



Park Hill School District

Building Successful Futures • Each Student • Every Day

High School Calculus Curriculum

Course Description: Designed for the serious and motivated college bound student. It consists of the basic topics in Calculus that includes limits, differentiation, and integration. *Graphing calculators are required. See instructor for recommendations.

Scope and Sequence:

Timeframe	Unit	Instructional Topics
10-12 class periods	Limits	Topic 1: Limits: Graphically and Numerically Topic 2: Limits Analytically (Including Trigonometry) Topic 3: Continuity Topic 4: Intermediate Value Theorem
14-16 class periods	Derivative Rules & Tangent/Normal Lines	Topic 1: Limit Definition of a Derivative Topic 2: Rules of Derivatives 1 (Power Rule, Product Rule, Quotient Rule, and Trig Rules) Topic 3: Rules of Derivatives 2 (Chain Rule and Implicit Differentiation) Topic 4: Tangent Line Topic 5: Average Rate of Change vs. Instantaneous Rate of Change
14-16 class periods	Applications of Derivatives	Topic 1: First Derivatives Applications: Max, Min, Extreme Value Theorem, Increasing/Decreasing Intervals Topic 2: Second Derivatives: Concavity, Points of Inflection, Second Derivative Test

		Topic 3: Graphing Implications Topic 4: Mean Value Theorem
12-14 class periods	Integrals	Topic 1: Riemann Sums Topic 2: Areas Under Curves Topic 3: Rules of Integration - Definite and Indefinite Topic 4: Second Fundamental Theorem Topic 5: Integration as Accumulation Topic 6: Rectilinear Motion
11-14 class periods	Transcendental Functions	Topic 1: Differentiating $y=\ln x$ and integrate $y=1/x$ Topic 2: Logarithmic Differentiation Topic 3: Differentiate and Integrate $y=e^x$ Topic 4: Differentiate and Integrate $y=a^x$ and $y=\log a^x$ Topic 5: Differentiate Inverse Functions Topic 6: Differential Equations and Slope Fields
6-8 class periods	Areas and Volumes	Topic 1: Area Between Two Curves Topic 2: Volume of Solids of Revolution Topic 3: Volume of Solids with Known Cross-Sections

Unit 1: Limits

Subject: Calculus

Grade: 11, 12

Name of Unit: Limits

Length of Unit: 10-12 class periods

Overview of Unit: Students will evaluate limits using graphs, tables, and analytic methods.

Students will use limits to describe the behavior of functions. Students will use limits to discuss continuity.

Priority Standards for unit:

- **EK 1.1A1:** Given a function f , the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \rightarrow c} f(x) = R$.
- **EK 1.1A2:** The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
- **EK 1.1A3:** A limit might not exist for some functions at particular values of x . Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
- **EK 1.1B1:** Numerical and graphical information can be used to estimate limits.
- **EK 1.1C1:** Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
- **EK 1.1C2:** The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.
- **EK 1.1D1:** Asymptotic and unbounded behavior of functions can be explained and described using limits.
- **EK 1.2A1:** A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim_{x \rightarrow c} f(x)$ exists, and $\lim_{x \rightarrow c} f(x) = f(c)$.
- **EK 1.2A2:** Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
- **EK 1.2A3:** Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
- **EK 1.2B1:** Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.

Essential Questions:

1. What is a limit?
2. How do you find limits analytically?
3. How do you use limits to show continuity?

Enduring Understanding/Big Ideas:

1. A limit is a y -value a function approaches as x goes to a given value (also including one-sided limits). This concept is best developed looking at functions graphically and numerically.
2. To find limits analytically, you must include factoring and canceling, rationalization, fraction simplification, and manipulation of certain trig functions.
3. For the x -value of interest, you must show the limit exists, the function is defined, and the two are equal.
 - a. Continuity is essential for using the IVT to prove functions equal certain values within a given interval.

Unit Vocabulary:

Academic Cross-Curricular Words	Content/Domain Specific
Limit Infinity Continuity	Composite Functions Vertical and Horizontal Asymptotes

Resources for Vocabulary Development: Lecture, text, and internet resources.

Topic 1: Limits: Graphically and Numerically

Engaging Experience 1

Title: Understanding Limits

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 1.1A1:** Given a function f , the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \rightarrow c} f(x) = R$.
- **EK 1.1A3:** A limit might not exist for some functions at particular values of x . Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
- **EK 1.1B1:** Numerical and graphical information can be used to estimate limits.

Detailed Description/Instructions: Students will complete problems using teacher provided tables and graphs to understand the concept of limits. Examples a teacher could include to explain this concept could be how a football is caught (if you try to catch the ball with your right and left hands out of alignment, it's less likely) or a real life video (<http://www.calculus-help.com/when-does-a-limit-exist/>).

Bloom's Levels: Analyze

Webb's DOK: 3

Topic 2: Limits Analytically (Including Trigonometry)

Engaging Experience 1

Title: Skills practice--algebra techniques to find limits

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 1.1C1:** Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules.
- **EK 1.1C2:** The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.

Detailed Description/Instructions: Students will practice their algebraic manipulation skills on teacher provided practice problems.

Bloom's Levels: Apply

Webb's DOK: 2

Engaging Experience 2

Title: Skills practice--trig techniques to find limits

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 1.1C2:** The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.

Detailed Description/Instructions: Students will practice their algebraic manipulation skills on trig functions using teacher provided practice problems.

Bloom's Levels: Apply

Webb's DOK: 2

Engaging Experience 3

Title: Skills practice--limit definitions of vertical and horizontal asymptotes

Suggested Length of Time: 2 class periods

Standards Addressed

Priority:

- **EK 1.1A1:** Given a function f , the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \rightarrow c} f(x) = R$.
- **EK 1.1C2:** The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions, or the squeeze theorem.

Detailed Description/Instructions: Students will practice skills on one-sided limits and limit definitions of vertical and horizontal asymptotes using teacher provided practice problems. Example of a question to check for student understanding of the limit definition of an HA (option D sees if they understand the limit definition of a VA if you wanted to tweak the question): For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f . Which one of the following statements must be true?

A. $f(0) = 2$ B. $f(x) \neq 2$ for all $x \geq 0$ C. $f(2)$ is undefined

D. $\lim_{x \rightarrow 2} f(x) = \infty$ E. $\lim_{x \rightarrow \infty} f(x) = 2$

Bloom's Levels: Apply

Webb's DOK: 2

Topic 3: Continuity

Engaging Experience 1

Title: Understanding Continuity at Points

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 1.2A1:** A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim_{x \rightarrow c} f(x)$ exists, and $\lim_{x \rightarrow c} f(x) = f(c)$.
- **EK 1.2A3:** Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.

Detailed Description/Instructions: Students will practice proving continuity or discontinuity at a point and identifying intervals of continuity on teacher provided practice problems.

Bloom's Levels: Evaluate

Webb's DOK: 3

Engaging Experience 2

Title: Understanding Overall Continuity

Suggested Length of Time: 1/2 of a class period

Standards Addressed

Priority:

- **EK 1.2A2:** Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
- **EK 1.2A1:** A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim_{x \rightarrow c} f(x)$ exists, and $\lim_{x \rightarrow c} f(x) = f(c)$.
- **EK 1.2A3:** Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.

Detailed Description/Instructions: Students will practice AP-writing to prove continuity at a point and overall, for the entire domain of a function on teacher provided practice problems.

Bloom's Levels: Evaluate

Webb's DOK: 3

Topic 4: Intermediate Value Theorem

Engaging Experience 1

Title: Applying the Intermediate Value Theorem

Suggested Length of Time: $\frac{1}{2}$ of a class period

Standards Addressed

Priority:

- **EK 1.2B1:** Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.

Detailed Description/Instructions: Students will practice using IVT to prove functions equal certain values on teacher provided example problem.

Here's a good sample problem:

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

 You need to know this means the function is continuous

Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

Bloom's Levels: Evaluate

Webb's DOK: 3

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Graphing from given information sheet. Reference Schoology. This sheet gives students several limits and from those, they must construct a function meeting all criteria.

Rubric for Engaging Scenario: See Schoology

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Limits: Graphically and Numerically	Understanding Limits	Students will complete problems using teacher provided tables and graphs to understand the concept of limits. Examples a teacher could include to explain this concept could be how a football is caught (if you try to catch the ball with your right and left hands out of alignment, it's less likely) or a real life video (http://www.calculus-help.com/when-does-a-limit-exist/).	1 class period
Limits Analytically (Including Trigonometry)	Skills practice-- algebra techniques to find limits	Students will practice their algebraic manipulation skills on teacher provided practice problems.	1 class period
Limits Analytically (Including Trigonometry)	Skills practice-- limit definitions of vertical and horizontal asymptotes	Students will practice skills on one-sided limits and limit definitions of vertical and horizontal asymptotes using teacher provided practice problems. Example of a question to check for student understanding of the limit definition of an HA (option D sees if they understand the limit definition of a VA if you wanted to tweak the question): For $x > 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f . Which one of the following statements must be true? (see diagram)	2 class periods
Continuity	Understanding Continuity at Points	Students will practice proving continuity or discontinuity at a point and identifying intervals of continuity on teacher provided practice problems.	1 class period

Continuity	Understanding Overall Continuity	Students will practice writing to prove continuity at a point and overall, for the entire domain of a function on teacher provided practice problems.	½ of a class period
Intermediate Value Theorem	Applying the Intermediate Value Theorem	Students will practice writing using IVT to prove functions equal certain values on teacher provided example problem.	½ of a class period

Unit 2: Derivative Rules & Tangent/Normal Lines

Subject: Calculus

Grade: 11, 12

Name of Unit: Derivative Rules/Tangential Line Normal

Length of Unit: 14-16 class periods

Overview of Unit: Students will find the derivative using the definition of the derivatives. Students will apply derivative rules to find the slope of a tangent line. Students will apply the chain rule to implicit differentiation. Students will apply first and second derivatives to motion along a line.

Priority Standards for unit:

- **EK 2.1A1:** The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.
- **EK 2.1A2:** The instantaneous rate of change of a function at a point can be expressed by $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.
- **EK 2.1A3:** The derivative of f is the function whose value at x is $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$ provided this limit exists.
- **EK 2.1A4:** For $y = f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, and y' .
- **EK 2.1A5:** The derivative can be represented graphically, numerically, analytically, and verbally.
- **EK 2.1B1:** The derivative at a point can be estimated from information given in tables or graphs.
- **EK 2.1C1:** Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- **EK 2.1C2:** Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- **EK 2.1C3:** Sums, differences, products, and quotients of functions can be differentiated using derivative rules.
- **EK 2.1C4:** The chain rule provides a way to differentiate composite functions.
- **EK 2.1C5:** The chain rule is the basis for implicit differentiation.
- **EK 2.1D1:** Differentiating f' produces the second derivative f'' , provided the derivative of f' exists; repeating this process produces higher order derivatives of f .

- **EK 2.1D2:** Higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$, and y'' . Higher order derivatives can be denoted $\frac{d^ny}{dx^n}$ or $f^n(x)$.
- **EK 2.2B1:** A continuous function may fail to be differentiable at a point in its domain.
- **EK 2.2B2:** If a function is differentiable at a point, then it is continuous at that point.
- **EK 2.3A1:** The unit for $f'(x)$ is the unit for f divided by the unit for x .
- **EK 2.3A2:** The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable.
- **EK 2.3B1:** The derivative at a point is the slope of the line tangent to a graph at that point on the graph.
- **EK 2.3C1:** The derivative can be used to solve rectilinear motion problems involving position, speed, velocity and acceleration.
- **EK 2.3C2:** The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change by relating it to other quantities whose rates of change are known.

Essential Questions:

1. How do you find a derivative using the limit definition?
2. How do you calculate derivatives with the rules?
3. How do you find a tangent line to a curve?

Enduring Understanding/Big Ideas:

1. Find the limit as h goes to 0 of the difference quotient.
2. You must include derivative of a constant, power rule, product rule, quotient rule, chain rule, implicit differentiation, and all trig derivatives.
3. First find the derivative at a point to calculate the slope. Then write the equation of the line in point slope form.

Unit Vocabulary:

Academic Cross-Curricular Words	Content/Domain Specific
Tangent lines Normal lines Position Velocity Acceleration	Difference Quotient Derivative Chain Rule Implicit Differentiation

Resources for Vocabulary Development: Lecture, text, and internet resources.

Topic 1: Limit Definition of a Derivative

Engaging Experience 1

Title: Limit Definition of Derivative--the basics

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.1A1:** The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.
- **EK 2.1A2:** The instantaneous rate of change of a function at a point can be expressed by $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.
- **EK 2.1A3:** The derivative of f is the function whose value at x is $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$ provided this limit exists.
- **EK 2.1A4:** For $y = f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, and y' .
- **EK 2.1D2:** Higher order derivatives are represented with a variety of notations. For $y = f(x)$, notations for the second derivative include $\frac{d^2y}{dx^2}$, $f''(x)$, and y'' . Higher order derivatives can be denoted $\frac{d^ny}{dx^n}$ or $f^n(x)$.
- **EK 2.1A5:** The derivative can be represented graphically, numerically, analytically, and verbally.
- **EK 2.1B1:** The derivative at a point can be estimated from information given in tables or graphs.

Detailed Description/Instructions: Students will practice applying the limit definition of the derivative to teacher created problems. Make sure to emphasize derivatives find instantaneous rates of change versus the difference quotient finds average rates of change; students have to apply this idea to graphs and tables.

Bloom's Levels: Analyze

Webb's DOK: 3

Engaging Experience 2

Title: Continuity and Differentiability Relationships

Suggested Length of Time: $\frac{1}{2}$ of a class period

Standards Addressed

Priority:

- **EK 2.2B1:** A continuous function may fail to be differentiable at a point in its domain.
- **EK 2.2B2:** If a function is differentiable at a point, then it is continuous at that point.

Detailed Description/Instructions: Students will practice applying continuity and differentiability relationships to teacher created problems. Make sure to clearly lay out which logic directions work.

Bloom's Levels: Analyze

Webb's DOK: 3

Topic 2: Rules of Derivatives 1 (Power Rule, Product Rule, Quotient Rule, and Trig Rules)

Engaging Experience 1

Title: Power Rule and Trig Derivatives

Suggested Length of Time: ½ of a class period

Standards Addressed

Priority:

- **EK 2.1C2:** Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- **EK 2.1C3:** Sums, differences, products, and quotients of functions can be differentiated using derivative rules.

Detailed Description/Instructions: Students will practice applying the derivative rules on teacher created problems.

Bloom's Levels: Apply

Webb's DOK: 2

Engaging Experience 2

Title: Product and Quotient Rules

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.1C3:** Sums, differences, products, and quotients of functions can be differentiated using derivative rules.

Detailed Description/Instructions: Students will practice applying the derivative rules on teacher created problems.

Bloom's Levels: Apply

Webb's DOK: 2

Engaging Experience 3

Title: Limit Definition of Derivative

Suggested Length of Time: ½ of a class period

Standards Addressed

Priority:

- **EK 2.1A3:** The derivative of f is the function whose value at x is $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$ provided this limit exists.

Detailed Description/Instructions: Students will practice teacher created problems making them use the limit definition of derivative backwards, i.e. they are given a limit, have to recognize it as a derivative, find the derivative, and plug in the appropriate x.

Here's an example: $\lim_{h \rightarrow 0} \frac{\sqrt[3]{8+h} - 2}{h}$; this is really $f'(8)$ for $f(x) = \sqrt[3]{x}$

Bloom's Levels: Analyze

Webb's DOK: 3

Topic 3: Rules of Derivatives 2 (Chain Rule and Implicit Differentiation)

Engaging Experience 1

Title: Chain Rule

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.1C4:** The chain rule provides a way to differentiate composite functions.

Detailed Description/Instructions: Students will practice applying the chain rule on teacher created problems with emphasis on the different outer and inner components with the students.

Bloom's Levels: Apply

Webb's DOK: 2

Engaging Experience 2

Title: Implicit Differentiation

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.1C5:** The chain rule is the basis for implicit differentiation.

Detailed Description/Instructions: Students will practice applying the chain rule on teacher created problems. Make sure you cover both first and second derivatives with implicit. Make sure to emphasize implicit differentiation applies to variables other than just x and y .

Bloom's Levels: Apply

Webb's DOK: 2

Engaging Experience 3

Title: Related Rates

Suggested Length of Time: 2 class periods

Standards Addressed

Priority:

- **EK 2.3C2:** The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change by relating it to other quantities whose rates of change are known.

Detailed Description/Instructions: Students will practice solving related rates on teacher created problems with emphasis that they only substitute unchanging quantities before differentiating and help them with strategies to identify an equation to use.

Bloom's Levels: Apply

Webb's DOK: 4

Topic 4: Tangent Line

Engaging Experience 1

Title: Writing Tangent Lines

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.3B1:** The derivative at a point is the slope of the line tangent to a graph at that point on the graph.

Detailed Description/Instructions: Students will practice writing tangent lines on teacher created problems. Make sure students know the difference between finding slope of a tangent line versus the actual tangent line itself. Also, make sure students know to leave lines in point slope form unless specifically asked to change it. Lastly, make sure you teach students how these tangent lines can approximate the curve near the point of tangency.

Bloom's Levels: Apply

Webb's DOK: 2

Topic 5: Average Rate of Change vs. Instantaneous Rate of Change

Engaging Experience 1

Title: Rates of Change--Average and Instantaneous

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.1A1:** The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.
- **EK 2.1A2:** The instantaneous rate of change of a function at a point can be expressed by $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.

Detailed Description/Instructions: Students will practice distinguishing between average and instantaneous rates of change on teacher created problems. Make sure to focus on graphical application examples.

Bloom's Levels: Analyze

Webb's DOK: 3

Engaging Experience 2

Title: Introduction to Rectilinear Motion

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.3C1:** The derivative can be used to solve rectilinear motion problems involving position, speed, velocity and acceleration.
- **EK 2.1A1:** The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.
- **EK 2.1A2:** The instantaneous rate of change of a function at a point can be expressed by $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$, provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.

Detailed Description/Instructions: Students will practice rectilinear motion from teacher created problems. Make sure to cover average velocity versus velocity at a point (same for acceleration), graphical applications, and speeding up/slowing down.

Bloom's Levels: Evaluate

Webb's DOK: 3

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Students will complete the Water Line activity from Desmos.com

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Limit Definition of a Derivative	Limit Definition of Derivative-- the basics	Students will practice applying the limit definition of the derivative to teacher created problems. Make sure to emphasize derivatives find instantaneous rates of change versus the difference quotient finds average rates of change; students have to apply this idea to graphs and tables.	1 class period
Limit Definition of a Derivative	Continuity and Differentiability Relationships	Students will practice applying continuity and differentiability relationships to teacher created problems. Make sure to clearly lay out which logic directions work.	½ of a class period
Rules of Derivatives 1 (Power Rule, Product Rule, Quotient Rule, and Trig Rules)	Power Rule and Trig Derivatives	Students will practice applying the derivative rules on teacher created problems.	½ of a class period
Rules of Derivatives 1 (Power Rule, Product Rule, Quotient Rule, and Trig Rules)	Product and Quotient Rules	Students will practice applying the derivative rules on teacher created problems.	1 class period
Rules of Derivatives 1 (Power Rule, Product Rule,	Limit Definition of Derivative	Students will practice teacher created problems making them use the limit definition of derivative backwards, i.e. they are given a limit, have to recognize it as a derivative, find the	½ of a class period

Quotient Rule, and Trig Rules)		derivative, and plug in the appropriate x.	
Rules of Derivatives 2 (Chain Rule and Implicit Differentiation)	Chain Rule	Students will practice applying the chain rule on teacher created problems with emphasis on the different outer and inner components with the students.	1 class period
Rules of Derivatives 2 (Chain Rule and Implicit Differentiation)	Implicit Differentiation	Students will practice applying the chain rule on teacher created problems. Make sure you cover both first and second derivatives with implicit. Make sure to emphasize implicit differentiation applies to variables other than just x and y.	1 class period
Rules of Derivatives 2 (Chain Rule and Implicit Differentiation)	Related Rates	Students will practice solving related rates on teacher created problems with emphasis that they only substitute unchanging quantities before differentiating and help them with strategies to identify an equation to use.	2 class periods
Tangent Line	Writing Tangent Lines	Students will practice writing tangent lines on teacher created problems. Make sure students know the difference between finding slope of a tangent line versus the actual tangent line itself. Also, make sure students know to leave lines in point slope form unless specifically asked to change it. Lastly, make sure you teach students how these tangent lines can approximate the curve near the point of tangency.	1 class period
Average Rate of Change vs. Instantaneous Rate of Change	Rates of Change-- Average and Instantaneous	Students will practice distinguishing between average and instantaneous rates of change on teacher created problems. Make sure to focus on graphical application examples.	1 class period

Average Rate of Change vs. Instantaneous Rate of Change	Introduction to Rectilinear Motion	Students will practice rectilinear motion from teacher created problems. Make sure to cover average velocity versus velocity at a point (same for acceleration), graphical applications, and speeding up/slowing down.	1 class period
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Unit 3: Applications of Derivatives

Subject: Calculus

Grade: 11, 12

Name of Unit: Applications of Derivatives

Length of Unit: 14-16 class periods

Overview of Unit: Students will use the first and second derivatives to describe the behavior of a function and graph it. Students will apply the derivative of a function to solve related rates problems and max/min problems. Students will apply the Mean Value Theorem, Intermediate Value Theorem, and Extreme Value Theorem. It is recommended to test students after curve sketching in order to break up the unit.

Priority Standards for unit:

- **EK 1.1D2:** Relative magnitudes of functions and their rates of change can be compared using limits.
- **EK 1.2B1:** Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.
- **EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
- **EK 2.2A2:** Key features of functions and their derivatives can be identified and are related to their graphical, numerical, and analytical representations.
- **EK 2.2A3:** Key features of the graphs of f , f' and f'' are related to one another.
- **EK 2.3B2:** The tangent line is the graph of a locally linear approximation of the function near the point of tangency.
- **EK 2.3C3:** The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
- **EK 2.3D1:** The derivative can be used to express information about rates of change in applied contexts.
- **EK 2.4A1:** If a function f is continuous over the interval $[a, b]$ and differentiable over the interval (a, b) , the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.

Supporting Standards for unit:

- ISTE-COMPUTATIONAL THINKER.5.C - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

Essential Questions:

1. How do you use the first and second derivatives to determine the behavior of a function?
2. How do you use derivatives in optimization problems?

Enduring Understanding/Big Ideas:

1. You must include that the first derivative tells you key information about relative maxs/mins and increasing/decreasing intervals. The second derivative tells you key information about points of inflection and concavity intervals.
2. You calculate the derivative, find critical numbers, and test critical numbers/endpoints in the original function to determine absolute maxs/mins.

Unit Vocabulary:

Academic Cross-Curricular Words	Content/Domain Specific
Concavity Optimization	L'Hopital's Rule Mean Value Theorem Intermediate Value Theorem

Resources for Vocabulary Development: Lecture, text, and internet resources.

Topic 1: First Derivatives Applications: Max, Min, Extreme Value Theorem, Increasing/Decreasing Intervals

Engaging Experience 1

Title: Absolute Extrema and Extreme Value Theorem

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 1.2B1:** Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.
- **EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.

Detailed Description/Instructions: Students will practice finding absolute extrema and justifying absolute extrema exist on teacher created problems with emphasis on checking endpoints and checking in the original function.

Bloom's Levels: Apply

Webb's DOK: 2

Engaging Experience 2

Title: Increasing/Decreasing and Relative Extrema

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.

Detailed Description/Instructions: Students will practice finding intervals of increasing/decreasing and relative extrema on teacher created problems. Make sure to include justifications of inc/dec/max/min. Really stress when to plug into the derivative and when to plug into the original.

Bloom's Levels: Apply

Webb's DOK: 2

Engaging Experience 3

Title: Advanced Horizontal Asymptote Limits

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 1.1D2:** Relative magnitudes of functions and their rates of change can be compared using limits.

Detailed Description/Instructions: Students will find limits at infinity of non-rational functions on teacher created problems. Make sure to emphasize how non-rational functions often have two horizontal asymptotes.

Bloom's Levels: Analyze

Webb's DOK: 3

Engaging Experience 4

Title: Optimization

Suggested Length of Time: 2 class periods

Standards Addressed

Priority:

- **EK 2.3C3:** The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
- **EK 2.3D1:** The derivative can be used to express information about rates of change in applied contexts.

Detailed Description/Instructions: Students will practice solving optimization problems on teacher created problems. Make sure students actually verify they have a max/min, checking endpoints where applicable.

Bloom's Levels: Evaluate

Webb's DOK: 3

Topic 2: Second Derivatives: Concavity, Points of Inflection, Second Derivative Test

Engaging Experience 1

Title: Concavity Intervals and Points of Inflection

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.

Detailed Description/Instructions: Students will practice finding intervals of concavity and points of inflection on teacher created problems. Make sure to include justifications of concavity/points of inflection.

Bloom's Levels: Apply

Webb's DOK: 2

Engaging Experience 2

Title: Second Derivative Test

Suggested Length of Time: $\frac{1}{2}$ of a class period

Standards Addressed

Priority:

- **EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.

Detailed Description/Instructions: Students will practice verifying extrema on teacher created problems. Make sure students realize the 2nd Derivative Test is a test to verify extrema using the 2nd derivative, NOT sign chart info. about concave up/down or inflection points.

Bloom's Levels: Apply

Webb's DOK: 2

Engaging Experience 3

Title: Linearization

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.3B2:** The tangent line is the graph of a locally linear approximation of the function near the point of tangency.

Detailed Description/Instructions: Students will practice solving linearization problems on teacher created problems. Make sure you cover whether the approximation is an over- or under-estimate of the true function value based on concavity.

Here are examples:

2010 AB, Form B #2

The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.

(d) Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

2009 AB, #5

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \leq x \leq 13$.

(d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \leq x \leq 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \leq 4$. Use the secant line for the graph of f on $5 \leq x \leq 8$ to show that $f(7) \geq \frac{4}{3}$.

Bloom's Levels: Evaluate

Webb's DOK: 3

Topic 3: Graphing Implications

Engaging Experience 1

Title: Connections Amongst f , f' , and f'' graphs

Suggested Length of Time: 2 class periods

Standards Addressed

Priority:

- **EK 2.2A1:** First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
- **EK 2.2A2:** Key features of functions and their derivatives can be identified and are related to their graphical, numerical, and analytical representations.
- **EK 2.2A3:** Key features of the graphs of f , f' and f'' are related to one another.

Detailed Description/Instructions: Students will practice graphical interpretation and curve sketching on teacher created problems. At first, you can begin with basic curve sketching (given a function, finding all asymptotes, intercepts, maxs/mins/inflection points, intervals of inc/dec/concave up/concave down), it's really important for students to match up graphs moving in all directions, for example, given a graph of f' and finding f .

Bloom's Levels: Analyze

Webb's DOK: 3

Topic 4: Mean Value Theorem

Engaging Experience 1

Title: Mean Value Theorem

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 1.2B1:** Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem
- **EK 2.4A1:** If a function f is continuous over the interval $[a, b]$ and differentiable over the interval (a, b) , the Mean Value Theorem guarantees a point within that open interval where the instantaneous rate of change equals the average rate of change over the interval.
- **EK 2.3D1:** The derivative can be used to express information about rates of change in applied contexts.
- **ISTE-COMPUTATIONAL THINKER.5.C** - break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

Detailed Description/Instructions: Students will practice applying the MVT on teacher created problems.

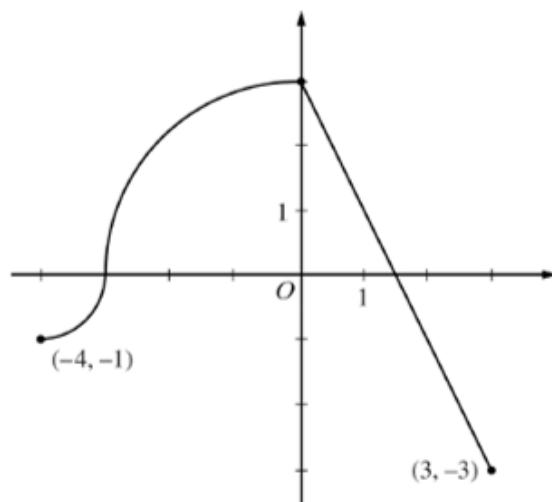
A real-life example is: A car leaves a toll booth on I-70. Fifteen minutes later and 20 miles down the road, it's clocked at 70 mph by a state trooper. Prove that the driver of the car exceeded the 70-mph speed limit at some time in between leaving the toll booth and being clocked by the officer.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(b) Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.

and



Graph of f

The continuous function f is defined on the interval $-4 \leq x \leq 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t) dt$.

- (d) Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c , $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

Bloom's Levels: Evaluate

Webb's DOK: 3

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Have students explore the graphical relationship between a function and its derivative using Desmos.com graphing example Calculus: Derivatives. Resources are available in Schoology.

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
First Derivatives Applications: Max, Min, Extreme Value Theorem, Increasing/Decreasing Intervals	Absolute Extrema and Extreme Value Theorem	Students will practice finding absolute extrema and justifying absolute extrema exist on teacher created problems with emphasis on checking endpoints and checking in the original function.	1 class period
First Derivatives Applications: Max, Min, Extreme Value Theorem, Increasing/Decreasing Intervals	Increasing/Decreasing and Relative Extrema	Students will practice finding intervals of increasing/decreasing and relative extrema on teacher created problems. Make sure to include justifications of inc/dec/max/min. Really stress when to plug into the derivative and when to plug into the original.	1 class period
First Derivatives Applications: Max, Min, Extreme Value Theorem, Increasing/Decreasing Intervals	Advanced Horizontal Asymptote Limits	Students will practice finding limits at infinity of non-rational functions on teacher created problems. Make sure to emphasize how non-rational functions often have two horizontal asymptotes.	1 class period
First Derivatives Applications: Max, Min, Extreme Value Theorem, Increasing/Decreasing Intervals	Optimization	Students will practice solving optimization problems on teacher created problems. Make sure students actually verify they have a max/min, checking endpoints where applicable.	2 class periods
Second Derivatives: Concavity, Points of Inflection, Second Derivative Test	Concavity Intervals and Points of Inflection	Students will practice finding intervals of concavity and points of inflection on teacher created problems. Make sure to include justifications of concavity/points of inflection.	1 class period

Second Derivatives: Concavity, Points of Inflection, Second Derivative Test	Second Derivative Test	Students will practice verifying extrema on teacher created problems. Make sure students realize the 2nd Derivative Test is a test to verify extrema using the 2nd derivative, NOT sign chart info. about concave up/down or inflection points.	1/2 of a class period
Second Derivatives: Concavity, Points of Inflection, Second Derivative Test	Linearization	Students will practice solving linearization problems on teacher created problems. Make sure you cover whether the approximation is an over- or under-estimate of the true function value based on concavity.	1 class period
Graphing Implications	Connections Amongst f , f' , and f'' graphs	Students will practice graphical interpretation and curve sketching on teacher created problems. At first, you can begin with basic curve sketching (given a function, finding all asymptotes, intercepts, maxs/mins/inflection points, intervals of inc/dec/concave up/concave down), it's really important for students to match up graphs moving in all directions, for example, given a graph of f' and finding f .	2 class periods
Mean Value Theorem	Mean Value Theorem	Students will practice applying the MVT on teacher created problems.	1 class period

Unit 4: Integrals

Subject: Calculus

Grade: 11, 12

Name of Unit: Integrals

Length of Unit: 12-14 class periods

Overview of Unit: Students will find the area under a curve using Riemann Sums, Trapezoidal Sums, and the Fundamental Theorem of Calculus. Students will find antiderivatives. Students will define function in terms of the definite integral. Students will apply the Second Fundamental Theorem of Calculus including chain rule applications.

Priority Standards for unit:

- **EK 2.3E1:** Solutions to differential equations are functions or families of functions.
- **EK 2.3E2:** Derivatives can be used to verify that a function is a solution to a given differential equation.
- **EK 3.1A1:** An antiderivative of a function f is a function g whose derivative is f .
- **EK 3.1A2:** Differentiation rules provide the foundation for finding antiderivatives.
- **EK 3.2A1:** A Riemann sum, which requires a partition of an interval I , is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
- **EK 3.2A2:** The definite integral of a continuous function f over the interval $[a, b]$, denoted by $\int_a^b f(x)dx$, is the limit of Riemann sums as the widths of the subintervals approach 0. That is,

$$\int_a^b f(x)dx = \lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n f(x_i^*) \Delta x_i,$$

where x_i^* is a value in the i th subinterval, Δx_i is the width of the i th subinterval, n is the number of subintervals, and $\max \Delta x_i$ is the width of the largest subinterval. Another form of the definition is

$$\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x_i,$$

where $\Delta x_i = \frac{b-a}{n}$ and x_i^* is a value in the i th subinterval.

- **EK 3.2A3:** The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a related Riemann sum can be written as a definite integral.

- **EK 3.2B1:** Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally.
- **EK 3.2B2:** Definite integrals can be approximated using a left Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.
- **EK 3.2C1:** In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
- **EK 3.2C2:** Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
- **EK 3.2C3:** The definition of the integral may be extended to functions with removable or jump discontinuities.
- **EK 3.3A1:** The definite integral can be used to define new functions; for example,

$$f(x) = \int_0^x e^{-t^2} dt .$$
- **EK 3.3A2:** If f is a continuous function on the interval $[a,b]$, then $\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$ where x is between a and b .
- **EK 3.3A3:** Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x) = \int_a^x f(t) dt$.
- **EK 3.3B1:** The function defined by $F(x) = \int_a^x f(t) dt$ is an antiderivative of f .
- **EK 3.3B2:** If f is continuous on the interval $[a,b]$ and F is an antiderivative of f , then

$$\int_a^b f(x) dx = F(b) - F(a).$$
- **EK 3.3B3:** The notation $\int f(x) dx = F(x) + C$ means that $F'(x) = f(x)$, and $\int f(x) dx$ is called an indefinite integral of the function f .
- **EK 3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables.
- **EK 3.4A1:** A function defined as an integral represents an accumulation of a rate of change.
- **EK 3.4A2:** The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
- **EK 3.4A3:** The limit of an approximating Riemann sum can be interpreted as a definite integral.
- **EK 3.4B1:** The average value of a function f over an interval $[a,b]$ is $\frac{1}{b-a} \int_a^b f(x) dx$.

- **EK 3.4C1:** For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.
- **EK 3.4D1:** Areas of certain regions in the plane can be calculated with definite integrals
- **EK 3.4E1:** The definite integral can be used to express information about accumulation and net change in many applied contexts.
- **EK 3.5A4:** The function F defined by $F(x) = c + \int_a^x f(t)dt$ is a general solution to the differential equation $\frac{dy}{dx} = f(x)$, and $F(x) = y_0 + \int_a^x f(t)dt$ is a particular solution to the differential equation $\frac{dy}{dx} = f(x)$ satisfying $F(a) = y_0$.

Essential Questions:

1. What is an antiderivative?
2. How do you find area under a curve?
3. How do you evaluate integrals, both definite and indefinite?
4. How do you take the derivative of an integrally-defined function?
5. How do you use a definite integral as an accumulation function?
6. How do you use calculus to describe a particle's rectilinear motion?

Enduring Understanding/Big Ideas:

1. An antiderivative is a family of curves that have the same derivative.
2. Methods that are relevant are Riemann sums, (right, left, and midpoint), trapezoidal rule, and the definite integral.
3. You should use the Power Rule, Trig Rules, and substitution methods. If it is a definite integral use the First Fundamental Theorem of Calculus.
4. Use the Second Fundamental Theorem of Calculus.
5. If you integrate f' , you accumulate information about f . This shows up in application examples most frequently, i.e. given a rate, find an accumulated amount.
6. You must make sure to include the difference between total distance traveled versus displacement. Also, make sure students understand the relationships between displacement, velocity, and acceleration.

Unit Vocabulary:

Academic Cross-Curricular Words	Content/Domain Specific
Partition Subinterval Accumulation	Riemann Sum Definite Interval Antiderivative Trapezoidal Sum Rectilinear

Resources for Vocabulary Development: Lecture, text, and internet resources.

Topic 1: Riemann Sums

Engaging Experience 1

Title: Riemann Sums

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 3.2A1:** A Riemann sum, which requires a partition of an interval I , is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
- **EK 3.2A3:** The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a related Riemann sum can be written as a definite integral.
- **EK 3.2B1:** Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally.
- **EK 3.2B2:** Definite integrals can be approximated using a left Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.

Detailed Description/Instructions: Students will practice finding Riemann Sums (including lower, upper, left, right, midpoint, and trapezoidal) on teacher created problems. Make sure to include: tables of data, intervals of equal and unequal widths, and whether the approximations are over-/under-estimates of the actual integral/area under the curve. A hw example can be found in Schoology.

Bloom's Levels: 4

Webb's DOK: Evaluate

Engaging Experience 2

Title: Integrals as Limits of Riemann Sums

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 3.2A2:** The definite integral of a continuous function f over the interval $[a, b]$, denoted by $\int_a^b f(x)dx$, is the limit of Riemann sums as the widths of the subintervals approach 0. That is,

$$\int_a^b f(x)dx = \lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n f(x_i^*) \Delta x_i,$$

where x_i^* is a value in the i th subinterval, Δx_i is the width of the i th subinterval, n is the number of subintervals, and $\max \Delta x_i$ is the width of the largest subinterval. Another form of the definition is

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x_i,$$

where $\Delta x_i = \frac{b-a}{n}$ and x_i^* is a value in the i th subinterval.

- **EK 3.2A3:** The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a related Riemann sum can be written as a definite integral.

Detailed Description/Instructions: Students will practice writing integrals as limits of Riemann Sums and vice versa on teacher created problems. Here are a couple of examples:

Write $\int_{-1}^1 x^3 dx$ as a limit of a related Riemann Sum.

Which of the following is a definite integral for the related Riemann Sum of

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(3 + \frac{4k}{n} \right)^2 \frac{4}{n} ?$$

- $\int_4^8 x^2 dx$
- $4 \int_3^4 x^2 dx$
- $4 \int_3^3 x^2 dx$
- $\int_3^3 x^2 dx$

Bloom's Levels: Analyze

Webb's DOK: 4

Topic 2: Areas Under Curves

Engaging Experience 1

Title: Using Integration to Find Exact Areas Under Curves

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 3.2C1:** In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area.
- **EK 3.2C2:** Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
- **EK 3.2C3:** The definition of the integral may be extended to functions with removable or jump discontinuities.
- **EK 3.3A3:** Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x) = \int_a^x f(t)dt$.
- **EK 3.4D1:** Areas of certain regions in the plane can be calculated with definite integrals.

Detailed Description/Instructions: Students will practice evaluating integrals by finding signed areas under curves with known geometric shapes on teacher created problems. Make sure to include integral properties here, discussing how students can split apart integrals.

Bloom's Levels: Apply

Webb's DOK: 2

Topic 3: Rules of Integration - Definite and Indefinite

Engaging Experience 1

Title: Indefinite Integration/Basic Differential Equations

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.3E1:** Solutions to differential equations are functions or families of functions.
- **EK 2.3E2:** Derivatives can be used to verify that a function is a solution to a given differential equation.
- **EK 3.1A1:** An antiderivative of a function f is a function g whose derivative is f .
- **EK 3.1A2:** Differentiation rules provide the foundation for finding antiderivatives.
- **EK 3.3B3:** The notation $\int f(x)dx = F(x) + C$ means that $F'(x) = f(x)$, and $\int f(x)dx$ is called an indefinite integral of the function f .
- **EK 3.5A4:** The function F defined by $F(x) = c + \int_a^x f(t)dt$ is a general solution to the differential equation $\frac{dy}{dx} = f(x)$, and $F(x) = y_0 + \int_a^x f(t)dt$ is a particular solution to the differential equation $\frac{dy}{dx} = f(x)$ satisfying $F(a) = y_0$.

Detailed Description/Instructions: Students will practice indefinite integration and solving basic differential equations (given $f'(x)$ and asked to find $f(x)$) on teacher created problems.

Bloom's Levels: Analyze

Webb's DOK: 3

Engaging Experience 2

Title: Definite Integration

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 3.3B2:** If f is continuous on the interval $[a,b]$ and F is an antiderivative of f , then $\int_a^b f(x)dx = F(b) - F(a)$.

Supporting:

- **EK 3.2C2:** Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.

Detailed Description/Instructions: Students will practice definite integration on teacher created problems. Make sure to include how to integrate when the integrand has absolute values.

Bloom's Levels: Analyze
Webb's DOK: 4

Engaging Experience 3

Title: U-substitution

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables.

Detailed Description/Instructions: Students will practice integration with u-substitution on teacher created problems. Make sure to cover both definite and indefinite integration.

Bloom's Levels: Analyze

Webb's DOK: 3

Topic 4: Second Fundamental Theorem

Engaging Experience 1

Title: Second Fundamental Theorem of Calculus

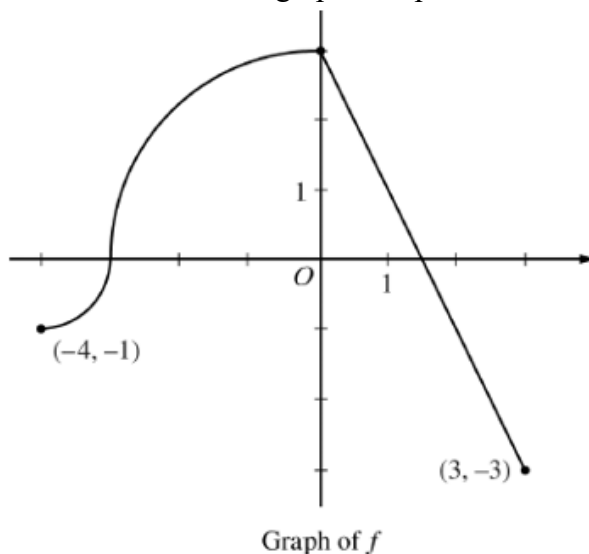
Suggested Length of Time: $\frac{1}{2}$ of a class period

Standards Addressed

Priority:

- **EK 3.3A2:** If f is a continuous function on the interval $[a,b]$, then $\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$ where x is between a and b .
- **EK 3.3B1:** The function defined by $F(x) = \int_a^x f(t) dt$ is an antiderivative of f .

Detailed Description/Instructions: Students will practice differentiating definite integrals with the 2nd Fundamental Theorem of Calculus on teacher created problems. Make sure to include questions that require students to do this with graph interpretation, such as:



The continuous function f is defined on the interval $-4 \leq x \leq 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t) dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

Bloom's Levels: Apply

Webb's DOK: 3

Topic 5: Integration as Accumulation

Engaging Experience 1

Title: Average Value of a Function

Suggested Length of Time: $\frac{1}{2}$ of a class period

Standards Addressed

Priority:

- **EK 3.4B1:** The average value of a function f over an interval $[a, b]$ is $\frac{1}{b-a}$

$$\int_a^b f(x) dx.$$

Detailed Description/Instructions: Students will practice finding average value of a function on teacher created problems. Make sure to do this in an applied sense, so students note the differences between average value (new) and average rate of change (old). Here are some examples:

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

Train A runs back and forth on an east-west section of railroad track. Train A 's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

- (a) Find the average acceleration of train A over the interval $2 \leq t \leq 8$.

For $0 \leq t \leq 6$, a particle is moving along the x -axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin(e^{t/4}) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos(e^{t/4})$ and $x(0) = 2$.

- (b) Find the average velocity of the particle for the time period $0 \leq t \leq 6$.

Grass clippings are placed in a bin, where they decompose. For $0 \leq t \leq 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

- Find the average rate of change of $A(t)$ over the interval $0 \leq t \leq 30$. Indicate units of measure.
- Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.
- Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \leq t \leq 30$.
- For $t > 30$, $L(t)$, the linear approximation to A at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

Bloom's Levels: Analyze

Webb's DOK: 3

Engaging Experience 2

Title: Integrally Defined Functions/Integrals as Accumulators

Suggested Length of Time: 2 class periods

Standards Addressed

Priority:

- EK 3.3A1:** The definite integral can be used to define new functions; for example, $f(x) = \int_0^x e^{-t^2} dt$.
- EK 3.3A3:** Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x) = \int_a^x f(t)dt$.
- EK 3.4A1:** A function defined as an integral represents an accumulation of a rate of change.
- EK 3.4A2:** The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
- EK 3.4E1:** The definite integral can be used to express information about accumulation and net change in many applied contexts.
- EK 3.5A4:** The function F defined by $F(x) = c + \int_a^x f(t)dt$ is a general solution to the differential equation $\frac{dy}{dx} = f(x)$, and $F(x) = y_0 + \int_a^x f(t)dt$ is a particular solution to the differential equation $\frac{dy}{dx} = f(x)$ satisfying $F(a) = y_0$.

Detailed Description/Instructions: Students will practice applied integration on teacher created problems. Make sure to do these problems in the context of graphical application too. Here are several examples:

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

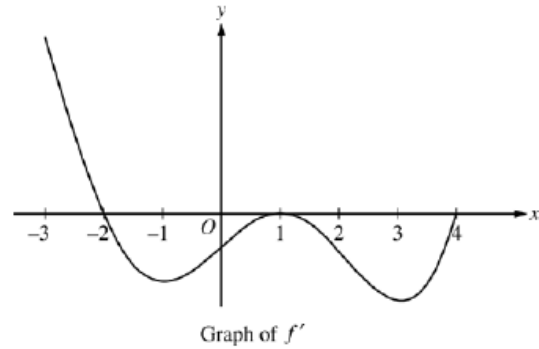
As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \leq t \leq 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

- (b) Using correct units, explain the meaning of $\frac{1}{10} \int_0^{10} H(t) dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10} \int_0^{10} H(t) dt$.
- (c) Evaluate $\int_0^{10} H'(t) dt$. Using correct units, explain the meaning of the expression in the context of this problem.

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45 \cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \leq t \leq 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \leq t \leq 8$, the plant processes gravel at a constant rate of 100 tons per hour.

- (a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.
- (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.
- (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.
- (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

The figure above shows the graph of f' , the derivative of a twice-differentiable function f , on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x -axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



- Find all x -coordinates at which f has a relative maximum. Give a reason for your answer.
- On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.
- Find the x -coordinates of all points of inflection for the graph of f . Give a reason for your answer.
- Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

Bloom's Levels: Evaluate

Webb's DOK: 3

Topic 6: Rectilinear Motion

Engaging Experience 1

Title: Distance/Displacement

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 3.4C1:** For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.
- **EK 3.4A2:** The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval.
- **EK 3.4E1:** The definite integral can be used to express information about accumulation and net change in many applied contexts.

Detailed Description/Instructions: Students will practice finding displacement and total distance traveled on teacher created problems. AP Calculus AB Free Response 2009 Test, Question 1 is a good applied example of this.

Bloom's Levels: Analyze

Webb's DOK: 3

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.)

Students drive for 20 minutes while varying their speed. Then have students calculate the distance travelled using left, right, midpoint, upper, lower, and trapezoidal Riemann sums.

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Riemann Sums	Riemann Sums	Students will practice finding Riemann Sums (including lower, upper, left, right, midpoint, and trapezoidal) on teacher created problems. Make sure to include tables of data, intervals of equal and unequal widths, and whether the approximations are over-/under-estimates of the actual integral/area under the curve. A hw example can be found in Schoology.	1 class period
Areas Under Curves	Using Integration to Find Exact Areas Under Curves	Students will practice evaluating integrals by finding signed areas under curves with known geometric shapes on teacher created problems. Make sure to include integral properties here, discussing how students can split apart integrals	1 class period
Rules of Integration - Definite and Indefinite	Indefinite Integration/Basic Differential Equations	Students will practice indefinite integration and solving basic differential equations (given $f'(x)$ and asked to find $f(x)$) on teacher created problems.	1 class period
Rules of Integration - Definite and Indefinite	Definite Integration	Students will practice definite integration on teacher created problems. Make sure to include how to integrate when the integrand has absolute values.	1 class period
Rules of Integration -	U-substitution	Students will practice integration with u-substitution on teacher created	1 class period

Definite and Indefinite		problems. Make sure to cover both definite and indefinite integration	
Second Fundamental Theorem	Second Fundamental Theorem of Calculus	Students will practice differentiating definite integrals with the 2nd Fundamental Theorem of Calculus on teacher created problems. Make sure to include questions that require students to do this with graph interpretation, such as: (see diagram).	1/2 of a class period
Integration as Accumulation	Average Value of a Function	Students will practice finding average value of a function on teacher created problems. Make sure to do this in an applied sense, so students really note the differences between average value (new) and average rate of change (old). Here are some examples: (see diagram)	1/2 of a class period
Integration as Accumulation	Integrally Defined Functions/Integrals as Accumulators	Students will practice applied integration on teacher created problems. Make sure to do these problems in the context of graphical application too. Here are several examples: (see diagram)	2 class periods
Rectilinear Motion	Distance/Displacement	Students will practice finding displacement and total distance traveled on teacher created problems. AP Calculus AB Free Response 2009 Test, Question 1 is a good applied example of this.	1 class period

Unit 5: Transcendental Functions

Subject: Calculus

Grade: 11, 12

Name of Unit: Transcendental Functions

Length of Unit: 11-14 class periods

Overview of Unit: Students will find derivatives and antiderivatives of transcendental functions. Students will find the solutions to simple differential equations. Students will draw simple slope fields. Students will solve application problems involving exponentials.

Priority Standards for unit:

- **EK 2.1C1:** Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- **EK 2.1C2:** Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- **EK 2.1C6:** The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.
- **EK 2.3F1:** Slope fields provide visual clues to the behavior of solutions to first order differential equations.
- **EK 3.5A2:** Some differential equations can be solved by separation of variables.
- **EK 3.5A3:** Solutions to differential equations may be subject to domain restrictions.
- **EK 3.5B1:** The model for exponential growth and decay that arises from the statement “The rate of change of a quantity is proportional to the size of the quantity” is $\frac{dy}{dt} = ky$.
- **EK 3.1A1:** An antiderivative of a function f is a function g whose derivative is f .
- **EK 3.1A2:** Differentiation rules provide the foundation for finding antiderivatives.
- **EK 3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables.

Essential Questions:

1. How do you differentiate and integrate with transcendental functions?
2. How do you differentiate inverse functions?
3. How do you interpret slope fields and solve separable differential equations?

Enduring Understanding/Big Ideas:

1. Students must know and apply the rules for differentiating and integrating exponential, logarithmic, and inverse trig functions.

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

2. The derivative of an inverse is
3. Students must understand slope fields show visual depictions of solutions to differential equations. To solve separable differential equations, separate your variables, integrate, and plug in your initial condition to find C (only if given an initial condition).

Unit Vocabulary:

Academic Cross-Curricular Words	Content/Domain Specific
Exponential Growth Logarithmic Growth	Slope field Differential equation Separation of variables

Resources for Vocabulary Development: Lecture, text, and internet resources.

Topic 1: Differentiating $y=\ln x$ and integrate $y= 1/x$

Engaging Experience 1

Title: Differentiating $y=\ln(x)$

Suggested Length of Time: 1/2 of a class period

Standards Addressed

Priority:

- **EK 2.1C1:** Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- **EK 2.1C2:** Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.

Detailed Description/Instructions: Students will practice differentiating functions involving natural logs on teacher created problems. If you introduce the natural log as an integrally-defined function, $\ln \int_1^x \frac{1}{t} dt$ where $x > 0$ then the derivative is an application of the 2nd Fundamental Theorem of Calculus. Make sure to review algebra skills of log properties and solving exponential and logarithmic equations.

Bloom's Levels: Apply

Webb's DOK: 2

Engaging Experience 2

Title: Integrating of the form $y=1/x$

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 3.1A1:** An antiderivative of a function f is a function g whose derivative is f .
- **EK 3.1A2:** Differentiation rules provide the foundation for finding antiderivatives.
- **EK 3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables.

Detailed Description/Instructions: Students will practice integrating functions of the form $y=1/x$ on teacher created problems. This is a good review of u-substitution from Unit 4. Remember to cover the strategy of using division to rewrite the integrand if the degree is bigger on top.

Bloom's Levels: Analyze

Webb's DOK: 3

Topic 2: Logarithmic Differentiation

Engaging Experience 1

Title: Logarithmic Differentiation

Suggested Length of Time: 1/2 of a class period

Standards Addressed

Priority:

- **EK 2.1C1:** Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- **EK 2.1C2:** Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.

Detailed Description/Instructions: Students will practice logarithmic differentiation on teacher created problems. Students need to know they must use this technique when the function they want to differentiate has a variable in both the base and the exponent. They can use this technique for exponential functions if they choose.

Bloom's Levels: Apply

Webb's DOK: 2

Topic 3: Differentiate and Integrate $y=e^x$

Engaging Experience 1

Title: Differentiation and Integration of $y = e^x$

Suggested Length of Time: 2 class periods

Standards Addressed

Priority:

- **EK 2.1C1:** Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- **EK 2.1C2:** Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- **EK 3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables.
- **EK 3.1A1:** An antiderivative of a function f is a function g whose derivative is f .
- **EK 3.1A2:** Differentiation rules provide the foundation for finding antiderivatives.

Detailed Description/Instructions: Students will practice differentiating and integrating $y = e^x$ on teacher created problems.

Bloom's Levels: Analyze

Webb's DOK: 3

Topic 4: Differentiate and Integrate $y=a^x$ and $y=\log a^x$

Engaging Experience 1

Title: Differentiating and Integrate functions of the form $y=a^x$ and $y=\log a^x$

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.1C1:** Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- **EK 2.1C2:** Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- **EK 3.3B5:** Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables.
- **EK 3.1A1:** An antiderivative of a function f is a function g whose derivative is f .
- **EK 3.1A2:** Differentiation rules provide the foundation for finding antiderivatives.

Detailed Description/Instructions: Students will practice differentiating and integrating functions of the form $y=a^x$ and $y=\log a^x$ on teacher created problems. Make sure to show students the proofs of the rules so they have the option to have less to memorize--ex: $y=\log_2(x)=\ln(X)\ln(2)=\ln(2)\ln(x)$. Now you can use natural log differentiation rules without having an extra one to memorize.

Bloom's Levels: Analyze

Webb's DOK: 3

Topic 5: Differentiate Inverse Functions

Engaging Experience 1

Title: Differentiating Inverse Functions

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.1C6:** The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.

Detailed Description/Instructions: Students will practice differentiating inverse functions on teacher created problems. Make sure to include questions for derivatives of inverses, such as:

(2008, #28) Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x . What is the value of $g'(3)$?

- a.) $-\frac{1}{2}$ b.) $-\frac{1}{8}$ c.) $\frac{1}{6}$ d.) $\frac{1}{3}$
- e.) The value of $g'(3)$ cannot be determine from the information given.

Ex: Let f be the integrally-defined function

$$f(x) = \int_2^x \frac{1}{\sqrt{1+t^4}} dt.$$

Find $(f^{-1})'(0)$.

If f and g are functions such that $f(g(x)) = x$ and $g(f(x)) = x$ for all x in their domains, and if $f(a) = b$ and $f'(a) = c$, then which of the following is true?

- a. $g'(a) = \frac{1}{c}$
- b. $g'(a) = -\frac{1}{c}$
- c. $g'(b) = \frac{1}{c}$
- d. $g'(b) = -\frac{1}{c}$
- e. $g'(b) = \frac{1}{a}$

2007 AP Calculus AB Free Response Exam, Question 3d

Bloom's Levels: Analyze

Webb's DOK: 3

Rubric: AP scoring guide

Engaging Experience 2

Title: Differentiating Inverse Trig Functions

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.1C2:** Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
- **EK 2.1C6:** The chain rule can be used to find the derivative of an inverse function, provided the derivative of that function exists.

Detailed Description/Instructions: Students will practice differentiating and integrating inverse trig functions on teacher created problems.

Bloom's Levels: Apply

Webb's DOK: 2

Topic 6: Differential Equations and Slope Fields

Engaging Experience 1

Title: Slope Fields

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 2.3F1:** Slope fields provide visual clues to the behavior of solutions to first order differential equations.

Detailed Description/Instructions: Students will practice creating and interpreting slope fields on teacher created problems. A slope fields hw worksheet can be found in Schoology. Easy real-life examples to discuss when introducing the idea include wind maps--see the following links:

<http://hint.fm/wind/index.html>,

http://www.mesonet.org/index.php/weather/map/wind_map/wind.

Bloom's Levels: Create

Webb's DOK: 3

Engaging Experience 2

Title: Solving Separable Differential Equations

Suggested Length of Time: 2 class periods

Standards Addressed

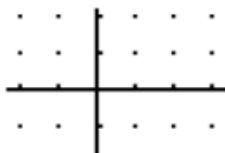
Priority:

- **EK 3.5A3:** Solutions to differential equations may be subject to domain restrictions.
- **EK 3.5B1:** The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is $\frac{dy}{dt} = ky$.

Detailed Description/Instructions: Students will practice solving differential equations on teacher created problems. Make sure to include examples where students have to write their own differential equations given proportionality/variation vocabulary (see below for examples). For domain discussion, 2006 AP Calculus AB Free Response Question #5 is good. Also make sure to include problems using their solutions to approximate function values, as seen here:

17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(A) On the axes provided, sketch a slope field for the given differential equation.



(B) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point $(1, 1)$. Then use your tangent line equation to estimate the value of $f(1.2)$.

(C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$. Use your solution to find $f(1.2)$.

(D) Compare your estimate of $f(1.2)$ found in part (b) to the actual value of $f(1.2)$ found in part

(E) Was your estimate from part (b) an underestimate or an overestimate? Use your slope field to explain why.

Examples for proportionality/variation:

Write and solve the differential equation:

2. The rate of change of y with respect to x varies jointly as x and $L - y$.
3. The rate of change of Q with respect to t is inversely proportional to the square of t .

Bloom's Levels: Analyze

Webb's DOK: 3

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Have students split into two groups and play the integral game from SMART exchange. See Schoology.

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Differentiating $y=\ln x$ and integrate $y=\frac{1}{x}$	Differentiating $y=\ln(x)$	Students will practice differentiating functions involving natural logs on teacher created problems. If you introduce the natural log as an integrally-defined function, $\ln x = \int_1^x \frac{1}{t} dt$, where $x > 0$, then the derivative is an application of the 2nd Fundamental Theorem of Calculus. Make sure to review algebra skills of log properties and solving exponential and logarithmic equations.	1/2 of a class period
Differentiating $y=\ln x$ and integrate $y=\frac{1}{x}$	Integrating of the form $y=1/x$	Students will practice integrating functions of the form $y=1/x$ on teacher created problems. This is a good review of u-substitution from Unit 4. Remember to cover the strategy of using division to rewrite the integrand if the degree is bigger on top	1 class period
Logarithmic Differentiation	Logarithmic Differentiation	Students will practice logarithmic differentiation on teacher created problems. Students need to know they must use this technique when the function they want to differentiate has a variable in both the base and the exponent. They can use this technique for exponential functions if they choose.	1/2 of a class period
Differentiate and Integrate $y=e^x$	Differentiation and Integration of $y = e^x$	Students will practice differentiating and integrating $y = e^x$ on teacher created problems.	2 class periods
Differentiate and Integrate $y=a^x$ and $y=\log a^x$	Differentiating and Integrate functions of the form $y=a^x$ and $y=\log a^x$	Students will practice differentiating and integrating functions of the form $y=a^x$ and $y=\log a^x$ on teacher created problems. Make sure to show students the proofs of the rules so they have the option to have less to	1 class period

		memorize--ex: $y = \log_2(x) = \ln(X)\ln(2) = 1\ln(2)\ln(x)$. Now you can use natural log differentiation rules without having an extra one to memorize.	
Differentiate Inverse Functions	Differentiate Inverse Functions	Students will practice differentiating inverse functions on teacher created problems. Make sure to include questions for derivatives of inverses, such as: (see diagram)	1 class period
Differentiate Inverse Functions	Differentiating Inverse Trig Functions	Students will practice differentiating and integrating inverse trig functions on teacher created problems.	1 class period
Differential Equations and Slope Fields	Slope Fields	Students will practice creating and interpreting slope fields on teacher created problems. A slope fields hw worksheet can be found in Schoology. Easy real-life examples to discuss when introducing the idea include wind maps--see the following links: http://hint.fm/wind/index.html, http://www.mesonet.org/index.php/weather/map/wind_map/wind.	1 class period
Differential Equations and Slope Fields	Solving Separable Differential Equations	Students will practice solving differential equations on teacher created problems. Make sure to include examples where students have to write their own differential equations given proportionality/variation vocabulary (see below for examples). For domain discussion, 2006 AP Calculus AB Free Response Question #5 is good. Also make sure to include problems using their solutions to approximate function values, as seen here: (see diagram)	2 class periods

Unit 6: Areas and Volumes

Subject: Calculus

Grade: 11, 12

Name of Unit: Areas and Volumes

Length of Unit: 6-8 class periods

Overview of Unit: Students will apply the definite integral to find the area between curves. Students will find volumes made by known cross-sections and by revolving a known area around a vertical or horizontal line.

Priority Standards for unit:

- **EK 3.4D1:** Areas of certain regions in the plane can be calculated with definite integrals
- **EK 3.4D2:** Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.

Essential Questions:

1. How do you find the area under a curve or between curves?
2. How do you find volumes of solids of revolutions or solids generated from known cross sections?

Enduring Understanding/Big Ideas:

1. You must integrate from one intersection point to the next of the most positive curve minus the most negative curve (make sure to cover in terms of x and y).
2. You accumulate (with integration) the areas of the faces of the slices you take.

Unit Vocabulary:

Academic Cross-Curricular Words	Content/Domain Specific
Area Volume	Disc Method Washer Method

Resources for Vocabulary Development: Lecture, text, and internet resources.

Topic 1: Area Between Two Curves

Engaging Experience 1

Title: Area Under a Curve and Between Two Curves

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 3.4D1:** Areas of certain regions in the plane can be calculated with definite integrals

Detailed Description/Instructions: Students will practice finding the area under curves and between two curves on teacher created problems. Make sure to focus on finding intersection points and on calculating areas both in terms of x and y .

Bloom's Levels: Analyze

Webb's DOK: 3

Topic 2: Volume of Solids of Revolution

Engaging Experience 1

Title: Volumes of Solids of Revolutions

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 3.4D1:** Areas of certain regions in the plane can be calculated with definite integrals

Detailed Description/Instructions: Students will practice finding volumes of solids of revolution on teacher created problems. Make sure to focus on calculating volumes both in terms of x and y . Also make sure to emphasize volumes are found by integrating the area of the face of a cross-section. Here's a link with a collection of animations showing solids of revolution:

<http://www.mathdemos.org/mathdemos/solids/gallery.html>

Bloom's Levels: Analyze

Webb's DOK: 3

Topic 3: Volume of Solids with Known Cross-Sections

Engaging Experience 1

Title: Volumes of Solids with Known Cross-Sections

Suggested Length of Time: 1 class period

Standards Addressed

Priority:

- **EK 3.4D2:** Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals.

Detailed Description/Instructions: Students will practice finding volumes of solids with known cross-sections on teacher created problems. Make sure to focus on calculating volumes both in terms of x and y . If you emphasize volumes are found by integrating the area of the face of a cross-section when you find volumes of revolution, students should see this really is the same idea. That being said, show them animations of the differences between a solid of revolution and one with known cross-sections.

Here are some helpful links:

<http://www.mathdemos.org/mathdemos/sectionmethod/sqrcross75.gif>

<http://www.mathdemos.org/mathdemos/sectionmethod/sqrcross75slab.gif>

Bloom's Levels: Analyze

Webb's DOK: 3

Engaging Scenario

Engaging Scenario (An Engaging Scenario is a culminating activity that includes the following components: situation, challenge, specific roles, audience, product or performance.) Have students rotate a circle about a line and find the volume of the torus created. Then have students use this concept to find the volume of a donut or bagel.

Summary of Engaging Learning Experiences for Topics

Topic	Engaging Experience Title	Description	Suggested Length of Time
Area Between Two Curves	Area Under a Curve and Between Two Curves	Students will practice finding the area under curves and between two curves on teacher created problems. Make sure to focus on finding intersection points and on calculating areas both in terms of x and y .	1 class period
Volume of Solids of Revolution	Volumes of Solids of Revolutions	Students will practice finding volumes of solids of revolution on teacher created problems. Make sure to focus on calculating volumes both in terms of x and y . Also make sure to emphasize volumes are found by integrating the area of the face of a cross-section. Here's a link with a collection of animations showing solids of revolution: http://www.mathdemos.org/mathdemos/solids/gallery.html	1 class period
Volume of Solids with Known Cross-Sections	Volume of Solids with Known Cross-Sections	Students will practice finding volumes of solids with known cross-sections on teacher created problems. Make sure to focus on calculating volumes both in terms of x and y . If you emphasize volumes are found by integrating the area of the face of a cross-section when you find volumes of revolution, students should see this really is the same idea. That being said, show them animations of the differences between a solid of revolution and one with known cross-sections. Here are some helpful links: http://www.mathdemos.org/mathdemos/sectionmethod/sqrcross75.gif http://www.mathdemos.org/mathdemos/sectionmethod/sqrcross75slab.gif	1 class period

Unit of Study Terminology

Appendices: All Appendices and supporting material can be found in this course's shell course in the District's Learning Management System.

Assessment Leveling Guide: A tool to use when writing assessments in order to maintain the appropriate level of rigor that matches the standard.

Big Ideas/Enduring Understandings: Foundational understandings teachers want students to be able to discover and state in their own words by the end of the unit of study. These are answers to the essential questions.

Engaging Experience: Each topic is broken into a list of engaging experiences for students. These experiences are aligned to priority and supporting standards, thus stating what students should be able to do. An example of an engaging experience is provided in the description, but a teacher has the autonomy to substitute one of their own that aligns to the level of rigor stated in the standards.

Engaging Scenario: This is a culminating activity in which students are given a role, situation, challenge, audience, and a product or performance is specified. Each unit contains an example of an engaging scenario, but a teacher has the ability to substitute with the same intent in mind.

Essential Questions: Engaging, open-ended questions that teachers can use to engage students in the learning.

Priority Standards: What every student should know and be able to do. These were chosen because of their necessity for success in the next course, the state assessment, and life.

Supporting Standards: Additional standards that support the learning within the unit.

Topic: These are the main teaching points for the unit. Units can have anywhere from one topic to many, depending on the depth of the unit.

Unit of Study: Series of learning experiences/related assessments based on designated priority standards and related supporting standards.

Unit Vocabulary: Words students will encounter within the unit that are essential to understanding. Academic Cross-Curricular words (also called Tier 2 words) are those that can be found in multiple content areas, not just this one. Content/Domain Specific vocabulary words are those found specifically within the content.