AP Calculus BC

Summer Assignment

Name: _____   Period: _

The purpose of this packet is to both convey to students the foundational skills needed to be successful in this course and to provide them an opportunity to self-assess and develop these skills prior to entering the class. In order to be successful in this and all subsequent math courses at Servite, students must master and retain the content and skills from all previous math courses. As such, we ask that you please work on this assignment with integrity and diligence always striving to meet the intended purpose and goal of this assignment.

Directions: Please print this packet. You **must show all work** in this packet in the space provided. For every word problem, write your answer in the form of a sentence. After you make an honest attempt at a problem, check your answer. If your answer is incorrect, try to identify where you went wrong, review the topic, and redo the problem correctly.

This packet will be **collected** on the **second day** of school. You will be given a homework grade for completing this packet.   Per Servite School policy, if this packet is not turned in on the second day of school, you will receive half credit if it is turned in the following day. After that, you will receive a zero for this packet. An assessment will be given at the beginning of the school year to make sure you have mastered all pre-requisites. This assessment will count as a quiz grade. If you earn less than a 75% on the assessment, a meeting with the counseling staff will be held to reevaluate your goals and potential success in the course.

Have a great summer and we are looking forward to seeing you in August!

I understand that I have to show all my work.

_____   _____
(Student Signature)   **(Date)**

I have checked to see that my child has shown all work and completed all problems.

_____   _____
(Parent/Guardian Signature)   **(Date)**

Topic A: Functions

1.) If $f(x) = 4x - x^2$, find:

    a.) $f(4) - f(-4)$ b.) $\sqrt{f\left(\dfrac{3}{2}\right)}$ c.) $\dfrac{f(x+h) - f(x)}{2h}$

2.) If $V(r) = \dfrac{4}{3}\pi r^3$, find:

    a.) $V\left(\dfrac{3}{4}\right)$ b.) $V(r+1) - V(r-1)$ c.) $\dfrac{V(2r)}{V(r)}$

3.) If $f(x) = \begin{cases} -x, & x < 0 \\ x^2 - 1, & 0 \le x < 2, \\ \sqrt{x+2} - 2, & x \ge 2 \end{cases}$ find:

    a.) $f(0) - f(2)$ b.) $\sqrt{5 - f(-4)}$ c.) $f(f(3))$

Topic B: Domain and Range

Find the domain of the following functions using interval notation:

1.) $y = \dfrac{x^3 - x^2 + x}{x}$

2.) $y = \dfrac{x - 4}{x^2 - 16}$

3.) $f(x) = \dfrac{1}{4x^2 - 4x - 3}$

4.) $y = \sqrt{2x - 9}$

5.) $y = \log(x - 10)$

6.) $y = \dfrac{\sqrt{2x + 14}}{x^2 - 49}$

Find the range of the following functions:

7.) $y = 100^x$

8.) $y = \sqrt{x^2 + 1} + 1$

Topic C: Special Factorization

Factor completely.

1.) $x^3 + 8$

2.) $(x - 3)^2 (2x + 1)^3 + (x - 3)^3 (2x + 1)^2$

3.) $27x^3 - 125y^3$

4.) $x^4 + 11x^2 - 80$

5.) $ac + cd - ab - bd$

6.) $2x^2 + 50y^2 - 20xy$

Topic D: Linear Functions

1.) Find the equation of the line in point-slope form, with the given slope, passing through the given point.

a.) $m = -\dfrac{1}{2}$, $(2, -8)$

b.) $m = \dfrac{2}{3}$, $\left(-6, \dfrac{1}{3}\right)$

2.) Find the equation of the line in point-slope form, where possible, passing through the given points.

a.) $(-5, 7)$, $(-5, 9)$

b.) $(2, -1)$, $(7, -1)$

c.) $\left(-2, \dfrac{2}{3}\right)$, $\left(\dfrac{1}{2}, 1\right)$

3.) Find k if the lines $3x - 5y = 9$ and $2x + ky = 11$ are a.) parallel and b.) perpendicular.

Topic E: Solving Polynomial Equations

Solve each equation for x over the real number system.

1.) $12x^2 - 5x = 2$

2.) $20x^2 - 56x + 15 = 0$

3.) $81x^2 + 72x + 16 = 0$

4.) $x + \dfrac{1}{x} = \dfrac{17}{4}$

5.) $x^3 - 5x^2 + 5x - 25 = 0$

6.) $2x^4 - 15x^3 + 18x^2 = 0$

Topic F: Negative and Fractional Exponents

Simplify and write with positive exponents.

1.) $-12^2 x^{-5}$

2.) $\left(-12x^5\right)^{-2}$

3.) $\left(4x^{-1}\right)^{-1}$

4.) $\left(\dfrac{5x^3}{y^2}\right)^{-3}$

5.) $\left(x^3 - 1\right)^{-2}$

6.) $\dfrac{\left(x^2 - 1\right)^{-\frac{1}{2}}}{\left(x^2 + 1\right)^{\frac{1}{2}}}$

7. $\left(x^{-2} + 2^{-2}\right)^{-1}$

Topic G: Complex Fractions

Simplify the complex fractions:

1.) $\dfrac{x - \dfrac{1}{x}}{x + \dfrac{1}{x}}$

2.) $\dfrac{1 + x^{-1}}{1 - x^{-2}}$

3.) $\dfrac{x^{-1} + y^{-1}}{x + y}$

4.) $\dfrac{2x(2x-1)^{\frac{1}{2}} - 2x^2(2x-1)^{-\frac{1}{2}}}{(2x-1)}$

5.) $\dfrac{\dfrac{1}{3}(3x-4)^{-\frac{3}{4}}}{-\dfrac{3}{4}}$

Topic H: Solving Inequalities

Solve the following inequalities:

1.) $\dfrac{3}{4} > x+1 > \dfrac{1}{2}$

2.) $x+7 \geq |5-3x|$

3.) $(x+2)^2 < 25$

4.) $x^3 < 4x^2$

Topic I: Exponential Functions and Logarithms

Simplify the following:

1.) $\log_2 \dfrac{1}{4}$

2.) $\log_8 4$

3.) $\ln \dfrac{1}{\sqrt[3]{e^2}}$

4.) $5^{\log_5 40}$

5.) $e^{\ln 12}$

Solve the following:

6.) $\log_9\left(x^2 - x + 3\right) = \dfrac{1}{2}$

7.) $\log(x-3) + \log 5 = 2$

8.) $3^{x-2} = 18$

Topic J: Using your graphing calculator

Use your calculator to find the zeros of each of the following functions. Make sure each equation is set equal to zero first.

1.) $3x^3 - x - 5 = 0$

2.) $2x^2 - 1 = 2^x$

3.) $2\ln(x+1) = 5\cos x$ on $[0, 2\pi)$

Use your calculator to find the solution (intersection) of the given system of equations.

4.) $\begin{cases} f(x) = x^4 - 6.5x^2 + 6x + 2 \\ g(x) = 1 + x + e^{x^2 - 2x} \end{cases}$

Use your calculator to find both a relative maximum and a relative minimum point of the given function.

5.) $h(x) = 2x^5 - 3x^4 + x - 4$

Topic K: Sequences and Series

1.)   Write the first five terms of the sequence generated by $a_n = \dfrac{3n}{n!}$

2.)   Write the first five terms of the sequence generated by $\left\{ \dfrac{2n+1}{n+3} \right\}$

3.)   Find $\displaystyle\sum_{n=1}^{5} \dfrac{n!}{n^2}$

4.)   Find the sum of $\dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \ldots + \dfrac{1}{3^n} + \ldots$

Topic L: Limits

Find each limit or function value using the graph.

1.) $\displaystyle\lim_{x \to -1^-} f(x) =$ _____

2.) $\displaystyle\lim_{x \to -1^+} f(x) =$ _____

3.) $\displaystyle\lim_{x \to -4^-} f(x) =$ _____

4.) $\displaystyle\lim_{x \to -4^+} f(x) =$ _____

5.) $\displaystyle\lim_{x \to -4} f(x) =$ _____

6.) $f(-4) =$ _____

7.) $\displaystyle\lim_{x \to 2^-} f(x) =$ _____

8.) $\displaystyle\lim_{x \to 2^+} f(x) =$ _____
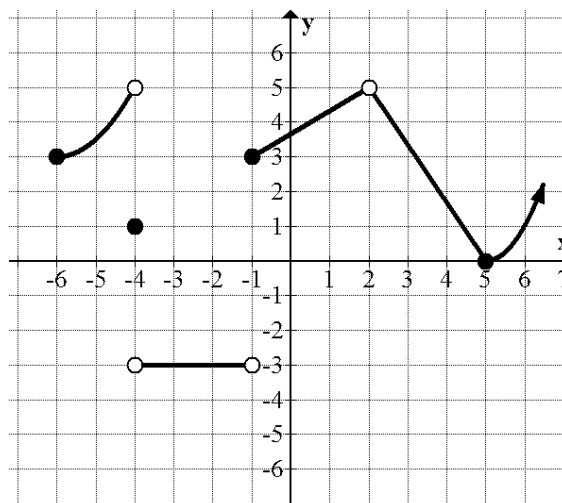
Topic M: Review of Calculus

1.)   A particle moves along the y-axis. Its position at time t is given by $s(t) = 2t^3 - 4t + 1$.

What is the position of the particle at time $t = 2$?

At what time(s), if any, is the velocity of the particle zero? Explain how you found your answer(s).

What was the average velocity of the particle on the interval $[-1,2]$?

Find a formula for the instantaneous velocity of the particle at any time t.

2.)   Estimate the area under the curve $y = x^2 + 1$ on the interval $[0,2]$ using Riemann Sums with **four** equally-spaced subdivisions with heights determined by:

Use LRAM (left endpoint)

Use RRAM (right endpoint)

Use MRAM (midpoint)