$$5^2 \div (48 - 43) \times (2 + 3)$$

(c)
$$(4^2 - 8 + 2 \times 10) \div (3^2 - 5)$$

(d)
$$(8 \times (64 \div 4^2 + 2)) + 6^2 \div 4$$

Exercises 1-3

EXERCISE 1.1

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BASICPRACTICE

- 1. Write an expression for each of the following statements. Then evaluate each expression.
 - (a) 8 more than the product of 14 and 7.
 - (b) 4 less than the quotient of 36 by 6.
 - (c) The quotient of 48 by 8 decreased by 6.
 - (d) The sum of 10 and 5 increased by the quotient of 45 by 5.

- 2. Express the following using exponents.
 - (a) $5 \times 5 \times 5 \times 5$
 - **(b)** 8×8
 - (c) $10 \times 10 \times 10 \times 10 \times 10 \times 10$
 - (d) $35 \times 35 \times 35$
- 3. Evaluate the following expressions.
 - (a) 25^2
- **(b)** 7×10^4
- (c) 5×3^2
- (d) $8^2 2^3$

Example [2

Find the least common multiple of 6 and 8.

- (a) List the first 10 multiples of 6.
- (b) List the first 10 multiples of 8.
- (c) In the multiples you have listed, what multiples are common to both 6 and 8?
- (d) What is the least common multiple of 6 and 8?

Solution

- (a) The first 10 multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, and 60.
- (b) The first 10 multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, and 80.
- (c) The multiples that are common to both 6 and 8 are 24 and 48.

6, 12, 18, 24, 30, 36, 42, 48, 54, and 60.

8, 16, 24, 32, 40, 48, 56, 64, 72, and 80.

(d) The least common multiple of 6 and 8 is 24.

Are 72 and 96 common multiples of 4 and 6? Explain.

Try It! 12

Find the least common multiple of 10 and 15.

- (a) List the first 10 multiples of 10.
- **(b)** List the first 10 multiples of 15.
- (c) In the multiples you have listed, what multiples are common to both 10 and 15?
- (d) What is the least common multiple of 10 and 15?

Exercises 1-3

EXERCISE 1.2



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- 1. List the factors of
 - (a) 64,
 - **(b)** 48.
- 2. List the first 10 multiples of
 - (a) 8,
 - **(b)** 16.



FURTHERPRACTULE

- 3. Find the following.
 - (a) The greatest common factor of 18 and 24.
 - (b) The greatest common factor of 36 and 54.
 - (c) The least common multiple of 5 and 6.
 - (d) The least common multiple of 8 and 12.

In earlier grades, we saw that when we multiply a number by 10, 100, 1,000,..., the digits move 1, 2, 3,... places to the left, respectively. This means we place 1, 2, 3,..., zeros at the end of the number we are multiplying.

Example 16

Multiply 4,000 by 26.

Solution

$$4,000 \times 26 = 4 \times 1,000 \times 26$$

= $4 \times 26 \times 1,000$
= $104 \times 1,000$
= $104,000$



Mentally calculate the following.

- (a) 300×45
- **(b)** $36 \times 4,000$
- (c) $20,000 \times 57$



Exercises 1-6

EXERCISE 1.3



BASICPRACTICE

- 1. Fill in the blanks below.
 - (a) $6 \times 15 = 6 \times 3 + 6 \times$
 - **(b)** $8 \times 18 = 8 \times (___ \times 9)$
 - (c) $7 \times (4+6) = 7 \times (___ \times 2)$
- 2. Multiply mentally.
 - (a) 6×10
 - **(b)** 7×100
 - (c) $11 \times 1,000$
- 3. Mentally calculate the following.
 - (a) 24×6
- **(b)** 48×7
- (c) 6×55
- (d) 23×99

FURTHERPRA

- 4. Evaluate the following expressions.
 - (a) $7 \times 4 \times 5$
 - (b) $3 \times 5 \times 9 \times 4$
 - (c) $7 \times 10 \times (4 \times 2)$
 - **(d)** $50 \times (3 \times 6) \times 2$

- Multiply the following expressions.
 - (a) 26×200
- **(b)** $3,000 \times 16$
- (c) 580×35
- (d) 307×89
- (e) $2,093 \times 72$



- 6. An outdoor furniture store wants to sell benches at \$158 each. At this price they expect to sell 300 benches each month.
 - (a) What is the amount of the total expected monthly sales for the benches?
 - (b) If the price of each bench increases by \$42, the store expects to sell only 200 benches. What would the total expected monthly sales be then?

EXERCISE 1.4





- Mentally divide the following.
 - (a) $84 \div 7$
- **(b)** $128 \div 8$
- (c) $276 \div 6$
- (d) $600 \div 10$
- (e) $7,000 \div 100$
- (f) $2850 \div 50$



URTHERPRACTICE

- Evaluate the following expressions.
 - (a) $28 \div 4 + 3 \times 2^4$
 - **(b)** $15 + 4^3 \div 2 \times 2^2 8$
 - (c) $5^3 \div (5 \times 3 10)$
 - (d) $175 (13 + 7^2) \div 2$
 - (e) $(56-2\times8) \div 2^3-3+5^2$
- Fill in the boxes with the appropriate numbers. Then check your answer by evaluating both sides of the equation.
 - (a) $48 \div 8 = (48 \div$
- $) \div (8 \div 4)$
- **(b)** $16 \div 4 = (16 \times 2) \div (4 \times 1)$
- Mentally calculate the following.
 - (a) $6.300 \div 700$
- **(b)** $9.720 \div 90$
- (c) $135 \div 15$
- (d) 306 ÷ 18

- (e) $22,500 \div 250$
- (f) $456 \div 24$



MATH WORK

- 5. Twelve friends are out for dinner at a restaurant. Their bill is \$216.
 - (a) How much does each person have to pay if they share the cost equally?
 - (b) They want to leave a \$24 tip. How much does each person have to pay including the tip?

A farmer plants one section of a rectangular field with corn and the other section with soybeans. The areas of each section and the width of the field are shown below. What is the length of the field?

Soybeans	Com	
32 km ²	28 km²	4



- Sarah said that to solve $90,000 \div 18,000$, she could just do $45 \div 9$.
 - (a) Does her method work? Explain why.
 - (b) Think of another division expression that can be solved using a similar method.
- You are given the the following expression.

$$144 \div 24 = (144 \div) \div (24 \div)$$

- (a) Think of at least 3 different numbers that can go in and write the expressions.
- (b) Check your expressions to make sure your answers are equal to the quotient of $144 \div 24$.
- (c) Which number in (a) makes the calculation of 144 ÷ 24 the easiest? Explain why.

IN A NUTSAN

Multiplication of Fractions

1. Multiply a fraction by a whole number.

$$\frac{a}{b} \times c = \frac{a \times c}{b}$$

Example
$$\frac{2}{3} \times 2 = \frac{2 \times 2}{3}$$
$$= \frac{4}{3}$$
$$= 1\frac{1}{3}$$

2. Multiply a fraction by a fraction.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$
$$= \frac{ac}{bd}$$

Example
$$\frac{2}{5} \times \frac{10}{11} = \frac{2 \times 10^{2}}{15 \times 11}$$
$$= \frac{2 \times 2}{1 \times 11}$$
$$= \frac{4}{11}$$

Division of Fractions

- 1. The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.
 - Example $\frac{2}{3}$ and $\frac{3}{2}$, $\frac{1}{4}$ and 4, and 5 and $\frac{1}{5}$ are reciprocals of each other.
- 2. Divide a whole number by a fraction.

$$\alpha \div \frac{b}{c} = \alpha \times \frac{c}{b}$$

Example
$$4 \div \frac{2}{7} = 4^2 \times \frac{7}{21}$$

$$= 2 \times \frac{7}{1}$$

$$= 14$$

3. Divide a fraction by a whole number.

$$\frac{a}{b} \div c = \frac{a}{b} \times \frac{1}{c}$$

Example
$$\frac{1}{5} \div 6 = \frac{1}{5} \times \frac{1}{6}$$
$$= \frac{1}{30}$$

4. Divide a fraction by a fraction.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Example
$$\frac{4}{3} \div \frac{2}{9} = \frac{\cancel{3}^2}{\cancel{3}} \times \frac{\cancel{3}^3}{\cancel{3}}$$
$$= \frac{2}{1} \times \frac{3}{1}$$

Method 2

Number of U.S. stamps =
$$\frac{4}{7} \times 336 = 192$$

Number of Canada stamps =
$$\frac{1}{2} \times 192 = 96$$

Number of China stamps =
$$336 - 192 - 96$$

= 48

Now,
$$192 - 48 = 144$$
.

There are 216 coins. $\frac{5}{9}$ of them are British coins, and the rest are Russian and Indian coins. There are $\frac{1}{2}$ as many Indian coins as British coins. How many more British coins than Russian coins are there?

Exercises 1-6,9

EXERCISE 2.1



BASICPRACTICE

- Evaluate the following expressions. Change products that are improper fractions to mixed numbers.
 - (a) $\frac{4}{5} \times 8$
- **(b)** $7 \times \frac{3}{10}$

- (c) $\frac{5}{6} \times \frac{1}{5}$ (d) $\frac{4}{7} \times \frac{3}{5}$ (e) $\frac{9}{10} \times \frac{4}{7}$ (f) $\frac{7}{8} \times \frac{2}{3}$
- **(g)** $\frac{2}{9} \times \frac{3}{11}$
- **(h)** $\frac{8}{25} \times \frac{15}{16}$



FURTHERPRACTICE

- Evaluate the following expressions. Express any improper fractions to mixed numbers.

 - (a) $\frac{8}{5} \times \frac{3}{4}$ (b) $2\frac{1}{10} \times 3$

- (c) $2\frac{1}{2} \times \frac{3}{5}$ (d) $3\frac{1}{3} \times \frac{9}{7}$ (e) $5\frac{1}{5} \times 1\frac{3}{8}$ (f) $2\frac{3}{4} \times 1\frac{1}{10}$

- Mrs. Lin had $\frac{3}{5}$ pound of flour. She used $\frac{5}{6}$ of it to make a cake. How much flour did she use?
- 4. A rectangular garden has a length of $3\frac{3}{5}$ m and a width of $2\frac{1}{2}$ m. What is the area of the garden?
- **5.** Each serving of sorbet is $\frac{3}{10}$ liter. How many liters will $2\frac{1}{2}$ servings be?
- A basketball weighs $1\frac{5}{8}$ pounds. How many pounds do 16 of these basketballs weigh?
- Jessica had \$460. She spent $\frac{3}{4}$ of her money on a computer. How much money did she have left?

8. Jayla had \$400. She spent $\frac{2}{5}$ of it on a game console and $\frac{3}{8}$ of the remainder on video games. How much money did she spend on video games?



- 9. Skye had $12\frac{1}{4}$ meters of ribbon. She used $\frac{2}{3}$ of it to wrap a present and $\frac{1}{2}$ of the remainder on a craft project. How many meters of ribbon did she have left?
- 10. Caitlin had \$185. She spent \$25 on a blouse and $\frac{3}{4}$ of the remainder on a dress. How much money did she have left?

- 11. Mr. Yan spent $\frac{1}{4}$ of his savings on a pair of trousers and $\frac{2}{5}$ of the remainder on a book.
 - (a) What fraction of his savings did he have left?
 - (b) If the book costs \$18, how much money did he have originally?



- 12. Esther had $\frac{3}{5}$ as many e-books as Tim. After Tim deleted 18 e-books, they both had the same number of e-books on their tablets. How many e-books did Tim have at the start?
- 13. Ian is $\frac{1}{4}$ the age of his mother. In 5 years, his age will be $\frac{1}{3}$ of his mother's age. How old is Ian's mother now?

2.2 Division of Fractions

A Division of a Whole Number by a Fraction

Two numbers that have a product of 1 are called **reciprocals**. For example, 2 and $\frac{1}{2}$ are reciprocals because $2 \times \frac{1}{2} = 1$ and $\frac{1}{2} \times 2 = 1$. Reciprocals are an important idea in dividing fractions, so let's explore this concept first.

To find the **reciprocal** of a number, switch the location of the numerator and of the denominator.





EXERCISE 2.2





BASICPRACTICE

- 1. Find the reciprocal of each of the following numbers.
 - (a) $\frac{5}{9}$
- **(b)** $\frac{1}{6}$
- **(c)** 7
- (d) $5\frac{2}{3}$
- 2. Evaluate the following expressions.
 - (a) $\frac{5}{6} \div 7$
- **(b)** $\frac{7}{9} \div 2$
- (c) $10 \div \frac{1}{2}$
- (d) $6 \div \frac{4}{7}$
- (e) $\frac{7}{8} \div \frac{3}{16}$
- **(f)** $1\frac{2}{3} \div \frac{10}{21}$
- (g) $\frac{14}{15} \div \frac{2}{3}$
- **(h)** $5\frac{4}{9} \div 3\frac{1}{2}$



FURTHERPRACTICE

- 3. Evaluate the following expressions.
 - (a) $\frac{9}{14} \times \frac{7}{9} \div 3$
 - **(b)** $\left(\frac{1}{2} + \frac{1}{4}\right) \times 6 \div \frac{1}{3}$
 - (c) $3\frac{1}{5} \div \frac{8}{15} \frac{3}{4} \times \frac{5}{6}$
 - (d) $5-3 \div 2\frac{1}{4} \times \frac{3}{8}$
- **4.** Jane baked a $\frac{4}{5}$ -kilogram butter cake. She gave each of the children in her class $\frac{2}{25}$ kilogram of the cake. How many children are there in Jane's class?
- 5. If $4\frac{1}{2}$ ounces of silver cost \$99, how much does 1 ounce of silver cost?
- 6. How much almonds will each person get if 20 people share $2\frac{2}{5}$ kilograms of almonds?



MATH@WORK

- 7. A large bottle holds $2\frac{1}{4}$ liters of juice and a small bottle holds $\frac{3}{8}$ liter. How many times as much juice does the large bottle hold as compared to the small bottle?
- 8. How many $\frac{3}{4}$ -cup servings can you get from $3\frac{3}{8}$ cups of sauce?
- 9. Jessica had $5\frac{1}{3}$ meters of ribbon. She used $\frac{1}{4}$ of it to wrap a present and cut the remainder of the ribbon into $\frac{1}{4}$ -meter-long pieces. How many pieces did she get?
- 10. A road crew can pave $\frac{2}{5}$ mile of road in $1\frac{1}{2}$ hours. How many miles of road can they pave in 10 hours?



BRAIN WORKS

- 11. Draw a model to support your answer to the division expression, $\frac{3}{5} \div \frac{1}{3}$.
- 12. In explaining why $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$, a student's proof is as follows:

$$\frac{a}{b} \div \frac{c}{d} = \frac{\frac{a}{b}}{\frac{c}{d}}$$

$$= \frac{\frac{a}{b} \times \frac{d}{c}}{\frac{c}{d} \times \frac{d}{c}}$$

$$= \frac{a}{b} \times \frac{d}{c}$$

Is his argument valid? Explain your answer.

IN A MUTSA

Convert Fractions to Decimals

Example (a)
$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25}$$

= $\frac{75}{100}$
= 0.75

(b)
$$1\frac{2}{5} = 1 + \frac{2 \times 2}{5 \times 2}$$

= $1 + \frac{4}{10}$
= $1 + 0.4$
= 1.4

Convert Decimals to Fractions

Example (a)
$$0.25 = \frac{25}{100}$$

= $\frac{1}{4}$

(b)
$$1.2 = 1 + \frac{2}{10}$$

= $1 + \frac{1}{5}$
= $1\frac{1}{5}$

Adding and Subtracting Decimals

- Write the numbers in vertical form by aligning the decimal points and the place
- Write zeros as needed to have the same number of places in each number.

Example

$$\begin{array}{r}
1 \\
2.80 \\
+ 3.41 \\
\hline
6.21
\end{array}$$

Multiplying Decimals

(a) Multiply by 10, 100, and 1,000

$$0.5\times10=5$$

$$0.5 \times 100 = 50$$

$$0.5 \times 1,000 = 500$$

(b) Multiply decimals

$$4.5 \times 0.12$$

$$\begin{array}{c} 4.5 \\ \times 0.12 \end{array}$$

$$\begin{array}{c}
4 & 5 \\
\times & 1 & 2 \\
\hline
9 & 0
\end{array}$$

Therefore, $4.5 \times 0.12 = 0.54$

Dividing Decimals

(a) Divide by 10, 100, and 1,000

$$0.5 \div 10 = 0.05$$

$$0.5 \div 100 = 0.005$$

$$0.5 \div 1,000 = 0.0005$$

(b) Dividing by a decimal

- Example 1.25 ÷ 0.4
 - Multiply the divisor by 10 to change it to a whole number. Then, multiply the numerator by the same number 10.

$$\frac{1.25}{0.4} = \frac{1.25 \times 10}{0.4 \times 10} = \frac{12.5}{4}$$

• Divide 12.5 by 4.

$$\begin{array}{r}
3.125 \\
4) 12.500 \\
-12 \\
\hline
0.5
\end{array}$$

$$\frac{20}{-20}$$

So,
$$1.25 \div 0.4 = 3.125$$
.

Conversion of Units

- 1 cm = 10 mm
- 1 dm = 10 cm
- 1 m = 10 dm
- 1 m = 100 cm
- $1 \, dm = 0.1 \, m$
- 1 cm = 0.01 m
- 1,000 m = 1 km
- 1,000 g = 1 kg
- 1,000 mL = 1 L

WRITE IN YOUR JOURNAL

After learning the rule "add a zero when you multiply by 10," a student did the following.

$$6.5 \times 10 = 6.50$$

Is the answer correct? Explain your reasoning.

Without performing any division, explain why 3.8 ÷ 0.11 and $380 \div 11$ have the same answer.

EXTEND YOUR LEARNING CURVE

The Magical 142857

- (a) Use long division or a calculator to verify that $\frac{1}{7} = 0.142857142857...$ Then compare the decimals of these fractions with that of $\frac{1}{7}$. What do you notice?
- (ii) $\frac{3}{7}$ (iii) $\frac{4}{7}$ (iv) $\frac{5}{7}$
- (b) Investigate some other interesting properties about the number 142857.

EXERCISE 3.1



BASICPRACTICE

- 1. Write the following as decimals.
 - (a) $\frac{6}{10}$
 - **(b)** $\frac{43}{100}$
 - (c) $\frac{5}{1,000}$
 - (d) 8 tenths 9 thousandths
 - (e) $1\frac{3}{100}$
 - **(f)** $4\frac{19}{1,000}$
 - (g) $\frac{38}{10}$
 - **(h)** $\frac{567}{100}$
- 2. Write the following as decimals.
 - (a) 9 tens 3 ones 9 tenths
 - (b) 2 ones 1 tenth 3 hundredths
 - (c) 5 ones 2 hundredths 1 thousandth
 - (d) 8 ones 6 tenths 6 thousandths
- 3. Calculate the following.
 - (a) 0.27 + 9.95
- **(b)** 24.3 + 8.78
- (c) 18.905 + 37.44
- (d) 6.03 -5.26
- (e) 36.2 -17.11
- (f) 15.103 - 9.47
- 4. Write in vertical form and then calculate the following expressions.
 - (a) 12.36 + 4.74
- **(b)** 2.305 + 5.5
- (c) 7 + 8.804
- (d) 3.756 + 0.28
- (e) 6.76 3.98
- (f) 0.8 0.47
- (g) 3.14 1.807
- **(h)** 8 0.179

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FURTHER PRACTICE

- Express each decimal as a fraction or mixed number in simplest form.
 - (a) 1.12
 - **(b)** 0.065
 - (c) 22.06
 - (d) 3.125
- 6. Express each fraction as a decimal.
 - (a) $\frac{3}{5}$
 - **(b)** $\frac{7}{4}$
 - (c) $4\frac{3}{50}$
 - (d) $2\frac{3}{8}$
- 7. Calculate the following.
 - (a) 1.23 + 45.6 + 0.789
 - **(b)** 3.46 + 12.8 9.23
 - (c) 15.3 1.268 + 3.76
 - (d) 8 0.35 2.009
 - **(e)** 137 (3.087 + 0.99)
 - **(f)** 65.4 (42.7 11.86)
 - (g) 2.86 + 3.057 (1.005 0.999)
 - **(h)** 21 9.64 (1.7 + 5.755)
- 8. Arrange these numbers in increasing order.

$$4\frac{2}{25}$$
, 4.1, 4.088, $4\frac{3}{100}$



MATH@ WORK

- 9. Mr. Bravo bought some tortillas for \$3.87 and some beans for \$2.67. He paid for the food using a \$10 bill. How much change did he receive?
- 10. Eliza, Abbie, and Jackie went shopping. They spent a total of \$200. Eliza spent \$87.50. Abbie spent \$58.36. How much did Jackie spend?

- 11. A baker has \$40. He spends \$14.50 to buy flour and \$12.86 to buy butter. How much money does he have left?
- 12. The thickness of a large brick is 3.000 cm and the thickness of a small brick is 1.908 cm less than the large brick. What is the combined thickness of the two bricks?



- **13.** Think about different ways to express the number 4.867.
 - (a) 4.867 = ____ ones + ___ tenths + ___ hundredths + ___ thousandths
 - **(b)** 4.867 =____ thousandths
 - (c) 4.867 = ____ hundredths + ____ thousandths
 - (d) 4.867 = ____ tenths + ___ thousandths
- **14.** In the repeating decimal 0.123451234512..., what is the 2018th digit to the right of the decimal point?

3.2 Multiplication of Decimals

Decimal Number System

In earlier grades, we learned how to multiply whole numbers and decimals by 10, 100, and 1,000, and by $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$.

Let's find the numbers that are 10 times, 100 times, and 1,000 times as large as 0.008.

$$0.008 \times 10 = \frac{8}{1,000} \times 10$$

$$= \frac{8}{100}$$

$$= 0.08$$

$$0.008 \times 100 = \frac{8}{1,000} \times 100$$

$$= \frac{8}{10}$$

$$= 0.8$$

$$0.008 \times 1,000 = \frac{8}{1,000} \times 1,000$$

$$= 8$$

EXERCISE 3.2



BASIC PRACTICE

- 1. What number is
 - (a) 10 times as large as 3.25?
 - **(b)** $\frac{1}{10}$ times as large as 58?
 - (c) 100 times as large as 0.123?
 - (d) $\frac{1}{100}$ times as large as 6?
 - (e) 1,000 times as large as 24.67?
 - (f) $\frac{1}{1.000}$ times as large as 1,940?
- 2. Without doing any calculations, determine which of the following multiplication expressions will give a product that is larger than 2.04? Explain your answers.
 - (a) 2.04×1.3
- **(b)** 2.04×0.678
- (c) 2.04×1.01
- (d) 2.04×0.9
- 3. Without doing any calculations, determine which of the following multiplication expressions will give a product that is less than 130.7? Explain your answers.
 - (a) 130.7×1.2
- **(b)** 130.7×0.05
- (c) 130.7 × 0.999
- (d) 130.7 × 1.001
- 4. Write in vertical form. Then multiply the following expressions.
 - (a) 2.39×0.4
- **(b)** 64.8×2.3
- (c) 140.2×6.5
- (d) 37.8×345
- (e) 0.0735×940
- (f) 8.6×0.325

Ó.

FURTHER PRACTICE

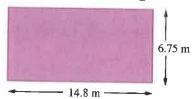
- Fill in the blanks.
 - (a) $3.45 \times$ = 34.5
 - **(b)** $567 \times \underline{\hspace{1cm}} = 5.67$
 - (c) $0.49 \times \underline{\hspace{1cm}} = 0.0049$

 - (e) $20 \times \underline{\hspace{1cm}} = 0.02$
 - **(f)** $0.37 \times \underline{\hspace{1cm}} = 370$

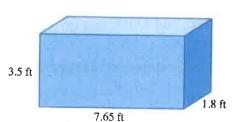
- 6. Nina did the following:
 - (a) $1.2 \times 100 = 1.200$
 - **(b)** $\frac{2.05}{10} = \frac{2.05}{10} = 2.5$

Were her methods valid? Explain your answers.

7. Find the area of the rectangle below.



- 8. Calculate the following.
 - (a) 0.007×0.8
 - **(b)** 0.0016×625
 - (c) 4.9×0.005
 - (d) $67.9 \times 0.32 \times 0.025$
 - (e) $(0.13 + 0.095) \times (53 52.66)$
 - **(f)** $(43.2 6.95) \times 6.4$
- 9. Find the volume of the rectangular prism below.



- **10.** One foot of copper tubing weighs 0.837 pound. How much does 3.5 feet of this tubing weigh?
- 11. One kilogram of apples costs \$4.79. How much does 5.7 kilograms of apples cost? Round your answer to the nearest cent.
- 12. A piece of rope is 12.65 meters long. We want a second piece that is 1.5 times longer than the first piece. How long should the second piece be?



- 13. Maya saved \$3,850. She spent $\frac{1}{10}$ of it on a dress and $\frac{1}{100}$ of the remainder on lunch. How much did she spend on the lunch?
- 14. A bag of coffee costs \$10.65 at Super Mart. The same bag of coffee costs 0.75 times as much at Valley Market. Which store sells coffee at a cheaper price and what is the price? Round your answer to the nearest cent.
- 15. Jeannie buys 3 pounds of cheese at \$7.99 per pound, 8 pounds of lemon at \$1.69 per pound, and some flowers for \$18.50. She pays with a \$100 bill. How much change should she receive?
- 16. The length of a red ribbon is 3.5 times the length of a blue ribbon. If the length of the blue ribbon is 3.05 m, how much longer is the red ribbon than the blue ribbon?



17. Mary multiplied 7.8 by 12.1 and got 94.38. She then checked whether her answer is reasonable or not, using estimation.

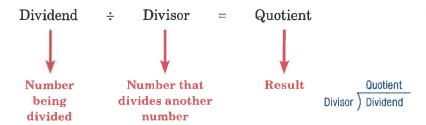
		Your answer	Estimate	Is your answer reasonable?
	7.8×12.1	94.38	8 × 12 = 96	Yes
(a)	4.15 × 24.7			
(b)	16.75×0.18			
(c)	510.5×0.06		1	

Calculate each of the products above. Then, use estimation to check whether your answer is reasonable or not.

3.3 Division of Decimals

A Divisor and Quotient

In a division, the three parts are defined as follows:



In Grade 5, we learned to divide decimals to hundredths, using models and strategies based on place value. We also learned that in a division, the size of a divisor affects the size of the quotient.

Example 12

Mrs. Jackson bought 6.2 pounds of oranges at Big Market and paid \$7.44. How much does 1 pound of oranges cost?

Solution

Since 6.2 lb of oranges cost \$7.44, 1 lb of oranges costs $7.44 \div 6.2$.

Multiply both the divisor and the dividend by 10 to make the divisor 6.2 a whole number.

Complete the division.

$$\begin{array}{r}
 1.2 \\
 62) \overline{74.4} \\
 -62 \\
 \hline
 124 \\
 -124 \\
 \hline
 0
\end{array}$$

Thus, 1 lb of oranges costs \$1.20.

Try It! 12

 Λ 3.5-meter-long metal pipe weighs 6.125 kilograms. How much does 1 meter of this pipe weigh?

Exercises 2,5,6

EXERCISE 3.3



BASICPRACTICE

- 1. Without doing any calculation, determine which of the following division expressions will give a quotient that is less than 3.87 and which will give a quotient that is larger than 3.87. Explain your answers.
 - (a) $3.87 \div 4$
 - **(b)** $3.87 \div 0.9$
 - (c) $3.87 \div 1.003$
 - (d) $3.87 \div 0.908$
 - (e) $3.87 \div 0.01$

- 2. Fill in the with the appropriate numbers to make the expressions
 - (a) $24 \div 6 = (24 \times 3) \div (6 \times 1)$
 - **(b)** $1.2 \div 4 = (1.2 \times 10^{-3}) \div (4 \times 2)$
 - (c) $81 \times 3 \div 9 \times 3 = 81 \div$

equivalent.

(d) $(13.8 \times 4) \div (2.3 \times 4) = \div 2.3$

with the appropriate Fill in the number to make each of the following equivalent. Then find the expressions quotient.

(a)
$$5.16 \div 1.2 =$$
 $\div 12$

(b)
$$0.207 \div 0.9 = \div 9$$

(c)
$$0.006 \div 0.04 = \div 4$$

(d)
$$39 \div 0.15 = \div 15$$

Divide completely.

(a)
$$6.54 \div 6$$

(b)
$$13.16 \div 2.8$$

(c)
$$42.16 \div 17$$

(d)
$$0.387 \div 0.03$$

(e)
$$5 \div 8$$

(f)
$$0.3 \div 4$$



JRTHER PRACTICE

Divide and round the quotient to two decimal places.

(a)
$$2.83 \div 0.56$$

(b)
$$16.79 \div 1.5$$

(c)
$$0.009 \div 0.64$$

(d)
$$5 \div 6$$

- A 2.8-meter-long ribbon costs \$4.90. How much would a 1-meter ribbon cost?
- A metal pipe measuring 3.5 meters weighs 12.67 kilograms. How much does 1 meter of this pipe weigh?
- A rectangular garden has an area of 25.3 m² and a width of 4.4 m. What is the length of the garden?
- A store wants to pack 36 pounds of coffee in 2.25-pound bags. How many bags of coffee will they be able to pack?
- 10. If 0.8 ounce of silver cost \$10.79 on a certain day, what was the cost per ounce of silver on that day? Round your answer to the nearest cent.
- 11. Shane ran 21 kilometers in 2.6 hours. How far did he run in 1 hour? Round your answer to 3 decimal places.



- 12. Zoltan weighs 62 kilograms. He weighs 2.5 times as much as his sister and 0.8 times as much as his father. What is the weight of his sister and of his father?
- 13. Jeannie spent \$14.50 on 5 pounds of oranges and 3 pounds of apples. The apples cost \$2.55 per pound. How much does 1 pound of oranges cost?
- 14. A 12.5-ounce bottle of coffee powder costs \$10.50. An 8.8-ounce bottle of the same coffee costs \$7.92. Which bottle of coffee offers a better deal? How much is the savings in cents?
- 15. At a fabric store, 12 yards of burlap fabric cost \$57 and 15 yards of cotton fabric cost \$78.75. Which fabric costs more per yard? How much more?



16. Julia's reasoning

In the repeating decimal 0.99999..., the number of 9's is infinite. If we do 1 - 0.99999..., we will get 0.000....01, where the last digit will be a 1. So, 0.99999... is less than 1.

Emily's reasoning

By division,

$$\frac{1}{9} = 0.11111...$$

$$\frac{1}{9} = 0.11111...$$

$$\frac{9}{9} = 9 \times 0.11111...$$

= 0.99999...

Since
$$\frac{9}{9} = 1$$
, so $0.99999... = 1$.

Who is right? Explain your answer.

17. Paul claimed that when 2 is divided by 0.3, the remainder is 2, as shown below.

$$\frac{2}{0.3} = \frac{20}{3}$$

Now,

$$20 = 3 \times 6 + 2$$

Hence, the reminder of $\frac{2}{0.3}$ is also 2.

Is his reasoning correct? Explain.

EXERCISE 3.4





BASICPRACTICE

- 1. Convert the following.
 - (a) 1.8 m to centimeters
 - **(b)** 4.567 kg to grams
 - (c) 4.08 L to liters and milliliters
- 2. Express the following as a decimal.
 - (a) 3,750 mL in liters
 - (b) 28.6 cm in meters
 - (c) 16 km 87 m in kilometers



FURTHER PRACTICE

- 3. There are two dogs in a pet store. Mila weighs 9.5 kilograms. Bailey weighs 850 grams less than Mila. What is Bailey's weight in kilograms?
- 4. Stephanie is 1.6 meters tall. Her sister, Jacqueline, is 4 centimeters taller than her. How tall is Jacqueline in meters?
- **5.** One bottle holds 350 milliliters of water. How much water do five of these bottles hold in liters?
- 6. One baseball weighs 150 grams. There is a bag of baseballs that weighs 2.7 kilograms. How many baseballs are in the bag? Assume the weight of the bag is negligible.



MATHWWORK

7. A store owner has two bags of peanuts, each weighing 2.4 kilograms. He wants to put the peanuts in smaller bags that weigh 60 grams each. How many of the smaller bags of peanuts can he make?

- 8. Danny has 3.8 meters of ribbon. He uses 1.4 meters to wrap presents, and cuts the remainder into 8 equal pieces for an art project. How long is each piece in centimeters?
- 9. A large bottle contains 750 milliliters of juice and a small bottle contains 400 milliliters of juice. A sixth grade class took 5 large bottles and 12 small bottles of juice on a field trip. How much juice did they take in liters?



BRAIN WORKS

- **10.** The speed of sound has been calculated at 343.2 meters per second.
 - (a) How many kilometers can sound travel in 1 minute? Round your answer to the nearest tenth of a kilometer.
 - (b) A person hears thunder 10 seconds after seeing lightning. About how many kilometers away is that person from where the lightning struck? Round your answer to the nearest tenth of a kilometer.
- 11. Given that $3.85 \times 4.24 = 16.324$, find the values of the following.
 - (a) $163.24 \div 0.004 24$
 - **(b)** $0.016324 \div 385$
 - (c) $0.0424 \times 3,850$
- 12. Samuel is thinking of a number. When he divides it by 6 and adds 2.3 to the answer, he gets 4. What number is he thinking of?
- 13. Rowan divided a 2-digit number by another 2-digit number and obtained 0.781 25 as the answer. What are the two numbers?

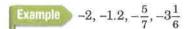
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Positive and Negative Numbers

A **positive number** has a value that is greater than zero.

A **negative number** has a value that is smaller than zero.

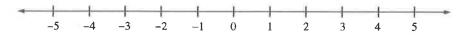
Example 4, 0.6, $\frac{3}{4}$, $2\frac{1}{3}$



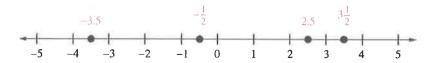
- Zero is neither positive nor negative.
- A positive or negative whole number is called an integer.
- Positive numbers and negative numbers include whole numbers, fractions, and decimals.

Number Line

A number line is divided into equal intervals. It can be used to represent numbers.



From the number line, we can see that the opposite of 4 is -4 and the opposite of -3 is 3. A number and its opposites are at an equal distance from the origin. For example, the opposite of 4 is -4, and both 4 and -4 are 4 units from the right and left of the origin 0 respectively.



Absolute Value

On a number line, the **absolute value** denotes the distance of a positive or negative number from zero.

|
$$87 | = 87$$

| $-225 | = 225$
| $0 | = 0$

- Numbers with opposite signs indicate that their locations are on opposite sides of 0, and are at an equal distance from 0 on the number line.
- To compare a negative number with another negative number, you can find the absolute value of each number. Negative numbers with greater absolute values are less than negative numbers with smaller absolute values.

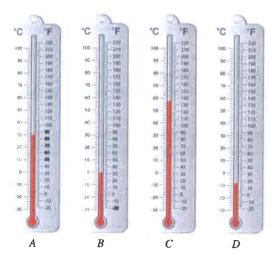
Example |-12| > |-5| but -12 < -5

EXERCISE 4.1



BASICPRACTICE

1. Express each temperature in degrees Celsius using a positive number, a negative number, or zero.



- 2. Express the following real-world situations using positive and negative numbers.
 - (a) An elevation of 333.75 meters above sea level
 - (b) A temperature of 5 degrees below 0°F
 - (c) A golf score of 10 under par
 - (d) An elevation of 427 meters below sea level
 - (e) A temperature of 20 degrees above 0°C
- **3.** Express the following situations using positive and negative numbers.
 - (a) A drop in temperature of 15°F
 - (b) A credit of \$85 to a checking account
 - (c) A bank deposit of \$100
 - (d) A car driving at 8 miles per hour above the speed limit
 - (e) A debit to a checking account of \$300.50
 - (f) A withdrawal of \$980.55 from a bank account
 - (g) A $3\frac{1}{2}$ -centimeter increase in the height of a plant
 - (h) A weight loss of $7\frac{3}{4}$ lb



FUR

FURTHERPRACTICE

The speed limit on a highway is 55 mph. The speeds of five cars clocked by radar are tabulated below.

Car	A	В	C	D	E
Speed (mph)	65	50	55	53	60.5

Express the difference between the speed limit and the speed of each car using a positive number, a negative number, or zero.

5. The table below shows the credit and debit amounts from Michelle's bank account in August.

Date	Debit	Credit
August 5	\$240.00	
August 9		\$125.50
August 21		\$320.00
August 23	\$192.40	
August 30		\$71.00

Use positive or negative numbers to express the balance on each day in the table below. The initial balance is \$100.

Date	Balance Amount
August 5	
August 9	
August 21	
August 23	
August 30	



- 6. At the beginning of the year, a dog weighed 10 pounds. At the end of the year, the dog weighed 40 pounds.
 - (a) How did the dog's weight change over the one-year period?
 - **(b)** Express the dog's weight change using a positive or negative number.
- 7. The heights in meters of some tall buildings in New York and Chicago are shown in the table below.

Building	Freedom Tower	Empire State Building	Willis Tower	Aon Center
Height (m)	541	381	442	346

A helicopter lands on top of Willis Tower. Express the position of the helicopter with reference to the top of each of the other buildings as the base, using positive or negative numbers.



- 8. Consider the sum $1-1+1-1+1-1\cdots$, where the terms continue alternating -1 and +1. The sum of the first two terms is 1-1=0.
 - (a) Find the sum of the first 10 terms
 - (b) Find the sum of the first 15 terms
 - (c) Find the sum of the first 231 terms

Explain how you get your answer.

- **9.** What is the value of 1 (-(-(-2)))?
- 10. A scuba diver dives 83 feet below sea level, and then rises 47 feet. What is the position with relation to sea level?
- 11. At 08:00, one winter morning, the temperature in Moscow was -3°C. Between 08:00 and 09:00, the temperature increased by 5°C. What was the temperature at 09:00?
- 12. The table shows the temperature of two towns at 10 P.M. on a certain day. Find the difference between the temperatures.

Town X	-2°F
Town Y	_6°F

Example 🦻

John's bank account balance shows -\$110. How much does he owe the bank? Explain your answer in terms of absolute value.

Solution

$$|-110| = 110$$

The size of the debt in dollars is 110. John owes the bank \$110.

Try It!

Amy's bank account balance shows -\$3,000. How much does she owe the bank? Explain your answer in terms of absolute value.

Exercises 1,3-4,7

EXERCISE 4.2



BASICPRACTICE

- 1. (a) Represent the numbers 7.5, 5, 0, and $4\frac{1}{2}$ on a horizontal number line.
 - (b) Arrange the numbers in ascending
 - (c) Arrange the numbers in descending order.
- 2. (a) Represent the numbers $10, -10, 0, 6\frac{3}{4}$, and 8.5 on a vertical number line.
 - **(b)** Arrange the numbers in ascending order.
 - (c) Arrange the numbers in descending order.
- 3. Fill in the blanks with >, <, or =.
 - (a) -57 ____ -185
 - **(b)** -4 ____ -300
 - (c) -7 ____ 0
 - (d) 44 ____ 444
 - (e) -60 ____ 60

- 4. Fill in each blank with >, <, or =.
 - (a) |0|___|-8|
 - **(b)** |17|___|5|
 - (c) |-3| ____|-7|
 - (**d**) |500| ____ |-1,000|



FURTHERPRACTICE

- 5. Fill in the blanks with >, <, or =.
 - (a) -41.5 ____ -41.05
 - **(b)** −28.6 ____ −30
 - (c) $0 = \left| -4\frac{1}{2} \right|$
 - (d) 57 ____ 50.35
 - (e) -0.007 = -0.02
- **6.** Fill in each blank with >, <, or =.
 - (a) $|0| = \left| -\frac{7}{8} \right|$
 - **(b)** $\left| -5\frac{5}{8} \right| = \left| -\frac{40}{8} \right|$
 - (c) $\left| -\frac{3}{5} \right| \left| -\frac{3}{7} \right|$
 - (d) $\left| -5\frac{1}{4} \right| = 5.2$



8.

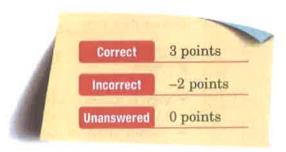
9.



- 7. Doris was on the 13th floor of a building. She got on the elevator and went two floors up, three down, five up, and seven down. Representing these trips as +2, -3, +5, and -7, what floor is Doris on now?
- 8. The price of a technology stock at the beginning of the week is \$21. The changes in price during the week in dollars are: +2, +1, -1, -2, and -1. What is the price of the stock at the end of the week?
- 9. The depth of the Mediterranean Sea is 1,500 m. The height of the Matterhorn, one of the highest peaks in the Swiss Alps, is 4,476 m.
 - (a) Express each elevation using positive or negative numbers.
 - (b) Find the absolute value of each elevation.
 - (c) Which is numerically greater, the height of the Matterhorn or the depth of the Mediterranean Sea, and by how much?
- **10.** In 1913, a record high temperature of 136°F was recorded in Libya. In 1983, a record low temperature of −129°F was recorded at the South Pole.
 - (a) Find the absolute value of each temperature.
 - (b) Which is numerically greater, the record high temperature or the record low temperature, and by how much?



11. A science quiz consists of five questions. The scoring scheme for each question is as follows:



- (a) What is the maximum score of the quiz?
- (b) What is the minimum score of the quiz?
- (c) Write a situation where a student scores 4 points for the quiz.
- (d) Write a situation where a student scores -5 points for the quiz.
- 12. Boadicea died 129 years after Cleopatra was born. Their combined ages were 100 years. If Cleopatra died in 30 B.C., when was Boadicea born?

Hint: Use a number line to represent the lifespans.

13. A math test has 20 questions with 7 points awarded for each correct answer, 2 points deducted for each wrong answer, and 0 points for each question omitted. If a student scores 87 points, how many questions did she omit?



The Meaning of Percent

The part of the whole expressed in hundredths is called the percent. The symbol for percent is "%".

$$1\% = \frac{1}{100}$$

$$23\% = \frac{23}{100}$$

$$100\% = 1$$

Conversion Between Percents, Fractions and Decimals

• Convert Fractions and Decimals to Percents

$$\frac{3}{4} = \frac{3}{4} \times 100\%$$

$$0.429 = 0.429 \times 100\%$$

= 42.9%

Move the decimal point two places to the right.

• Convert Percents to Fractions and Decimals

$$36\% = \frac{36}{100}$$

Divide by 100.

$$=\frac{9}{25}$$

$$7.89\% = \frac{7.89}{100}$$

Move the decimal point two places to the left.

$$= 0.0789$$

Percentage of a Quantity

We can use 10%, 25%, and 1% to find other percentages of a quantity.

- To find 10% of a number, divide the number by 10.
- To find 25% of a number, divide the number by 4.
- To find 1% of a number, divide the number by 100.

We can find the percentage of a quantity by multiplying the quantity by a fraction with 100 as the denominator or by its equivalent decimal.

Example

To find 67% of 500, we can multiply 500 by $\frac{67}{100}$ or multiply 500 by 0.67.

Method 2

$$1 - \frac{2}{5} = \frac{3}{5}$$

$$\frac{3}{5} \times 100\% = 60\%$$

60% of the students are non-sixth graders.

Try It! 7

Out of 25 apples in a basket, 18 are green apples and the rest are red apples. What percent of the apples are red apples?

Exercises 1-4,11

EXERCISE 7.1



BASICPRACTICE

- 1. Express each fraction as a percent.
 - (a) $\frac{7}{20}$
- **(b)** $\frac{12}{25}$
- (c) $1\frac{3}{5}$
- (d) $\frac{9}{4}$
- 2. Express each decimal as a percent.
 - (a) 0.7
- **(b)** 0.07
- (c) 1.8
- (**d**) 0.354
- 3. Express each percent as a fraction in simplest form.
 - (a) 42.5%
- **(b)** 0.8%
- (c) 240%
- (d) $3\frac{1}{2}\%$



- 4. Express each percent as a decimal.
 - (a) 30.7%
- **(b)** 9.05%
- (c) $1\frac{3}{4}\%$
- (d) 166%

EURTHER PRACTIC

- 5. Express $\frac{2}{7}$ as a percent rounded to one decimal place.
- 6. Of all the beads in a box, $\frac{9}{16}$ are red beads. What percent of the beads in the box are red beads?
- 7. Madison wants to buy a dress at a store that sells it at "25% off" the regular price. What percent of the regular price does she have to pay?

- 8. Skye spent 30% of her money on a pair of pants and 45% of her money on a shirt. What percent of her money did she have left?
- Tony expressed 0.3 as 3%. Do you agree with Tony's answer? Explain your answer.
- MATH@WORK
- 10. Shari has to walk 1 mile to the library. She has already walked 0.8 mile. What percent of the distance has she walked so far?
- 11. Marvin had \$25. He spent \$10 on his lunch and saved the rest. What percent of his money did he save?
- **12.** The boys make up 60% of the students on the soccer team. What fraction of the students on the team are girls?

- 13. Peterson spent $\frac{1}{8}$ of his money on a new coat. What percent of his money did he have left?
- **14.** Would you rather have 10% of \$20, or 20% of \$10? Why?

BRAINWORKS

- 15. Maya downloaded some math and science apps from Google Play. The number of math apps is 25% of the number of science apps. What percent are the math apps to the total number of apps?
- 50% off on all items
 Shop A Shop B

For a similar item costing the same price before any discount or promotion, which shop offers a better deal?

Method 2

 $75\% = \frac{3}{4}$ and the number of boys is equal to $\frac{3}{4}$ of the number of girls.

$$\frac{3}{4} \times 24 = 18$$

Method 3

100% - 75% = 25%

There are 25% of 24 more girls than boys in the class.

$$25\% \times 24 = 6$$

There are 6 more girls than boys in the class.

Since there are 24 girls in the class, there are 24 - 6 = 18 boys in the class.

There are 18 boys in the class.

Try It! 18

There are 31.2% fewer cows than goats on a farm. If there are 86 cows, how many goats are there on the farm?

Exercises 1,3,8

EXERCISE 7.2



BASICPRACTICE

- 1. Find the following amounts.
 - (a) 10% of 40 mi
 - **(b)** 50% of \$300
 - (c) 30% of 180 kg
 - (d) 25% of 68 lb
 - (e) 70% of 260 m
 - **(f)** 75% of 292 kg
- 2. Find the following amounts.
 - (a) 110% of \$875
 - **(b)** 1.2% of 3,000 mi
 - (c) $\frac{3}{4}$ % of 760 m
 - (d) $1\frac{15}{25}\%$ of 54 L
 - (e) 65% of \$680
 - (f) 14% of 750 L
 - (g) 27% of 540 kg
 - (h) 99% of 7,000 km



FURTHERPRACTICE

- 3. Brandon had \$500. He spent 47% of his money on a tablet computer. How much did the tablet computer cost?
- 4. Maya bought 350 grams of peanuts. She used 58% of the peanuts to make peanut butter. How many grams of peanuts did she use to make peanut butter?
- **5.** A sound system originally costs \$412. Angie bought it on sale for 75% of its original cost. How much did she pay for the sound system?
- 6. In a box, 72% of the beads are red beads and the rest are blue beads. There are 576 red beads. How many beads are there altogether?
- 7. 42% of a number is 231. What is the number?



- 8. In Jay's sports card collection, 73% of the cards are baseball cards and the rest are football cards. If he has 1,200 cards altogether, how many of them are football cards?
- 9. At a concert hall, 36% of the seats are orchestra seats. If there are 720 orchestra seats, how many seats are not orchestra seats?
- **10.** In a chess club, 55% of the students are girls. If there are 44 girls in the chess club, how many boys are there in the club?
- 11. Cole spent 55% of his money on a coat and the rest on food. If he spent \$187 on the coat, how much did he spend on food?
- 12. In an art museum, 80% of the paintings are impressionist paintings, and the rest are cubist paintings. If there are 380 cubist paintings, how many impressionist paintings are there?
- 13. Molly spent 35% of her money on a hat, 55% of her money on a sweater, and the rest on a pair of shoes. She spent \$154 on the hat.
 - (a) How much did she spend on the shoes?
 - (b) How much did she spend on the sweater?
- 14. After spending 68% of her money on guitar lessons, Mary had \$192 left. How much money did she spend on guitar lessons?



15. Your parents give you a choice regarding your pocket money.Option 1: An increase of 30% this year

Option 1: An increase of 30% this year followed by a 40% increase next year.

Option 2: An increase of 40% this year followed by a 30% increase next year.
Which one will you choose? Why?

16. A shepherd loses 60% of his sheep and then finds 60% of those he lost. What percent of his flock is still missing?

IN A MUTA

Writing Algebraic Expressions

We can use algebra to write expressions when a value is not known. A letter representing an unknown value in an algebraic expression is called a **variable**.

For example: x + 3 means "3 more than a number x."

3y - 2 means "2 less than the product of 3 and y." The variable in the expression x + 3 is x and the

variable in 3y - 2 is y.

Evaluating Algebraic Expressions

If we know the value of the variable in an algebraic expression, we can evaluate the expression by substituting the value for the variable.

For example: Evaluate n + 2, when n = 5.

When n = 5, n + 2 = 5 + 2 = 7.

Simplifying Expressions

We can simplify an expression by adding or subtracting the like terms.

Like terms: 3x and 2x, 5 and 12

Unlike terms: 5 and 3y, 2x and y, y^2 and 2y

Equivalent Expressions

When two expressions have the same values for all possible numbers of all their variables, they are **equivalent expressions**.

We can show whether two expressions are equivalent by simplifying one or both expressions to see if they are identical.

EXERCISE 8. I



BASICPRACTICE

- Write an algebraic expression for each of the following statements.
 - (a) 7 more than y
 - (b) 5 less than x
 - (c) w more than 9.1
 - (d) p less than $\frac{3}{4}$
- Write an algebraic expression for each of the following statements.
 - (a) The product of n and 17
 - (b) The product of 8 and p
 - (e) n to the sixth power
 - (d) The product of q and $\frac{5}{9}$
- Write an algebraic expression for each of the following statements.
 - (a) The quotient when 3x is divided by 5
 - **(b)** The quotient when x is divided by 3
 - (c) The quotient when 2y is divided by 7
 - (d) The quotient when a is divided by 4
- Evaluate the following expressions when a = 3.
 - (a) a + 7
- (c) 12 a

- (f) $\frac{5}{6}a$

FURTHER PRACTICE

- Write an algebraic expression for each of the following statements.
 - (a) 12 plus the quotient of 7x by 10
 - (b) $\frac{3}{5}$ more than the product of x and $\frac{1}{3}$
 - (c) 6 minus the product of m and 0.4
 - (d) 8.2 less than the quotient of n by 8
 - (e) 2 more than x squared

- Evaluate the following expressions when x = 5.
 - (a) 30 2x
 - **(b)** $x + \frac{2}{3}$
 - (c) $8 + \frac{x}{10}$
 - (d) $\frac{4x}{5} 4$
 - (e) $\frac{3}{4}x + \frac{1}{4}$
 - (f) 0.7x + 2.3
 - (g) $\frac{3}{10}x + 0.8$
 - (h) $x^2 5$
- Evaluate the following expressions when $x=\frac{3}{4}$.

 - **(a)** 4x + 3 **(b)** $x \frac{1}{2}$

 - (c) 7x 3 (d) 0.5 + 12x
 - (e) $\frac{2}{3}x \frac{1}{4}$ (f) $\frac{4x}{5}$

 - (g) $\frac{x}{3} + 1$ (h) $\frac{8x}{3} 0.5$

MATH@WORK

Nicole and Grace share a birth date, but

- Grace is 5 years older. (a) If Nicole is 15 years old, how old is
- Grace? (b) If Nicole is p years old, how old is
- (c) If Grace is q years old, how old is Nicole?

- **9.** A fruit seller is selling bags of oranges. In every bag, there are 5 oranges.
 - (a) Complete the table.

Number of Bags	Total Number of Oranges
1	5
2	10
3	
4	
5	

If 15 bags of oranges were sold, how many oranges were there in total?

- (b) If the fruit seller has k oranges, how many bags can be possibly pack?
- (c) How many bags can the fruit seller possibly pack if he has a stock of
 - (i) 400 oranges?
 - (ii) 403 oranges?
- (d) (i) What do you observe from your answers for (c)(i) and (c)(ii)?
 - (ii) What assumption do you think you had made in (b) ealier?
 - (iii) If k is any whole number, how would your answer to (b) change?
- 10. Riya observed that a swinging pendulum made 9 cycles every 15 seconds.
 - (a) If she counted 21 cycles after setting the pendulum in motion, what was the duration of the swing?
 - (b) Write an expression for the number of cycles a pendulum swings in t seconds.
 - (c) Using the same pendulum as Riya, Emma counted the cycles of the pendulum for 55 seconds. How many cycles did she count?

11. Jamie charges \$24 per hour for coaching. Complete the table and answer the question below.

Number of Hours	Amount Jamie Charges in Dollars
1	24
2	Car sun a little
5	
p	

Write an expression describing his earning for coaching p hours.

- (a) How much will he earn if he works for $2\frac{1}{2}$ hours?
- (b) How long will it take Jamie to earn \$108?
- **12.** Ian evaluated the expression $x^3 + 4$ when x = 2 and got the answer 10.
 - (a) Is his answer correct? If not, what is the correct answer?
 - (b) If you think his answer is not correct, explain what he may have done wrongly.

BRAINWORKS

13. "At a restaurant, for every four people who order cheesecake, five people order chocolate cake." Write an equation using the variables *c* and *s* to represent the above statement, where *c* stands for the number of cheesecakes ordered, and *s* stands for the number of chocolate cakes ordered.

Example 23

Use the distributive property to write an equivalent expression for each of the following.

(a)
$$2x + 6$$

(b)
$$15 - 10y$$

Solution

(a) 2x and 6 have a common factor of 2.

$$2x + 6 = 2 \times x + 2 \times 3$$

$$= 2 \times (x + 3)$$

$$= 2(x + 3)$$

$$2x + 6 = 2 \times x + 2 \times 3$$

$$2x + 6 = 2 \times x + 2 \times 3$$

$$2x + 6 = 2 \times x + 2 \times 3$$

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$$2x + 6 = 2 \times x + 2 \times 3$$

(b) 15 and 10y have a common factor of 5.

$$15 - 10y = 5 \times 3 - 5 \times 2y
= 5 \times (3 - 2y)
= 5(3 - 2y)$$

$$15 & 10y
1 & 10y
5 \times 3 \times 5 \times 2y$$

Try It! 23

Use the distributive property to write an equivalent expression for each of the following.

(a)
$$6n + 8$$

(b)
$$9 - 3x$$

(c)
$$8y + 4$$

(d)
$$14y - 21$$

Exercises 1-6,10

EXERCISE 8.2



BASICPRACTICE

1. Simplify the following expressions.

(a)
$$4p + 2p$$

(b)
$$c + 7c$$

(c)
$$3n + 2n + n$$

(d)
$$x + x + x + x + x$$

4. Determine if the following expressions are equivalent.

(a)
$$5p + 8$$
 and $2p + 6 + 3p + 2$

(b)
$$3x + 5 - x + 2$$
 and $7x + 10 - 5x - 3$

(c)
$$8 + 5y - 6 - y$$
 and $8y + 9 - 5y - 7$

(d)
$$5t + 9 - t - 6 + 2t$$
 and $11t + 11 - 7 - 5t$

2. Simplify the following expressions.

(a)
$$10y - 2y$$

(b)
$$12x - x - 4x$$

(c)
$$2m - m + 5m$$

(d)
$$11k - 7k - 4k$$

3. Simplify the following expressions.

(a)
$$3y + 2y + 7$$

(b)
$$15 + y - y$$

(c)
$$7a - 2a - 3$$

(d)
$$13 + 3x - x$$

0

FURTHER PRACTICE

5. Simplify the following expressions.

(a)
$$3y + 7 - y$$

(b)
$$6a + 5 + a - 4$$

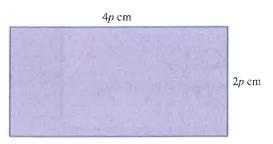
(c)
$$15 + 7q - 10 - q$$

(d)
$$5x + 2 - x + 3$$

- Determine if the following expressions are equivalent.
 - (a) 2x + 14 and 2(x + 7)
 - **(b)** m(5+8) and 12m
 - (c) $5n n + 3 \times 4$ and $2 \times 2n + 6 \times 2$
 - (d) 3y + 15 and $3 \times (y + 4)$
- Use the distributive property to write an equivalent expression for each of the following.
 - (a) $5 \times (2 + r)$
- **(b)** 4(x-2)
- (c) 0.5(x-4) (d) $\frac{2}{3} \times (y+6)$
- Use the distributive property to write an equivalent expression for each of the following.
 - (a) 5x + 5
- **(b)** 3 6y
- (c) 6x + 15
- (d) 10y 5

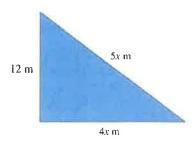


- A formula to find the perimeter of a rectangle is 2(l + w), where l is the length of the rectangle and w is its width. Use this formula to find the perimeter of a rectangle with a length of 12 in. and a width of 4 in.
- 10. A rectangle is 4p cm long and 2p cm wide.



- (a) Express the perimeter of the rectangle in terms of p.
- (b) Find the perimeter when p = 8.

11. The sides of a triangle are 12 m, 5x m, and 4x m long.



- (a) Express the perimeter of the triangle in terms of x.
- (b) Find the perimeter of the triangle when
- (c) The figure above corresponds to the case when x = 4. Sketch the corresponding triangles for the cases when x = 3, $x = \frac{12}{5}$ and x = 1.
- 12. Stephanie is y years old, and Kalama is twice as old as Stephanie. How old will each person be in two years? Will Kalama still be twice as old as Stephanie? Explain how you got your answer.



- 13. Write two expressions that are equivalent to 4x - 8.
- 14. Two formulas that can be used to find the perimeter of a rectangle are 2(l + w) and 2l + 2w. Prove that these two formulas are equivalent expressions.
- 15. The upper part of a flagpole is painted blue, the lower part red, and the middle part white. If you are told the sum of lengths of the blue and the white parts, the sum of lengths of the red and the white parts, and the length of the pole, show how you can find the length of the pole which is painted white.

IN A NUTO

Algebraic Equations

An equation is a statement of equality between two expressions. For example,

$$x + 7 = 11$$
, $1.2y = 24$, $\frac{n}{10} = \frac{3}{5}$.

A solution of an equation is a numerical value which, when substituted for the variable in the equation, makes the equation true.

Example: Determine whether $x = \frac{2}{3}$ is

a solution of the equation

$$9x = 6.$$
$$9 \times \frac{2}{3} = 3 \times 2$$

$$\frac{7}{3} = 6$$

Thus, $x = \frac{2}{3}$ is a solution of 9x = 6.

Methods of Solving Linear Equations in One Variable, x

Add the same number to both sides

For example,
$$x - 3 = 6$$

$$x - 3 + 3 = 6 + 3$$
$$x = 9$$

Subtract the same number from both sides
For example,
$$x + 2 = 14$$

$$x + 2 - 2 = 14 - 2$$

$$x = 12$$

Multiply both sides by the same number For example,

$$\frac{x}{5} = 6$$

$$\frac{x}{5} = 6$$

$$\frac{x}{5} \times 5 = 6 \times 5$$

$$x = 30$$

Divide both sides by the same number For example,

$$\frac{4x = 12}{4x} = \frac{12}{4}$$

$$x = 3$$

Algebraic Inequalities

• Examples of simple inequalities are:

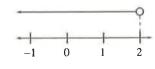
$$x < 2$$
 and $x \ge -5$

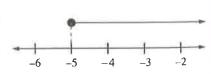
 An inequality has an infinite number of solutions that will make it true. For example, 1, 0, -2.6, and -7 are all solutions for x < 2.

$$-5$$
, -1 , 0, and $4\frac{1}{3}$ are all solutions for $x \ge -5$.

Graphing Inequalities

- It is not possible to write every solution for an inequality, but we can represen all of the solutions by graphing them on a number line.
- The number line diagram on the right shows the solution to x < 2.
- The open circle indicates that 2 is not part of the solution.
- The arrow indicates that every number to the left is a solution to x < 2.
- The number line diagram on the right shows the solution to $x \ge -5$.
- The closed circle indicates that -5 is part of the
- The arrow indicates that evry number to the right is a solution to $x \ge -5$.





Method 2

$$\frac{3}{5}x = 36$$

$$\frac{5}{3} \times \frac{3}{5} x = \frac{5}{3} \times 36$$

Multiply both sides of the equation by the reciprocal of $\frac{3}{6}$.

The coat cost \$60.

x = 60

Ping took $\frac{5}{8}$ as much time as Daniel to solve a puzzle. She took 45 minutes. Write an equation for the problem and solve it to find how much time Daniel took to solve the puzzle.

Exercises 1-5,9,11

EXERCISE 9. I



BASICPRACTICE

Determine whether x = 3 is a solution of each equation.

(a)
$$x + 12 = 15$$

(b)
$$21 - x = 17$$

(c)
$$7 = 11 - x$$

(d)
$$48 = x + 45$$

Determine whether y = 12 is a solution of each equation.

(a)
$$7y = 82$$

(b)
$$72 = 6y$$

(c)
$$\frac{y}{3} = 4$$

(d)
$$8 = \frac{y}{2}$$

Determine whether x = 8 is a solution of each equation.

(a)
$$\frac{35}{4} = x + \frac{3}{4}$$
 (b) $\frac{3}{2}x = 14$

(b)
$$\frac{3}{2}x = 14$$

(c)
$$2.3x = 18.2$$
 (d) $6 = \frac{3}{4}x$

(d)
$$6 = \frac{3}{4}x$$

4. Solve each equation and check your answer.

(a)
$$x + 19 = 52$$

(b)
$$y - 12 = 14$$

(c)
$$61 = n + 37$$

(d)
$$26 = p - 38$$

Solve each equation and check your answer.
(a)
$$8x = 104$$
 (b) $\frac{n}{} = 23$

(b)
$$\frac{n}{7} = 23$$

(c)
$$112 = \frac{x}{4}$$

(d)
$$105 = 15x$$

Determine whether y = 0.6 is a solution of each equation.

(a)
$$5y = 2$$

(b)
$$4.2 = 7v$$

(c)
$$\frac{2}{3}y = 0.4$$

(d)
$$17.4 - y = 18$$

Solve each equation and check your answer.

(a)
$$x + 8 = 11.4$$

(a)
$$x + 8 = 11.4$$
 (b) $n + \frac{3}{10} = \frac{4}{5}$

(c)
$$p + 0.7 = 18.5$$
 (d) $12 = x + \frac{1}{3}$

(d)
$$12 = x + \frac{1}{3}$$

(e)
$$y - \frac{3}{7} = \frac{9}{14}$$
 (f) $\frac{5}{8} = y - 6$

(f)
$$\frac{5}{8} = y - 6$$

(g)
$$2.5 = q - 1.7$$

8. Solve each equation and check your answer.

(a)
$$4n = 11.2$$

(b)
$$\frac{4}{5}x = 8$$

(c)
$$3.5w = 7$$

(d)
$$\frac{y}{5} = \frac{4}{3}$$

(e)
$$3.5 = 0.7n$$

(f)
$$\frac{3}{4}y = 1.5$$

(g)
$$8.5 = \frac{x}{2}$$



Write an equation for each problem and solve the problem.

- **9.** Carlos had some money. After earning \$150 from a part-time job, he now has \$325. How much money did he have initially?
- 10. A blue ribbon is twice as long as a red ribbon. If the blue ribbon is 128 cm, how long is the red ribbon?
- 11. Janice walked to raise money for charity. Peter walked $\frac{2}{3}$ of the distance that Janice walked. If Peter walked 10 miles, how far did Janice walk?
- 12. A big tank holds 3 times as much water as a small tank. If the big tank holds 74 gallons more than the small tank, how much water does the small tank hold?
- 13. Mei goes to her gym which is 4.5 miles from her home. She cycles some distance of the journey and then runs the rest to get there. If Mei runs $\frac{1}{4}$ of the distance that she cycles, how far does she cycle?

- 14. Mr. Wong sold cakes at his shop. The number of cakes he sold in May was 18 more than that sold in June. If the total number of cakes sold is 150, how many cakes did he sell in June?
- 15. In a box, there are five more quarters than dimes. If the total amount of money in the box is \$8.25, how many dimes are there?



- 16. The sum of three consecutive odd numbers is 129. Find the product of the three numbers.
- 17. A textbook contains the following two problems on algebraic simplification. How would you correct the printing errors to make them correct?

(a)
$$8x + 3 = 11x$$

(b)
$$11y - 4 = 7y$$

18. A student solved the equation 4x - 20 = 16 as follows:

$$4x - 20 = 16$$

$$x-20=16\div 4$$

$$x - 20 = 4$$

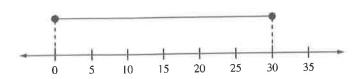
$$x = 20 + 4$$

$$x = 24$$

Where did he go wrong? Show one correct way to solve this equation.

19. Three boys want to buy a present for their friend. The present costs \$120. Peter and Andrea have \$70 altogether. Andrea and Russell have \$80 altogether. Peter and Russell have \$90 altogether. Do they have enough money to buy the present?

(c) $0 \le t \le 30$, where t represents the number of minutes Kini can read.



Why is the inequality not $t \le 30$? What is the reason for specifying $t \ge 0$?

Try It! 22

For each situation, write an inequality, and graph the solution for the inequality on a number line.

- (a) Jeremy has at least \$12 in his wallet.
- (b) Tom has at most 50 stamps in his album.
- (c) Aisha can run no faster than 8 km per hour.

Exercises 1-3,7

EXERCISE 9.2



BASICPRACTICE

- 1. Determine if 3 is a solution of the following inequalities.
 - (a) x > -4
 - **(b)** x < 3.1
 - (c) x > 3
- 2. Determine if -7 is a solution of the following inequalities.
 - (a) x < -6.8
 - **(b)** x > 0
 - (c) $x > -7\frac{2}{3}$
- **3.** Graph the solution for each of the following inequalities on a number line.
 - (a) x > -8
- **(b)** $x \ge 1$
- (c) $x \leq -4$
- (d) x < 9



FURTHERPRACTICE

- 4. Determine if $-\frac{3}{5}$ is a solution of the following inequalities.
 - (a) $x < \frac{3}{5}$
 - **(b)** $x \le -0.999$
 - (c) $x > -\frac{4}{3}$
- **5.** Determine if 0.8 is a solution of the following inequalities.
 - (a) $x > \frac{1}{2}$
 - **(b)** x < -800
 - (c) $x \ge -1.86$
- **6.** Graph the solution for each of the following inequalities on a number line.
 - (a) $x > -3\frac{1}{2}$
 - **(b)** $x \le 1.75$



- 7. Pablo has been mowing lawns to save up money for a play. He earns \$12 per hour and needs at least \$72 for the play ticket. How many hours should he work?
- 8. Theresa saves \$60 each week. She needs to save at least \$2,400 for a trip to Singapore. How many weeks will she need to save?
- 9. Colton has less than \$80. He wants to buy 4 pairs of pants. What price of pants can he afford if all the pants are of the same price? Write an inequality to represent the situation, and then graph the solution.



- 10. Mandy has been working part-time at a fast food outlet to save money for a new outfit for her graduation. She will need to work at least 6 hours to save enough, but she must work less than 20 hours a week. Write an inequality to represent the situation, and then graph the solution.
- 11. Josephine milks the cows each morning. She has never milked less than 5 gallons of milk, however, she always gets less than 10 gallons. Write an inequality to represent this situation, and then graph the solution.

- 12. A club charges its members \$10 an hour for the use of the badminton hall in addition to the \$80 monthly membership fees. Taking *h* hours be the number of hours a member uses the badminton hall in a month,
 - (a) write an expression in terms of h for his total monthly expenses for using the badminton hall in the club.
 - (b) If a member has only \$120 each month for club expenses, write an inequality to represent the situation, and then graph the solution on a number line.
 - (c) How many hours at most can this member use the badminton hall in a month?
- 13. Rachel can bake 6 cakes each day. She has received orders for more than 30 cakes.
 - (a) Write an inequality to represent the number of days, which Rachel needs to clear the cake orders. Then graph the solution.
 - (b) If Rachel must clear the current cakes orders in 8 days or less, how will the inequality and the graph change?
- **14.** What is the least number of presents a group of sixth grade students need to wrap to make more than \$250
 - (a) if they charge \$2.00 per present?
 - (b) if they charge \$1.50 per present?