AP Calculus AB Summer Work 2021

CONGRATULATIONS! You have enrolled in a very challenging but rewarding math course for next year. In order to make the most out of the class time we have, it is very important that you come to your first class prepared to learn calculus with your pre-calculus skills and knowledge fresh in your mind. Hence the required summer work. There are two assignments. The first is due on Friday, July 23rd – please mail the first one to me along with a self-addressed, stamped envelope and I will return it to you with comments and suggestions for improvement, if needed. The second assignment is due on Friday, August 13th. I will return the second assignment to you on the first day of class in September.

There are many websites that may be useful while you review the pre-calculus concepts. http://www.jamesrahn.com/PreCalculus/pages/table_of_contents.htm is an excellent review with many exercises and worked out solutions. http://www.khanacademy.org/#browse is a collection of mini-lectures with pre-calculus and trigonometry topics included. Check these out as you complete the summer work.

The two assignments are attached. Be sure to show all work and ***READ*** *any information included with the questions. You are allowed to work with a classmate or classmates but the work shown should be your own, and you should include the names of anyone you worked with or from whom you received help. There should be room to show all required work on the work sheets, but feel free to show any additional work on separate pages as well.*

In addition, you should come to class on the first day with the following materials:

1) *A three-ring binder or a spiral notebook*
2) *Summer work assignment #1.*
3) *TI – 83/84 calculator*

Please mail the assignments to: Mrs. Lisa DeFanti
192 River Road
Pawcatuck, CT 06379

Don't hesitate to e-mail me at ldefanti@stoningtonschools.org with questions.
I'm looking forward to working with you next school year in AP Calculus.

AP Calculus Summer Work 2021—Assignment #1*—Show all work on these pages, or, if you need extra space, use white unlined paper clearly marked with the question number. DUE DATE: Friday, JULY 23rd*

Use the point slope form of the equation of a line $y = m(x - x_1) + y_1$ *to write the equation of each of the following lines. DO NOT use the "solving for b" method.*

1) *With slope -2, containing the point (3,4)*

2) *Containing the points (1,-3) and (-5,2)*

3) *With slope 0, containing the point (5,2)*

4) *Parallel to* $2x - 3y = 7$ *and passing through (5,4)*

5) *Perpendicular to the line in question #4, containing the point (-3,2)*

6) *Passing through the point* $\left(\frac{\pi}{6}, \frac{1}{3}\right)$ $with\ slope$ $\frac{\sqrt{3}}{2}$.

Simplify each of the following:

7) $\dfrac{x-4}{x^2-3x-4}$	8) $\dfrac{5-x}{x^2-25}$
9) $\dfrac{1}{x+2}-\dfrac{1}{x}$	10) $\dfrac{1}{x+h}-\dfrac{1}{x}$
11) $\dfrac{(x+h)^2\ -x^2}{h}$	12) $\dfrac{(x+h)^3\ -x^3}{h}$

13) Multiply: $x^{\frac{1}{2}}\left(x + x^{\frac{1}{2}} + x^{\frac{3}{2}}\right)$	*14) Multiply:* $x^{-\frac{1}{2}}\left(x^{-\frac{1}{2}} + x^{\frac{1}{2}} + x^{\frac{3}{2}}\right)$

15) Rationalize the denominator: $\frac{2}{\sqrt{x+4}-2}$	*16) Rationalize the numerator:* $\frac{\sqrt{x+9}-3}{3}$
17) Solve for z: $y^2 + 3yz - 8z - 4x = 0$	*18) Solve for x:* $\lvert 5x - 2\rvert = 8$

Simplify—these are no calculator questions.

19) $\left(4b^{5/3}\right)^{3/2}$	20) $\left(5c^{2/3}\right)\left(4c^{3/2}\right)$
21) e^{ln3}	22) $\ln 1$
22) e^{3lnx}	23) lne^{8}
24) $log_2\, 4\sqrt{2}$	25) $log_{1/2}\, 8$

Rewrite as a single logarithm:

26) $2\ln 4 - ln3$	27) $log_2 5 + log_2(x^2 - 1) - log_2(x - 1)$

Solve each equation for x where x is a real number. Show the work.

28) $x^2 + 3x - 4 = 14$	29) $e^{4x} = 3$
30) $\log x + \log (x - 3) = 1$	31) $log_2(5x + 8) - log_2(x - 5) = 3$

Complete the following identities: (you should have these memorized)

32) $\sin^2 x + \cos^2 x =$	33) $\sin 2x =$
34) $\cos 2x =$	35) $\tan^2 x + 1 =$

Give the domain and range of each of the following trig functions:

36) $y = sinx$ *Domain:* *Range:* *Do you have a mental picture of this function?*	37) $y = \cos x$ *Domain:* *Range:* *Do you have a mental picture of this function?*	38) $y = \tan x$ *Domain:* *Range:* *Do you have a mental picture of this function?*
39) $y = \arcsin x$ *Domain:* *Range:*	40) $y = \arccos x$ *Domain:* *Range:*	41) $y = arctanx$ *Domain:* *Range:*

Without a calculator, determine the exact value of each expression. Note that in this course, no distinction is made between arcin θ and $\sin^{-1}\theta$. These functions are one-to-one and are derived from the restricted sin function. The same applies to all of the trig inverse functions.

43) $\sin 0$	44) $\sin\frac{\pi}{6}$	45) $\sin\frac{3\pi}{4}$
46) $\cos\pi$	47) $\cos\frac{7\pi}{6}$	48) $\cos -\frac{\pi}{3}$
49) $\tan\frac{7\pi}{4}$	50) $\tan\frac{\pi}{6}$	51) $\sec\frac{\pi}{3}$
52) $\arcsin\left(\frac{1}{2}\right)$	53) $\arctan 0$	54) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$
55) $\sin^{-1}(-\frac{1}{2})$	56) $\cos\ (\sin^{-1}\left(\frac{1}{2}\right)$	57) $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$ *(think about this one carefully!)*

Solve each equation for $\theta, 0 \le \theta < 2\pi$

58) $3sin^2\theta = cos^2\theta$	*59)* $\sin 2\theta = \sin\theta$

60) Find the remainder on division of $x^5 - 4x^4 + x^3 - 7x - 1$ *by* $x + 1$

61) *Is* $x + 1$ *a factor of* $x^5 - 4x^4 + x^3 - 7x - 1$*? Explain your reasoning.*

Let $f(x) = \sqrt{x-3}$ *and* $g(x) = x^3 + x + 3$*. Determine each of the following: [Recall* $(f \circ g)(x) = f(g(x))$*. This the operation of composition.*

62) $(f \circ g)(4)$	63) $(g(f(4))$
64) $(f \circ g)(x)$	*65)* $f^{-1}(x)$

AP Calculus Summer Work 2021—Assignment #2—*Show all work on these pages, or, if you need extra space, use white unlined paper clearly marked with the question number. DUE DATE: Friday, AUG 13th*

For each function below, determine the domain and range.

Function	*Domain*	*Range*
1) $f(x) = \sqrt{x-4}$		
2) $y = \sqrt{4-x^2}$		
3) $g(x) = \sqrt{x^2 + 4}$		

PIECEWISE DEFINED FUNCTIONS—Read this carefully before doing #4 - 6

The function $y = |x|$ *can be written* ***piecewise*** *as follows:* $y = \begin{cases} x & for\ x \geq 0 \\ -x & for\ x \leq 0 \end{cases}$

Notice that the function uses both $\geq$ *and* $\leq$ *at x = 0 since f(0) = 0 in either case. The function could also be written in either of the following ways:* $y = \begin{cases} x & for\ x > 0 \\ -x & for\ x \leq 0 \end{cases}$ *or* $y = \begin{cases} x & for\ x \geq 0 \\ -x & for\ x < 0 \end{cases}$

In this course it will be very important that you can rewrite absolute value functions piecewise. The method is outlined below and then you will be asked to write some similar absolute value function as piecewise functions.

First, remind yourself of what the graph of $y = |x|$ *looks like. Notice that solving f(x) for zero results in x=0, and that is where the two "pieces" of the graph come together. Shifting and/or dilating (stretching/shrinking) the graph does not change that fact—the two "pieces" of the graph connect at the zero of the absolute value function.*

Example 1: $f(x) = |x-3|$. *Notice that this is the original absolute value function shifted three units to the ________. Solving f(x) = 0 yields x = _____. So, at that x-value, the two linear pieces of the function connect. When x>3, x – 3 >0 and* $|x-3| = x-3.$ *On the other hand, when x < 3, x – 3 is negative and* $|x-3| = -(x-3).$

Therefore: $f(x) = |x-3|$ can be written piecewise as $y = \begin{cases} x-3 & for\ x \geq 3 \\ -(x-3) & for\ x < 3 \end{cases}$

(Note that it doesn't matter in which "piece" the equal to sign in the inequality symbol is attached to; it could be in the second one or both.)

Example 2: $g(x) = |x+3|$. *x + 3 >0 implies x > -3, that is where the two "pieces" connect.*

So g(x) = $\begin{cases} x+3 & for\ x \geq -3 \\ -(x+3) & \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{cases}$

Example 3: $y = |5-x|$. *Since 5 - x > 0 implies x < 5,* $y = \begin{cases} 5-x & for\ x < 5 \\ -(5-x) & for\ x \geq 5 \end{cases}$

Example 4: $h(x) = |-8+4x|$. *-8 + 4x > 0 implies x >2, h(x) =* $\begin{cases} -8+4x \\ -(-8+4x) \end{cases}$ *??*

Write each of the following piecewise:

4) $f(x) = \lvert 3x+6 \rvert$
5) $g(x) = \lvert 7-4x \rvert$
6) $y = \lvert x^2-4 \rvert$

FACTORING GREATEST COMMON FACTORS—Read this carefully before doing # 7 - 11

Another skill that will be very useful in your study of calculus is factoring using GCF. Read the examples below carefully, working along with the solution and then complete the problems that follow.

Example 1: *Factor:* $21x^3y^5 + 12x^2y^7$. *Since the GCF is* $3x^2y^5$ *the expression can be written as* $3x^2\,y^5(7x+4y^2)$

Example 2: *Factor* $(x-4)^6(2x+3)^3 + (x-4)^5(2x+3)^4$. *GCF is* $(x-4)^5(2x+3)^3$, *the expression can be rewritten as* $(x-4)^5(2x+3)^3[(x-4)+(2x+3)] = (x-4)^5(2x+3)^3(3x-1)$

Can you think of a reason why you might want to factor the original expression?

Example 3: *Factor* $(x+5)^{-2}(x+1)^4 + (x+5)^{-1}(x+1)^{3.}$ *GCF is* $(x+5)^{-2}(x+1)^3$.

$(x+5)^{-2}(x+1)^4 + (x+5)^{-1}(x+1)^{3.} = (x+5)^{-2}(x+1)^3[(x+1)+(x+5)]$

Notice the following: $(x+5)^{-2}$ *times* $(x+5)^1 = (x+5)^{-1}$ *since you add exponents when multiplying.*

Example 4: *Factor* $(2x+7)^{1/2}(x-6)^3 + (2x+7)^{3/2}(x-6)^2$

Notice that $(2x+7)^{1/2}$ *times* $(2x+7)^1 = (2x+7)^{3/2}$ *by addition of exponents.*

So, since GCF = $(2x+7)^{1/2}(x-6)^2$*, therefore*

$(2x+7)^{1/2}(x-6)^3 + (2x+7)^{3/2}(x-6)^2 = (2x+7)^{1/2}(x-6)^2[(2x+7)+(x-6)]$

One last note: remember that $a^{-1/2} \cdot a = a^{1/2}$ *and* $a^{-3/2} \cdot a = a^{-1/2}$ *(since you add exponents when multiplying expressions with the same base.)*

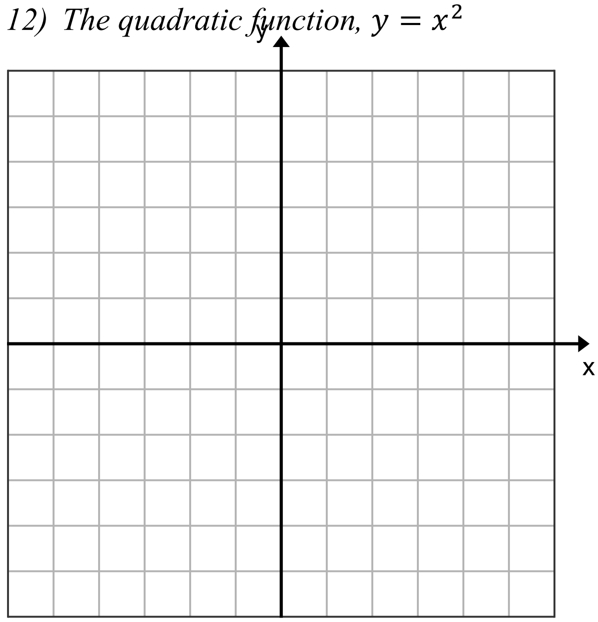
Factor each of the following:

7) $18x^4y^9 + 30x^3y^{10}$
8) $(x+5)^3(3x-7)^5 + (x+5)^2(3x-7)^6$
9) $(3x+5)^{-4}(x-2)^3 + (3x+5)^{-5}(x-2)^4$
10) $4(2x-7)^{1/2}(x+6)^5 + 6(2x-7)^{-1/2}(x+6)^4$
11) $x^{-3/2}y^{1/2} + x^{-1/2}y^{-1/2}$

<u>THE LIBRARY OF FUNCTIONS:</u> *These are the basic function that you must know. For each you should be able to sketch an accurate graph and you should know the domain and range. Study these!*

Graph each and give the domain and range:

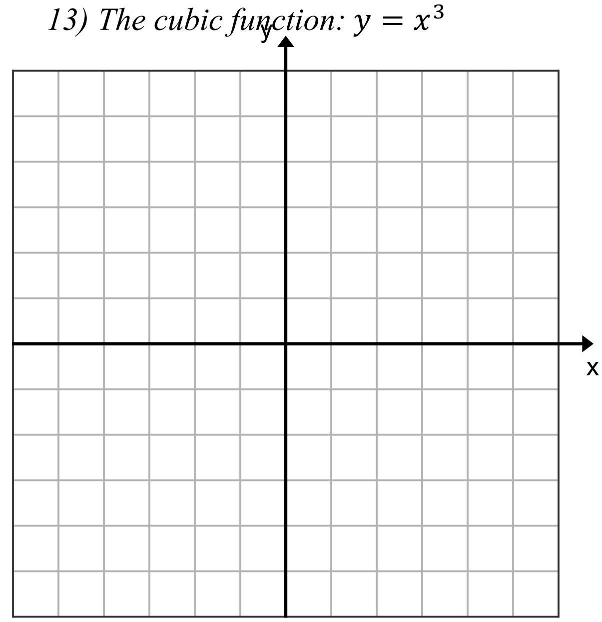
12) The quadratic function, $y = x^2$



Domain: ________________________

Range: ________________________

13) The cubic function: $y = x^3$



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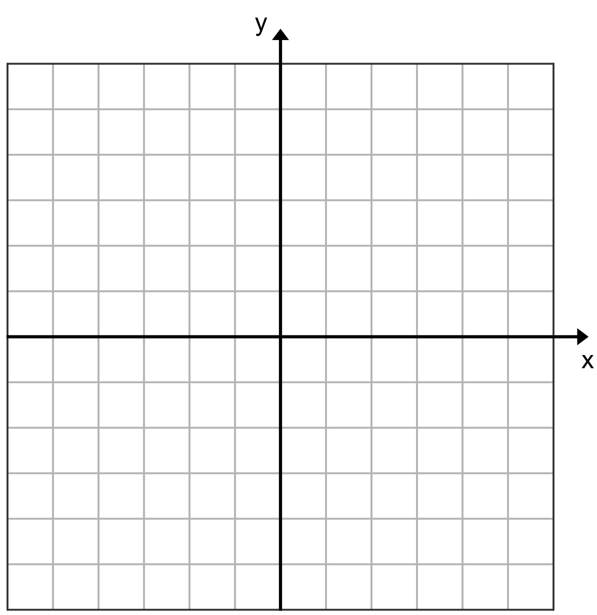
14) The square root function, $y = \sqrt{x}$



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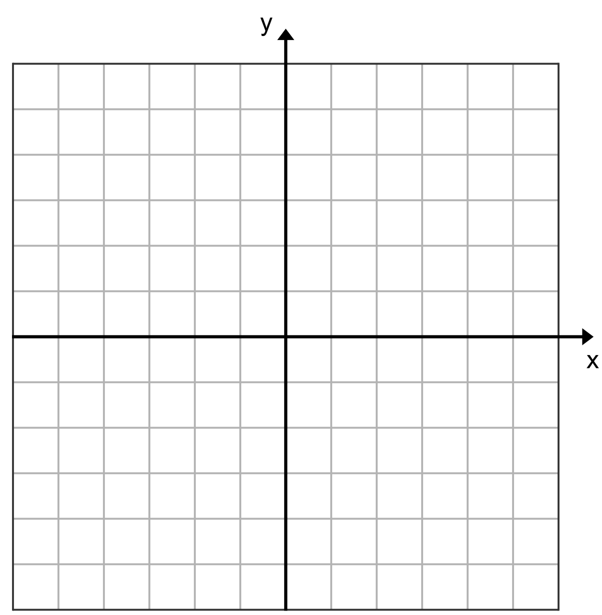
15) The absolute value function, $y = |x|$



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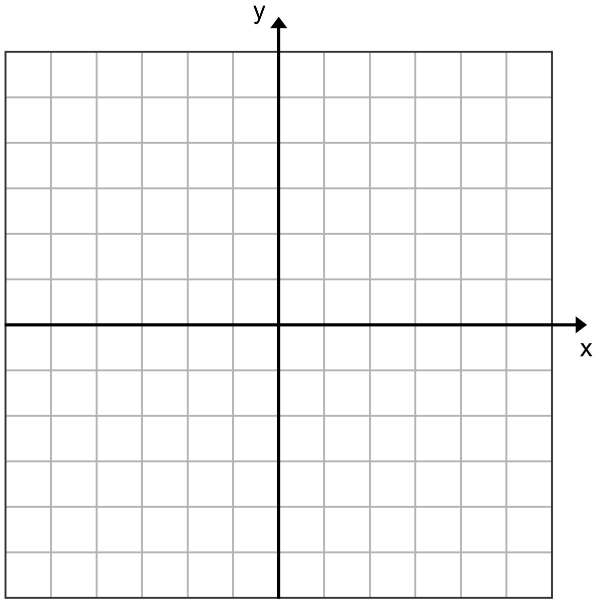
16) The identity function, $y = x$



Domain: ________________________

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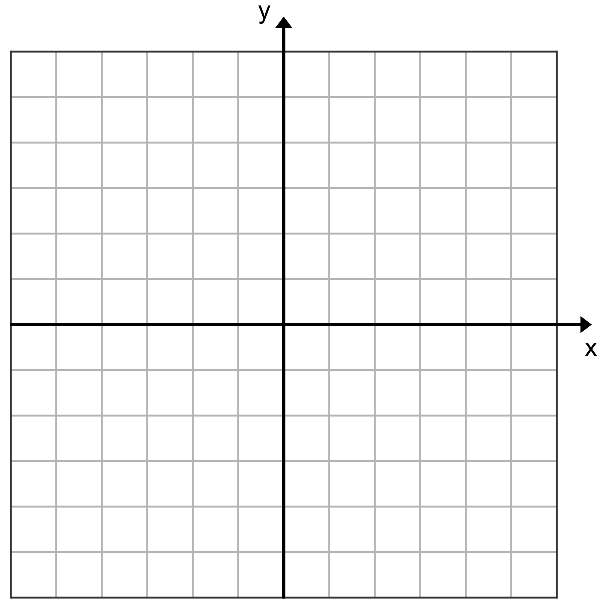
17) The cube root function: $y = \sqrt[3]{x}$



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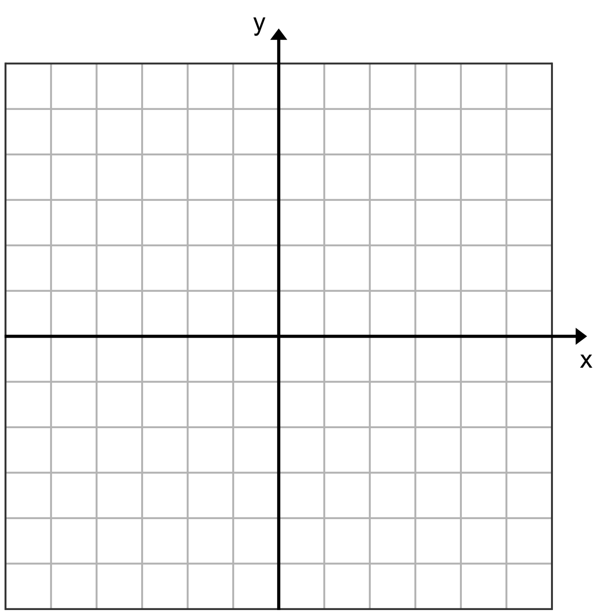
18) The exponential function, $y = e^{x}$



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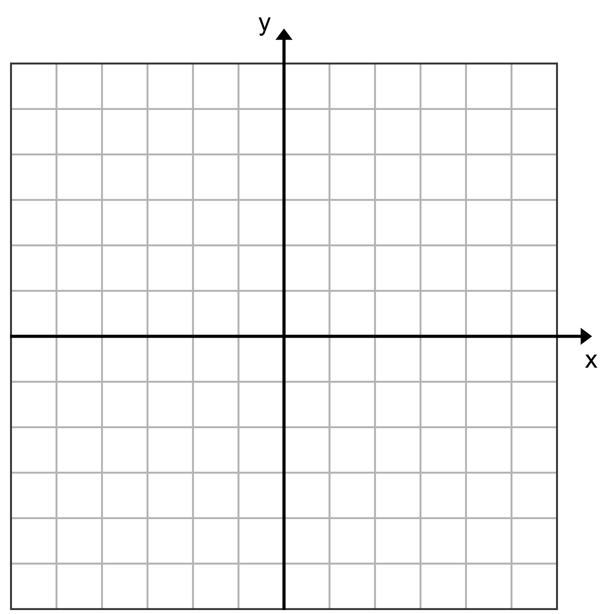
19) The natural log function, $y = \ln x$



Domain: ________________________

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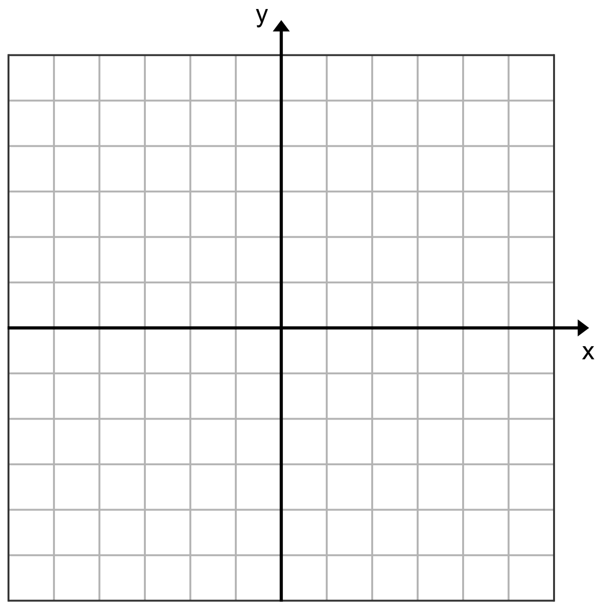
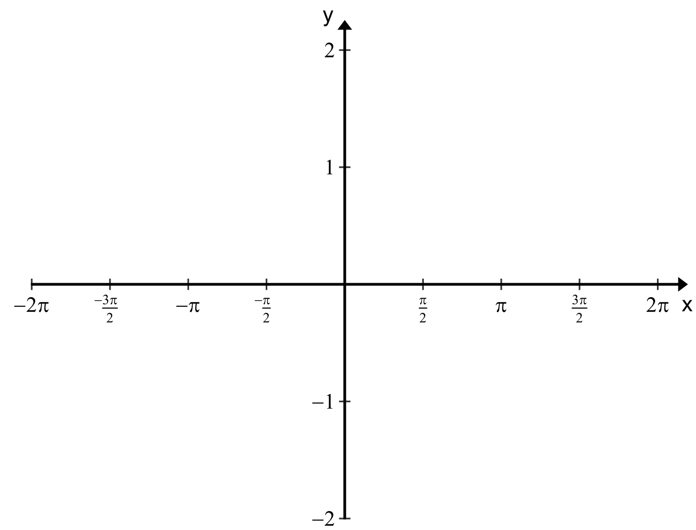
20) The inverse function, $y = \frac{1}{x}$



21) The sine function, y = sin x



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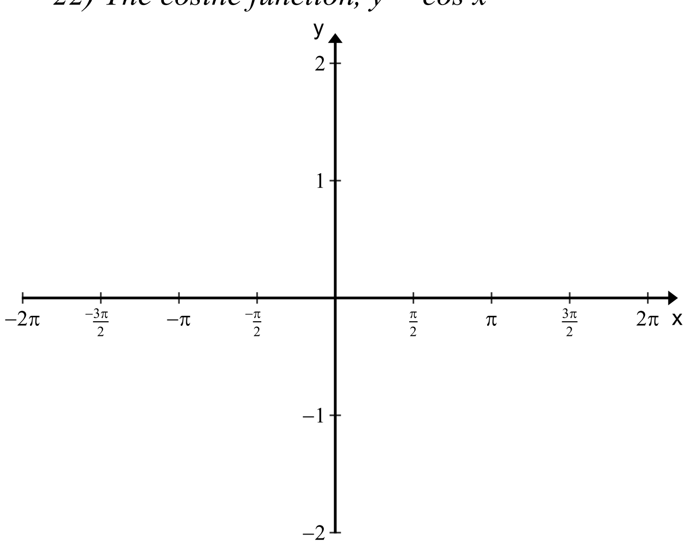
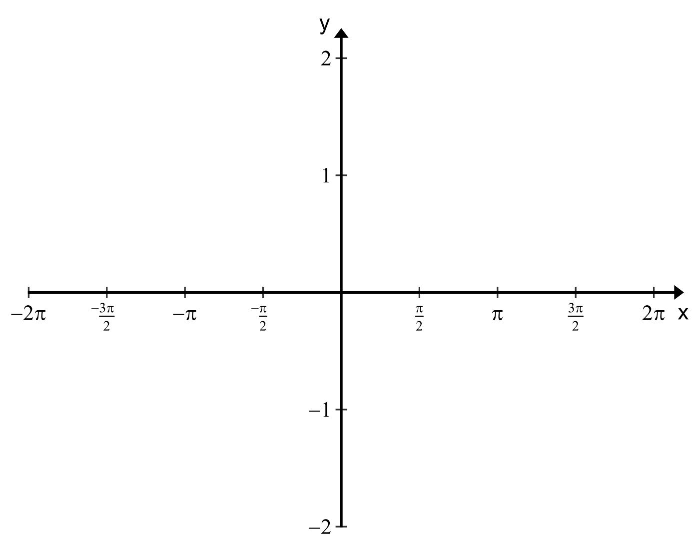
22) The cosine function, y = cos x



23) The tangent function, y = tan x



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