AP Calculus Summer 2021 Assignment

Welcome to Advanced Placement Calculus! This course will demand more of you than any other mathematics course you have taken. You will be asked to reason, justify, and expand your learning and do more than perform memorized computations.

Before being ready for AP Calculus, you should be competent in algebraic concepts taught in Algebra 1, Algebra 2, and Pre-Calculus. Also, you should be prepared that successful AP Calculus students possess the following characteristics:

- daily review of new content material taught in class
- diligent completion of homework on a daily basis
- participation in study groups or working with a study buddy
- understanding concepts vs. cramming details
- organizing notes and materials, grouping of similar concepts, discerning differences between concepts, and knowing and understanding major theorems and concepts
- asking questions in class and out of class before the next concept is introduced
- proficient in translating mathematical expressions into necessary and sufficient English justifications
- plans college visits sparingly and wisely and does not slack off in the spring of the school year

You will need to have your own calculator and bring it to class every day. AP Calculus relies on the use of a calculator. I highly recommend you have a TI -84 or TI-89 Titanium as does the AP College Board. If you choose to use a different edition or type, I will likely not be able to help you with all topics.

Before starting this course, I ask that you commit to being an active participant in the course. This means you must be willing to work with your fellow classmates and me. We will have a lot of fun; however, the ultimate goal of this course is to get you college credit in Calculus by getting a good score on the AP exam. If you are a student who likes to sit and do nothing during class time, not complete homework, or does not enjoy working through good mathematics questions, this course is probably not for you.

Now that you have a little information about the course, you are ready to start your summer assignment. Make sure you read all directions thoroughly and complete each part of the assignment. Remember that this assignment will be your first big grade in the course as well as set the tone for the type of student you will be in the class. I look forward to working with and teaching all of you over the course of the next year!

The assignment is due on the first day of class.

Enjoy your summer and see you at the start of school!

Your Assignment

Use the four links below to watch 4 lessons on Limits.
There is work to complete for each lesson.
You will have a test on this lesson when you return to school in September.

Here are the links to the 4 lessons:

- Lesson 1: Limits Graphically
  - https://www.youtube.com/watch?v=6hXKaIlV-sM
- Lesson 2: Limits Analytically
  - https://www.youtube.com/watch?v=S9JEUNTQNDE&list=PLxRSX8UqzWceo7f2qH6YEUbQrqfre0q8Z
- Lesson 3: Asymptotes
  - https://www.youtube.com/watch?v=rTT2X594jjQ
- Lesson 4:
  - https://www.youtube.com/watch?v=aDZAQGEVJNg

The assignments for each lesson are below. If you have any questions please email me at marie-claire.guidoux@saintjosephprep.org . I will do my best to get back to you quickly.
You may write work on additional paper and attach if necessary.

Calculus

Write your questions and thoughts here!

1.1 Limits Graphically

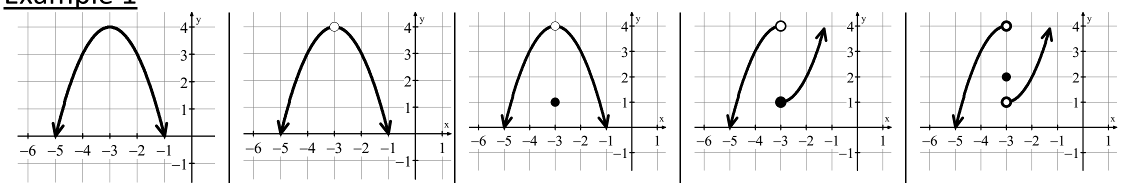
Name:______________________________

Notes

What is a limit?

A ***limit*** is the ___________ a function _______________ from *both* the left and the right side of a given ____________.

Example 1



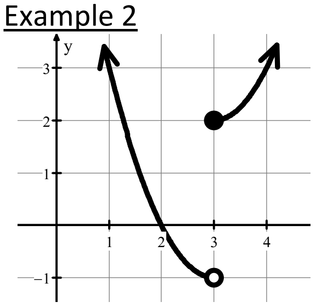
$f(-3) =$	$f(-3) =$	$f(-3) =$	$f(-3) =$	$f(-3) =$
$\lim_{x\to -3} f(x) =$	$\lim_{x\to -3} f(x) =$	$\lim_{x\to -3} f(x) =$	$\lim_{x\to -3} f(x) =$	$\lim_{x\to -3} f(x) =$

Limit: (*geeky math definition for Mr. Kelly*)
Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\boldsymbol{\lim_{x\to c} f(x) = R}$.

What is a *one-sided* limit?

A ***one-sided limit*** is the ____________ a function approaches as you approach a given ____________ from either the ______ or ______ side.

Example 2



"The limit of f as x approaches 3 from the left side is -1." $\lim_{x\to 3^-} f(x) = -1$

"The limit of f as x approaches 3 from the right side is 2." $\lim_{x\to 3^+} f(x) = 2$

Write your questions and thoughts here!

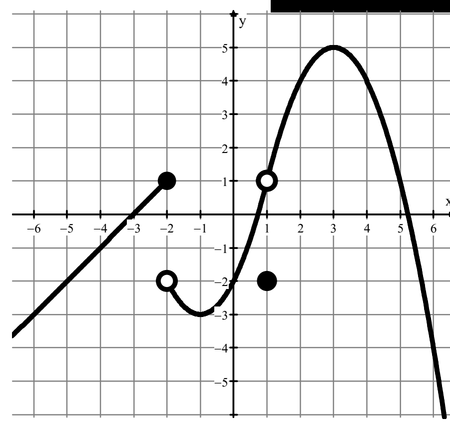
1.1 Limits Graphically

Notes

Example 3

a. $\lim_{x\to -2^-} f(x) =$	b. $\lim_{x\to -2^+} f(x) =$	c. $\lim_{x\to -2} f(x) =$
d. $\lim_{x\to 1} f(x) =$	e. $\lim_{x\to 0} f(x) =$	f. $\lim_{x\to 3^-} f(x) =$
g. $\lim_{x\to -1} f(x) =$	h. $\lim_{x\to -3} f(x) =$	i. $f(-2) =$

j. $f(1) =$



When does a limit not exist?

1.
2.
3.

Example 4

Sketch a graph of a function g that satisfies all of the following conditions.

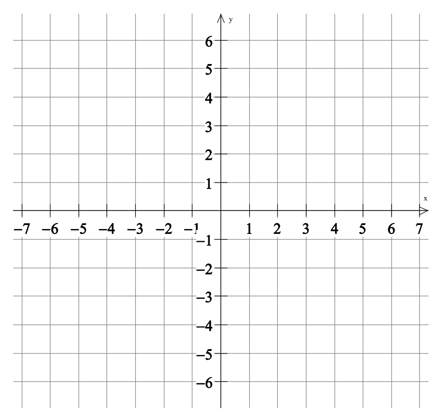
a. $g(3) = -1$

b. $\lim_{x\to 3} g(x) = 4$

c. $\lim_{x\to -2^+} g(x) = 1$

d. g is increasing on $-2 < x < 3$

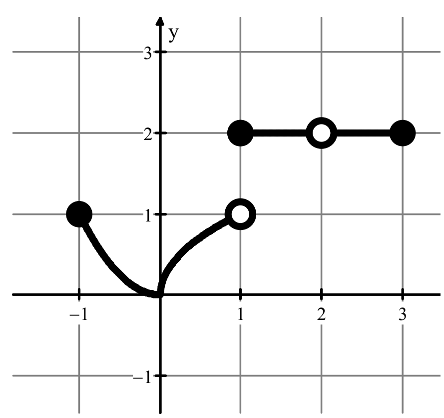
e. $\lim_{x\to -2^-} g(x) > \lim_{x\to -2^+} g(x)$



Example 5

Write **T** (true) or **F** (false) under each statement. Use the graph on the right.

a. $\lim_{x\to -1^+} f(x) = 1$	b. $\lim_{x\to 2} f(x) = 2$	c. $\lim_{x\to 1^-} f(x) = 1$
d. $\lim_{x\to 1^+} f(x) = 2$	e. $\lim_{x\to 1} f(x) =$ does not exist	
f. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$	g. $\lim_{x\to 2} f(x) =$ does not exist	



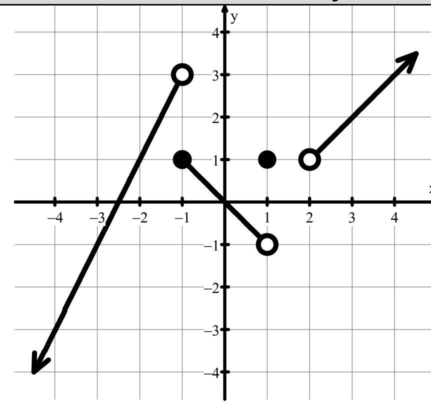
Now summarize what you learned!

1.1 Limits Graphically

Calculus Name: ____________________ **Practice**

For 1-5, give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined*."

1.

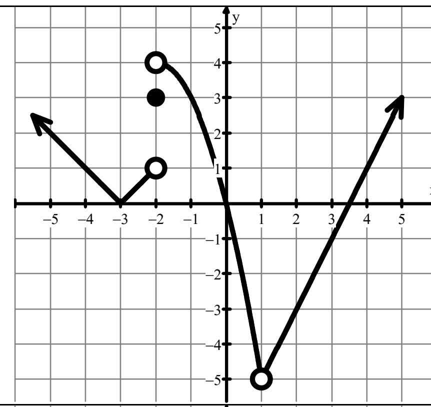


a. $\lim_{x\to -1^-} f(x) =$ b. $f(1) =$ c. $\lim_{x\to 0} f(x) =$

d. $\lim_{x\to 2^+} f(x) =$ e. $f(-1) =$ f. $f(2) =$

g. $\lim_{x\to -1^+} f(x) =$ h. $\lim_{x\to 1^-} f(x) =$ i. $\lim_{x\to 2} f(x) =$

2.

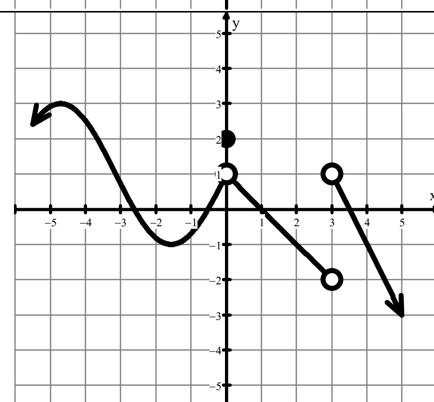


a. $\lim_{x\to -3} f(x) =$ b. $f(1) =$ c. $\lim_{x\to 1} f(x) =$

d. $\lim_{x\to -2^+} f(x) =$ e. $f(3) =$ f. $\lim_{x\to -2^-} f(x) =$

g. $\lim_{x\to -2} f(x) =$ h. $f(-2) =$ i. $f(4) =$

3.

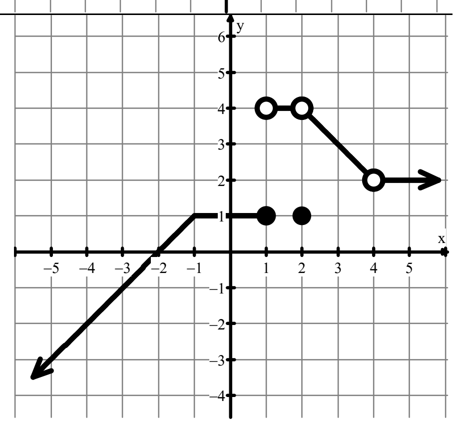


a. $\lim_{x\to 3^+} f(x) =$ b. $f(3) =$ c. $\lim_{x\to 0} f(x) =$

d. $\lim_{x\to 3} f(x) =$ e. $f(0) =$ f. $\lim_{x\to 3^-} f(x) =$

g. $\lim_{x\to 0^+} f(x) =$ h. $f(1) =$ i. $f(-1.6) =$

4.



a. $\lim_{x\to -1^-} f(x) =$ b. $f(2) =$ c. $\lim_{x\to 2} f(x) =$

d. $\lim_{x\to -1} f(x) =$ e. $f(4) =$ f. $\lim_{x\to 1^-} f(x) =$

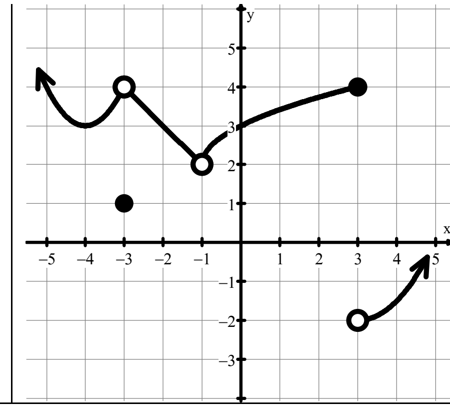
g. $\lim_{x\to -1^+} f(x) =$ h. $f(1) =$ i. $\lim_{x\to 4} f(x) =$

5.

a. $\lim_{x \to 3^-} f(x) =$ b. $f(-1) =$ c. $\lim_{x \to -3} f(x) =$

d. $\lim_{x \to -1} f(x) =$ e. $f(-3) =$ f. $\lim_{x \to 3^+} f(x) =$

g. $f(3) =$ h. $\lim_{x \to 0} f(x) =$ i. $f(-4) =$



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6. Sketch a graph of a function f that satisfies all of the following conditions.

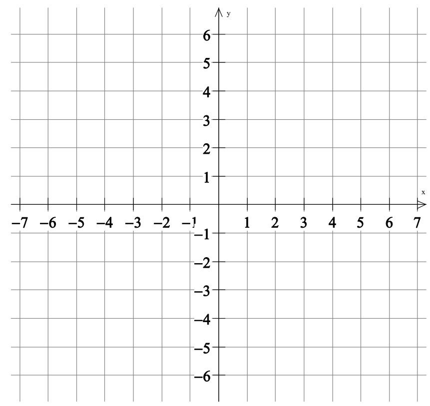
a. $f(-2) = 5$

b. $\lim_{x \to -2} f(x) = 1$

c. $\lim_{x \to 4^+} f(x) = 3$

d. f is increasing on $x < -2$

e. $\lim_{x \to 4^-} f(x) < \lim_{x \to 4^+} f(x)$



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7. Sketch a graph of a function g that satisfies all of the following conditions.

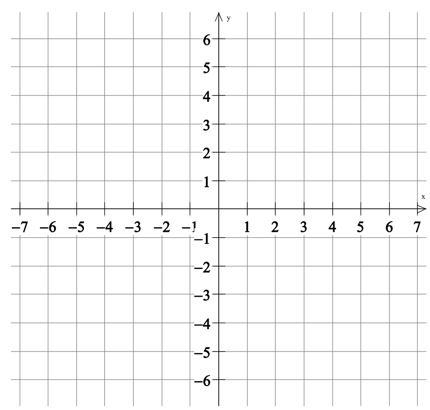
a. $g(1) = 3$

b. $\lim_{x \to 1} g(x) = -2$

c. $\lim_{x \to -3^+} g(x) = 5$

d. g is increasing only on $-5 < x < -3$ and $x > 1$

e. $\lim_{x \to -3^-} g(x) > \lim_{x \to -3^+} g(x)$



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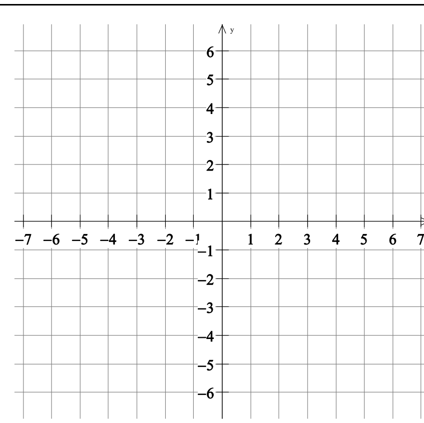
8. Sketch a graph of a function h that satisfies all of the following conditions.

a. $\lim_{x \to 3} h(x) = h(-2) = 1$

b. $h(3)$ is undefined.

c. $\lim_{x \to -2^-} h(x) < \lim_{x \to -2^+} h(x)$

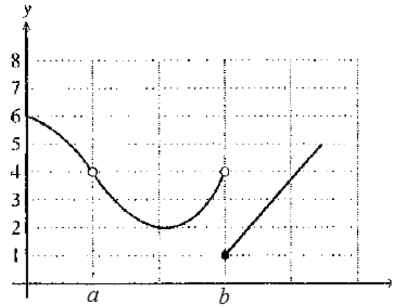
d. h is constant on $-2 < x < 3$ and decreasing everywhere else.



1.1 Limits Graphically

Test Prep

1. The graph of the function f is shown. Which of the following statements about f is true?



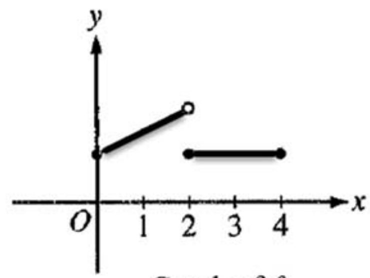
(A) $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$ (B) $\lim_{x\to a} f(x) = 4$

(C) $\lim_{x\to b} f(x) = 4$ (D) $\lim_{x\to b} f(x) = 1$

(E) $\lim_{x\to a} f(x)$ does not exist.

2. The figure below shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x\to 2^-} f(x)$ exists.
II. $\lim_{x\to 2^+} f(x)$ exists.
III. $\lim_{x\to 2} f(x)$ exists.



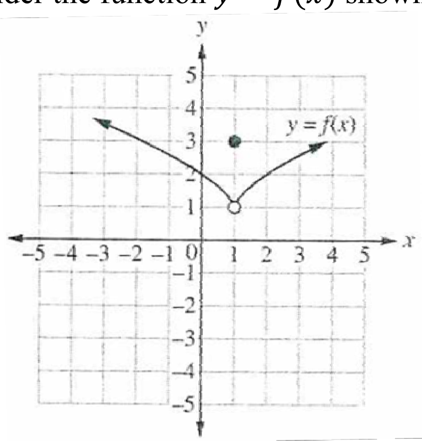
Graph of f

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

3. If $[x]$ represents the greatest integer that is less than or equal to x, then $\lim_{x\to 0^-} \frac{2}{[x]} =$

(A) -2 (B) -1 (C) 0 (D) 2 (E) the limit does not exist

4. Consider the function $y = f(x)$ shown below. Which of the following statements is true?



(A) $\lim_{x\to 1} f(x) = 3$

(B) $f(1) = 1$

(C) $f(x)$ is continuous for all x.

(D) $\lim_{x\to 1} f(x) = f(1)$

(E) None of the above

Calculus

Write your questions and thoughts here!

1.2 Limits Analytically

Name:__________________________

Notes

Recall: What is a limit?

Finding a limit:

1.

2.

a.

b.

3.

Direct Substitution		Factor and Cancel	
1. $\lim_{x \to -1} (x^2 + 2x - 4)$	2. $\lim_{x \to 2} \sqrt{3x - 2}$	3. $\lim_{x \to 0} \frac{4x^2 - 5x}{x}$	4. $\lim_{x \to -7} \frac{2x^2 + 13x - 7}{x + 7}$

Rationalize	Two variables
5. $\lim_{x \to 5} \frac{\sqrt{x+4} - 3}{x - 5}$	6. $\lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - (x^2 - 3x)}{h}$

Write your questions and thoughts here!

1.2 Limits Analytically

Notes

Piecewise defined functions and limits

$$f(x)=\begin{cases}\sqrt{11-x}, & x<-5\\ \dfrac{x+3}{5-x^2}, & x\geq -5\end{cases}$$

$$g(x)=\begin{cases}\sqrt{10-x^2}, & x<-1\\ \dfrac{26-5x^2}{7} & -1<x\leq e\\ \ln x^3, & x>e\end{cases}$$

7. $\lim\limits_{x\to -5^-} f(x)=$

8. $\lim\limits_{x\to -5^+} f(x)=$

9. $\lim\limits_{x\to -5} f(x)=$

10. $\lim\limits_{x\to -1} g(x)=$

11. $\lim\limits_{x\to e^+} g(x)=$

12. $\lim\limits_{x\to e} g(x)=$

Special Trig Limits:

$$\lim_{x\to 0}\frac{\sin x}{x}= \quad \text{or} \quad \lim_{x\to 0}\frac{x}{\sin x}=$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}= \quad \text{or} \quad \lim_{x\to 0}\frac{\cos x-1}{x}=$$

13. $\lim\limits_{x\to 0}\frac{\sin 3x}{x}$

14. $\lim\limits_{x\to 0}\frac{\tan 4x}{8x}$

15. $\lim\limits_{x\to 0}\frac{\cos^2 x-1}{x(\cos x+1)}$

Now summarize what you learned!

1.2 Limits Analytically

Calculus Name: ______________________________ **Practice**

Evaluate each limit.

1. $\lim_{x\to 2}(x - x^2)$
2. $\lim_{x\to 5}(x+1)^2$
3. $\lim_{x\to 1}\frac{x^2-5x}{x-1}$
4. $\lim_{x\to 1}\frac{x^2+x-30}{x-1}$
5. $\lim_{x\to 0}\frac{3x}{\sin x}$
6. $\lim_{x\to 0}\frac{\sin(2x)}{3x}$
7. $\lim_{x\to 0}\frac{\sqrt{x+7}-\sqrt{7}}{x}$
8. $\lim_{x\to 7}\frac{\sqrt{x+9}-4}{x-7}$
9. $\lim_{x\to -2}(3x^2 - x + 1)$
10. $\lim_{x\to 3}(2x^2 + 5x - 6)$
11. $\lim_{x\to -7}\frac{2x^3+11x^2-21x}{x^2+7x}$
12. $\lim_{x\to 8}\frac{x^2+2x-80}{x-8}$
13. $\lim_{x\to 0}\frac{\sqrt{x+9}-3}{x}$
14. $\lim_{x\to 0}\frac{\sqrt{x+11}-\sqrt{11}}{x}$
15. $\lim_{x\to 5}\sqrt{4x-9}$

16. $\lim_{x\to -1} \sqrt{3-x}$

17. $\lim_{h\to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h}$

18. $\lim_{h\to 0} \frac{5\sqrt{x+h}-5\sqrt{x}}{h}$

19. $\lim_{x\to \frac{1}{3}} \frac{6x^2+13x-5}{3x-1}$

20. $\lim_{x\to 0} \frac{7x^2+x}{x}$

21. $\lim_{x\to 2} \frac{\sqrt{5x-6}}{x}$

22. $\lim_{x\to \frac{\pi}{2}} \tan\left(\frac{x}{2}\right)$

23. $\lim_{x\to 1} 3$

24. $\lim_{x\to -3} 14$

25. $\lim_{x\to 0} \frac{\frac{1}{x+3}-\frac{1}{3}}{x}$

26. $\lim_{x\to 0} \frac{\frac{1}{(x+2)^2}-\frac{1}{4}}{x}$

27. $\lim_{x\to 0}(-2)$

28. $\lim_{x\to 1} \frac{\sqrt{x+5}+\sqrt{6}}{x}$

29. $\lim_{x\to 0} \frac{x^2+2x-8}{x-4}$

30. $\lim_{x\to -2} \frac{x^2-4x-10}{x}$

31. $\lim_{x\to 0} \frac{3x^2+5x}{x}$

32. $\lim_{x\to 4} \frac{5x^2-21x+4}{x-4}$

33. $\lim_{x\to \frac{1}{2}} \frac{1-x-2x^2}{2x-1}$

34. $\lim_{x\to \pi} \cos x$

35. $\lim_{x\to \frac{\pi}{8}} \sin(4x)$

36. $\lim_{x\to 2} \frac{x^2+6x-16}{2-x}$

37. $\lim_{x\to 5} \frac{2x^2-17x+35}{5-x}$

38. $\lim_{x\to 0} \frac{(1-\cos^2 x)\sin x}{x^2}$

39. $\lim_{h\to 0} \frac{(x+h)^2+6(x+h)-(x^2+6x)}{h}$

40. $\lim_{h\to 0} \frac{4(x+h)^2-5(x+h)-2-(4x^2-5x-2)}{h}$

On the AP exam, there will be questions where you must find the hole of a function. This is basically finding the limit as you approach the discontinuity. The problems on the exam will be worded differently, but if you can recognize what to do, they are not that difficult.

The following functions have a removable discontinuity (hole). If we fill in this hole to make the function continuous, what is the coordinate point to fill in?			
41. $\frac{x^2-x-12}{x-4}$	42. $\frac{x^2+7x}{2x}$	43. $\frac{2x-1}{2x^2+x-1}$	44. $\frac{3x^2+13x+4}{x+4}$

Using the following piecewise functions, find the given values.

$g(x)=\begin{cases}\sqrt{5-x}, & x<-4\\ x^2-5, & -4\le x<2\\ x-3, & x\ge 2\end{cases}$	$h(x)=\begin{cases}-\lvert x\rvert, & x\le -5\\ 20-x^2, & -5<x\le 3\\ 4x-1, & x>3\end{cases}$	$w(\theta)=\begin{cases}\sin\theta, & \theta\le\pi\\ \cos\theta, & \pi<\theta<2\pi\\ \tan\theta, & \theta>2\pi\end{cases}$
$\lim_{x\to 2^-} g(x)=$	$\lim_{x\to -5^+} h(x)=$	$\lim_{x\to \pi^-} w(\theta)=$
$\lim_{x\to -4^+} g(x)=$	$\lim_{x\to -5} h(x)=$	$w(\pi)=$
$g(2)=$	$h(3)=$	$\lim_{x\to \pi^+} w(\theta)=$
$\lim_{x\to -4^-} g(x)=$	$\lim_{x\to -5^-} h(x)=$	$\lim_{x\to 2\pi^-} w(\theta)=$
$\lim_{x\to 2^+} g(x)=$	$\lim_{x\to 3^+} h(x)=$	$\lim_{x\to \pi} w(\theta)=$
$\lim_{x\to 2} g(x)=$	$\lim_{x\to 3} h(x)=$	$\lim_{x\to 2\pi^+} w(\theta)=$
$\lim_{x\to -4} g(x)=$	$h(-5)=$	$\lim_{x\to 2\pi} w(\theta)=$
$g(-4)=$	$\lim_{x\to 3^-} h(x)=$	$w(2\pi)=$

Test Prep

1.2 Limits Analytically

1. $\lim_{x\to -1} \cos(\pi x) =$

(A) π (B) 1 (C) 0 (D) -1 (E) The limit does not exist

2. If $f(x) = \begin{cases} \ln 3x, & 0 < x \leq 3 \\ x \ln 3, & 3 < x \leq 4 \end{cases}$, then $\lim_{x \to 3} f(x)$ is

(A) $\ln 9$ (B) $\ln 27$ (C) $3 \ln 3$ (D) $3 + \ln 3$ (E) nonexistent

3. Evaluate $\lim_{x \to 1} \frac{\ln x}{3x}$ is

(A) 0 (B) $\frac{3}{e}$ (C) e (D) 3 (E) The limit does not exist.

4. $\lim_{x \to 0} 4 \frac{\sin x \cos x - \sin x}{x^2}$ is

(A) 2 (B) $\frac{40}{3}$ (C) ∞ (D) 0 (E) undefined

5. $\lim_{x \to a} \frac{x^2 - 2ax + a^2}{x - a} =$

(A) $-\infty$ (B) a (C) 0 (D) ∞ (E) The limit does not exist.

6. Let $f(x) = \begin{cases} \frac{\sin x}{x}, & x \geq 0 \\ \cos x, & x < 0 \end{cases}$ Which of the following statements about $f(x)$ is true?

I. $\lim_{x \to 0^+} f(x) = 1$

II. $\lim_{x \to 0^-} f(x) = 1$

III. $\lim_{x \to 0} f(x) = 1$

(A) None of these statements are true. (B) I only (C) II only (D) I and II only (E) I, II, and III

7. $\lim_{x \to 0} \left(\frac{3x^2 + 5\cos x - 5}{2x} \right) =$

(A) 0 (B) $\frac{5}{2}$ (C) 3 (D) 4 (E) Does not exist

8. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

Calculus

Write your questions and thoughts here!

1.3 Asymptotes

Name:__________________________

Notes

Recall: Vertical Asymptote Horizontal Asymptote

Vertical Asymptotes:

True or False. If you have the function $f(x) = \frac{blah,\ blah,\ blah}{x-a}$ then there must be a vertical asymptote at $x = a$.

Use the function $f(x) = \frac{x^2+2x-8}{x^2+x-12}$ to answer the following.

1. Identify all vertical asymptotes.
2. Evaluate $\lim_{x\to 3^-} f(x)$
3. Evaluate $\lim_{x\to 3^+} f(x)$

Horizontal Asymptotes: (End-behavior)

What does the y-value approach as the x-value approaches negative infinity AND positive infinity? Does it approach a specific number, or is it growing without bound?

Basic Rules for Horizontal Asymptotes:

________________ grows faster means $\frac{\text{not as big}}{\text{super duper BIG number!}} = 0$

If the numerator and denominator grow __________ fast, then you have $\frac{\text{BIG number!}}{\text{BIG number!}} = 1$

If the ____________ grows faster than the denominator, then you have $\frac{\text{BIG number!}}{\text{not as big}} = \infty$

First, you need to recognize which functions grow faster as x-values get larger and larger.

Rank Fastest to Slowest	$f(x)$	$x = 1$	$x = 10$	$x = 100$	$x = 1000$
	x^2				
	x^3				
	x^{10}				
	2^x				
	e^x				
	4^x				
	$\ln x$				

Write your questions and thoughts here!

1.3 Asymptotes

Notes

Find the horizontal asymptote(s) of each function.

4. $y = \frac{x^2+4}{3x-5}$	5. $y = \frac{x+4}{3x-5}$	6. $y = \frac{x+4}{3x^2-5}$

What about weird ones like this: $y = \frac{\sqrt{x^2+2}}{x-1}$

Evaluate the limit.

7. $\lim_{x\to\infty} \frac{\sqrt{4x^2+x-2}}{3x-1}$	8. $\lim_{x\to-\infty} \frac{\sqrt{4x^2+x-2}}{3x-1}$	9. $\lim_{x\to\infty} -4e^{\frac{1}{x}}$	10. $\lim_{x\to\infty} 5e^{-x}$

Trig Function's Horizontal Asymptotes:

Evaluate the limit.

11. $\lim_{x\to-\infty} \frac{\sin x}{x}$	12. $\lim_{x\to\infty} -3\cos\frac{1}{x}$	13. $\lim_{x\to\infty} \sin x$	14. $\lim_{x\to\infty} 5x\cos x$

Squeeze Theorem: a.k.a. "Sandwich Theorem" or "Pinching Theorem"

If $(x) \leq \quad (x) \leq \quad (x)$

and if $\lim f(x) =$ and $\lim h(x) =$

then $\lim g(x) =$

Use the Squeeze Theorem to evaluate the limit.

15. $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right)$

Now summarize what you learned!

1.3 Asymptotes

Calculus Name: ______________________________ **Practice**

Identify all vertical asymptotes of each function.			
1. $f(x) = \frac{x^2-x-12}{x+7}$	2. $f(x) = \frac{x^3+4x^2-24x}{x^2-1x}$	3. $f(x) = \frac{7x^2+4x-3}{7x-3}$	4. $f(x) = \frac{3x^2-11x+10}{x-2}$

Identify all horizontal asymptotes of each function.			
5. $f(x) = \frac{\sqrt{25x^4+2x}}{x^2}$	6. $f(x) = \frac{\sqrt{7x^6+3x^2+x}}{x^3+4x^2}$	7. $f(x) = \frac{\sqrt{9x^8-2x^3-6x}}{2x^4-10x} + 3$	8. $f(x) = \frac{3x^2}{\sqrt{3x^4-2x}}$

Using the Squeeze Theorem, evaluate each limit. SHOW WORK!		
9. $\lim_{x\to 0} x\cos\left(\frac{1}{x}\right)$	10. $\lim_{x\to 0} x^2\sin\left(\frac{1}{x^2}\right)$	11. $\lim_{x\to 0} x\sin\left(\frac{1}{x^2}\right)$

Evaluate each limit.			
12. $\lim_{x\to\infty} \frac{-x+2}{x^2+2x+2}$	13. $\lim_{x\to\infty} \left(\sin\frac{1}{x} - \frac{6x^2+2x}{3x^2}\right)$	14. $\lim_{x\to\infty} \left(5\cos\frac{1}{x}\right)$	15. $\lim_{x\to\infty} \frac{x^7}{4^x} - 5$
16. $\lim_{x\to\infty} 3^{-x} + 2$	17. $\lim_{x\to\infty} -3x\cos x$	18. $\lim_{x\to\infty} 2x\sin x$	19. $\lim_{x\to\infty} \frac{9x^4+4x^3+3}{x^7+2x^4+2x^3}$
20. $\lim_{x\to-\infty} \frac{3x^2-5x+11}{x^2-2x}$	21. $\lim_{x\to\infty} \cos\left(\frac{2x-\pi x^2}{x^2}\right)$	22. $\lim_{x\to\infty} \left(\frac{\sin x}{x} - 4\right)$	23. $\lim_{x\to\infty} \frac{-x^4-3x^2-8}{5x^4+7x+13}$
24. $\lim_{x\to\infty} \frac{x^3-7x^2+8}{x^2+7x-2}$	25. $\lim_{x\to\infty} x^2 2^{-x}$	26. $\lim_{x\to\infty} \frac{e^7}{9^x}$	27. $\lim_{x\to-\infty} \frac{3x^2-5x^7+6}{x^7-15x^4}$

28. $\lim_{x\to\infty} \frac{2x^4+3x^2+10}{5x^2+6x-1}$

29. $\lim_{x\to\infty} \left(\frac{\sin x}{x} + 2\right)$

30. $\lim_{x\to\infty} \cos\left(\frac{x^5}{e^x}\right) + 4$

31. $\lim_{x\to\infty} \frac{3x^6-5x^3+6}{x^3+x^8-2x^4}$

32. $\lim_{x\to\infty} \sin(2x)$

33. $\lim_{x\to\infty} \cos\left(\frac{\pi x^2+\frac{\sqrt{2}}{2}x}{5-2x^2}\right)$

34. $\lim_{x\to\infty} \cos\left(\frac{\frac{\sqrt{2}}{2}x-\pi x^2}{x^2-x^3+2}\right)$

1.3 Asymptotes

Test Prep

1. $\lim_{x\to 0^+} \frac{\cos x}{x} =$

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

---

2. Which of the following functions grows the fastest?

(A) $a(u) = \left(\frac{1}{2}\right)^u$ (B) $b(u) = u^{100} + u^{99}$ (C) $c(u) = 4^u$

(D) $d(u) = 200e^u$ (E) $e(u) = 3^u + u^3$

---

3. Consider the functions $f(x) = \frac{1}{x}, x \neq 0$, and $g(x) = x \sin\frac{1}{x}, x \neq 0$. Which of the following describes the behavior of f and g as $x \to 0$?

(A) $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} g(x) = 0$

(B) $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist.

(C) $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} g(x)$ does not exist.

(D) $\lim_{x\to 0} f(x)$ does not exist and $\lim_{x\to 0} g(x) = 0$

(E) $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} g(x) = 0$

4. Suppose that $g(x) = \sin^2 x\sqrt{x^6+4}$, and $\lim_{x\to\infty}\frac{g(x)}{f(x)} = 0$. Which of the following functions could be f?

(A) x (B) x^2 (C) x^3 (D) x^4 (E) $\ln x$

---

5. Which of the following statements are true for the function $f(x) = \frac{2x^3+3x+1}{2^x}$

I. $f(x)$ has a horizontal asymptote of $y = 1$
II. $f(x)$ has a horizontal asymptote of $y = 0$
III. $f(x)$ has a vertical asymptote of $x = 0$

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

---

6. Which of the following functions has both a vertical and horizontal asymptote?

(A) $f(x) = \frac{1}{1+e^{-x}}$ (B) $f(x) = \tan x$ (C) $f(x) = \frac{x}{x^2+2}$

(D) $f(x) = \frac{x}{x^2-2}$ (E) $f(x) = \frac{x^2+2}{x}$

---

7. The function $f(x) = \begin{cases} \frac{x^2+2x+3}{x^2-1}, & x \geq 0 \\ \frac{x}{e^x}, & x < 0 \end{cases}$ has which of the following asymptotes?

(A) $y = 0$ only. (B) $y = 1$ only. (C) $y = 1,\ x = 1$ only.

(D) $y = 1,\ x = \pm 1$ only. (E) $y = 0,\ y = 1,\ x = \pm 1$.

---

8. If the function $f(x) = \frac{-ax^3+bx^2+cx+d}{e^{-x}-wx^3+w}$ has a horizontal asymptote of $y = 2$ and a vertical asymptote of $x = 0$, then $w - a =$

(A) -1 (B) 0 (C) 1 (D) ∞ (E) The limit does not exist.

---

9. What are all horizontal asymptotes of the graph of $y = \frac{5+2^x}{1-2^x}$ in the xy-plane?

(A) $y = -1$ only (B) $y = 0$ only (C) $y = 5$ only

(D) $y = -1$ and $y = 0$ (E) $y = -1$ and $y = 5$

Calculus

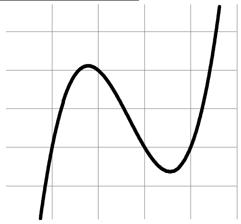
Write your questions and thoughts here!

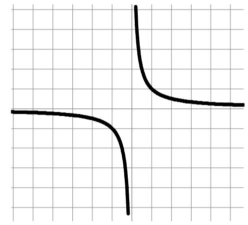
1.4 Continuity

Name:____________________________

Notes

Defining Continuity:





Formal Definition of Continuity:

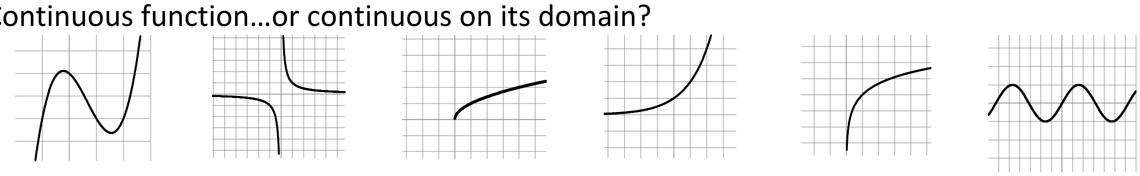
For $f(x)$ to be continuous at $x = c$, the following three conditions must be met:

1.

2.

3.

Continuous function…or continuous on its domain?



Types of Discontinuities:

1.

2.

3.

For each function identify the x value and type of each discontinuity.

1. $f(x) = \frac{x^2-8x+12}{x^2+3x-10}$	2. $f(x) = \sqrt{2x-3}$	3. $g(x) = \begin{cases} x^2 - 2x + 1, & x < -1 \\ x + 2, & -1 \le x < 2 \\ 2^x, & x \ge 2 \end{cases}$

Write your questions and thoughts here!

1.4 Continuity

Notes

Finding the Domain

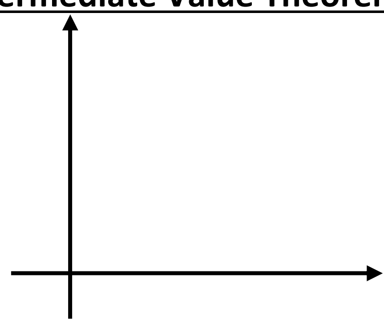
Two scenarios to watch for when looking for a **restriction** on the domain.	
1.	$f(x) = \dfrac{x-5}{x+1}$
2.	$f(x) = \sqrt{7x+3}$

Find the domain of each function.

4. $f(x) = \dfrac{3x}{x\sqrt{x+5}}$

5. $h(x) = \dfrac{5}{2-\sqrt{x}}$

Intermediate Value Theorem (for continuous functions) - IVT



6. Use the IVT to answer the following questions if $f(x) = x^3 - 2x - 5$.
   a. Find $f(1)$.

   b. Find $f(2)$.

   c. Find $f(3)$.

   d. Does the function have a zero? How do you know?

Now summarize what you learned!

1.4 Continuity

Calculus Name: ______________________________ **Practice**

Identify and classify each point of discontinuity of the given function.

1. $f(x) = \frac{x}{x+1}$

2. $f(x) = \frac{x^2}{x^2+3x}$

3. $f(x) = \frac{2x}{2x-5}$

4. $f(x) = \sqrt{2-6x}$

5. $f(x) = \frac{x+2}{x^2-2x-8}$

6. $f(x) = \frac{4x+5}{3}$

7. $f(x) = \begin{cases} 3-2x, & x < 2 \\ x-3, & x \geq 2 \end{cases}$

8. $f(x) = \begin{cases} 5x+1, & x \leq -1 \\ x+3, & x > -1 \end{cases}$

9. $f(x) = \begin{cases} \frac{x^2-1}{x+11}, & x < 4 \\ x-3, & x > 4 \\ 5, & x = 4 \end{cases}$

10. $f(x) = \begin{cases} \frac{x}{e}+3, & x < e \\ \ln x^4, & x \geq e \end{cases}$

Find the domain of each function.

11. $s(x) = \frac{\sqrt{6x-2}}{5}$

12. $w(t) = \frac{6}{\sqrt{2t+10}}$

13. $f(x) = \frac{x}{\sqrt{6-2x}}$

14. $v(t) = \frac{3t}{t\sqrt{t+7}}$

15. $g(x) = \frac{x+1}{x^2+5x+4}$

16. $g(w) = \frac{2}{4-\sqrt{w}}$

17. $s(t) = \sqrt[3]{t-8}$

18. $h(t) = \frac{\sqrt{4-t}}{t-5}$

19. $g(x) = x^2 + 11x + 30$

Below is a table of values for a continuous function f. Use this table to answer questions 20-22.

x	3	4	5	6	7
$f(x)$	4	1	-3	-1	6

20. On the interval $3 \le x \le 7$, must there be a value of x for which $f(x) = 5$? Explain.

21. On the interval $3 \le x \le 7$, ***could*** there be a value of x for which $f(x) = 7$? Explain.

22. What is the minimum number of zeros f must have on the interval $3 \le x \le 7$?

Below is a table of values for a continuous function g. Use this table to answer questions 23-26.

x	0	2	15	32	50
$g(x)$	-1	10	17	-10	8

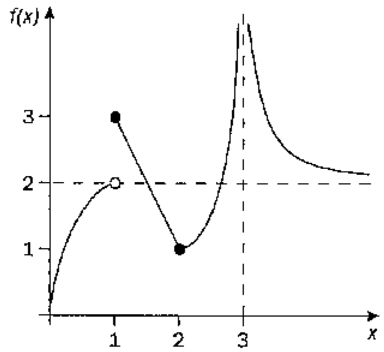
23. On the interval $0 \le x \le 15$, must there be a value of x for which $g(x) = -3$? Explain.

24. On the interval $0 \le x \le 50$, must there be a value of x for which $g(x) = 11$? Explain.

25. What is the minimum number of zeros g must have on the interval $15 \le x \le 50$?

26. What is the minimum number of zeros g must have on the interval $0 \le x \le 50$?

1. The graph of the function $f(x)$ is shown below:



Which of the following statements is true about f?

I. f is undefined at $x = 1$.

II. f is defined but not continuous at $x = 2$.

III. f is defined and continuous at $x = 3$.

(A) Only I (B) Only II (C) I and II (D) I and III (E) None of the statements are true.

---

2. Let $y = \frac{x^2+4x-21}{x^2-9}$. This function has a hole. What is the y-value of the hole?

(A) $\frac{5}{3}$ (B) 3 (C) $-\frac{10}{3}$ (D) 0 (E) -3

---

3. For which value of k is the following function continuous at $x = 4$?

$$f(x) = \begin{cases} \sin\frac{\pi}{x}, & x \le 4 \\ k\sqrt{\frac{x}{2}}, & x > 4 \end{cases}$$

(A) $k = 2$ (B) $k = 1$ (C) $k = -1$ (D) $k = \frac{1}{2}$ (E) $k = -\frac{1}{2}$

4.

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $g(x) = \frac{1}{2}$ must have at least two intersections with f in the interval $[0, 2]$ if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

---

5. For what value of k will the function $f(x) = \frac{x^2-(k+2)x+6}{x-k}$ have a point discontinuity at $x = k$?

(A) $k = -1$ (B) $k = 0$ (C) $k = 1$ (D) $k = 2$ (E) $k = 3$

---

6. Suppose f is continuous on the closed interval [0,4] and suppose $f(0) = 1, f(1) = 2, f(2) = 0,$ $f(3) = -3, f(4) = 3$. Which of the following statements about the zeros of f on [0,4] is always true?

(A) f has exactly one zero on $[0, 4]$.

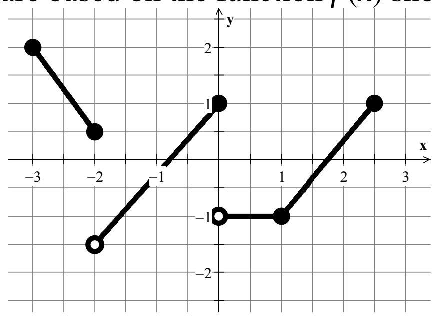
(B) f has more than one zero on $[0, 4]$.

(C) f has more than two zeros on $[0, 4]$.

(D) f has exactly two zeros on $[0, 4]$.

(E) None of the statements above is true.

Questions 7 through 9 are based on the function $f(x)$ shown in the graph below:



7. The function $f(x)$ has a removable discontinuity at:

(A) $x = -2$ only (B) $x = 0$ only (C) $x = 1$ only

(D) $x = -2$ and $x = 0$ only (E) $f(x)$ has no removable discontinuities.

---

8. On what intervals is $f(x)$ continuous?

(A) $[-3, -2] \cup [-2, 0] \cup [0, 2.5]$ (B) $[-3, -2] \cup (-2, 0] \cup [0, 2.5]$

(C) $[-3, -2] \cup (-2, 0] \cup (0, 2.5]$ (D) $[-3, -2] \cup [-2, 0] \cup (0, 2.5]$

(E) $[-3, -2] \cup (-2, 0] \cup (0, 1) \cup (1, 2.5]$

---

9. The function has a jump discontinuity at:

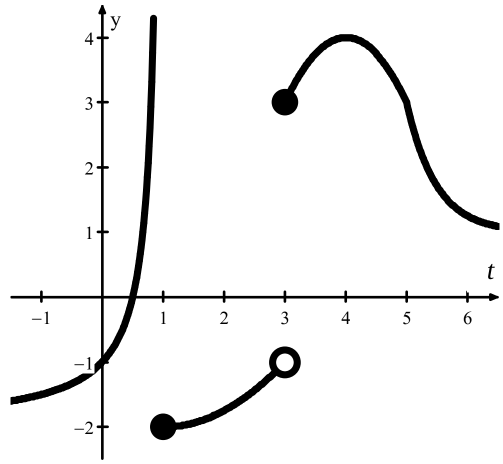
(A) $x = -2$ only (B) $x = 0$ only (C) $x = 1$ only

(D) $x = -2$ and $x = 0$ only (E) $f(x)$ has no jump discontinuities.

FREE RESPONSE ON THE BACK!

For this Free Response problem, answer each question as completely as possible. **Do NOT look at the answers until completed!** When done, use the Solution Key to grade your work. Put your score in the box below.

The graph of a function f is shown below and describes the position of a particle as it moves along the y-axis with respect to time.



a. Describe the movement of the particle on the interval $[1,3]$.

b. Assume $f(t) > 1$ for $t > 6$, and $y = 1$ is an asymptote. Describe the movement of the particle as t approaches infinity.

c. Can we use the Intermediate Value Theorem on the interval $[-1,2]$ to show that f has a zero in that interval? On the interval $[2,5]$? Explain your reasoning.