AP Calculus AB

Summer Packet

This packet is intended to prepare you for AP Calculus AB by reviewing prerequisite algebra and pre-calculus skills and covering the first chapter calculus textbook. (With the amount of curriculum to be covered before the AP Exam in May, we need to cover this review chapter over the summer.)

It is due on the first day of school and worth homework points based on completion. At the end of the first week of school, there will be a Summer Packet quiz covering this review content.

The packet is lengthy, so please start early! While many of the exercises cover basic algebra skills, you will encounter a few tough exercises. Embedded in this packet are short videos to guide you if you get stuck. If you need further assistance, please email me (stephen.kemp@district196.org) and we can schedule a meeting in-person or via Zoom.

Have a wonderful and relaxing summer! I am looking forward to delving into the study and exploration of a branch of mathematics which has been referred to as "one of the supreme accomplishments of the human intellect."

Calculus truly is a fascinating course—you will love it!

— Mr. Kemp

The following formulas and identities will help you complete this packet. You are expected to know ALL of these for the course.

LINES

Slope-intercept: $y = mx + b$

Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$

Horizontal line: $y = b$ (slope = 0)

Vertical line: $x = a$ (slope = undefined)

Parallel → same slope

Perpendicular → opposite reciprocal slopes

QUADRATICS

Standard: $y = ax^2 + bx + c$

Vertex: $y = a(x - h)^2 + k$

Intercept: $y = a(x - p)(x - q)$

Parabola opens: up if $a > 0$
down if $a < 0$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXPONENTIAL PROPERTIES

$x^a \cdot x^b = x^{a+b}$ $\qquad (xy)^a = x^a y^a$

$\frac{x^a}{x^b} = x^{a-b}$ $\qquad \sqrt[n]{x^m} = x^{m/n}$

$x^0 = 1 \ (x \neq 0)$ $\qquad \left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$

$x^{-n} = \frac{1}{x^n}$

In general, it is fine to have negative exponents in your answers!

LOGARITHMS

$y = \log_a x$ is equivalent to $a^y = x$

$\log_b(mn) = \log_b m + \log_b n$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \log_b m$

TRIGONOMETRIC IDENTITIES

$\csc x = \frac{1}{\sin x}$ $\qquad \sec x = \frac{1}{\cos x}$ $\qquad \cot x = \frac{1}{\tan x}$ $\qquad \tan x = \frac{\sin x}{\cos x}$ $\qquad \cot x = \frac{\cos x}{\sin x}$

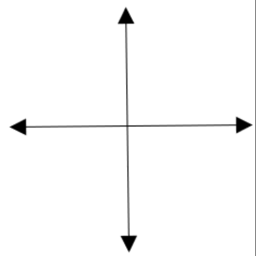
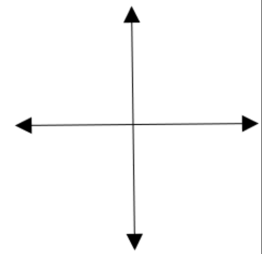
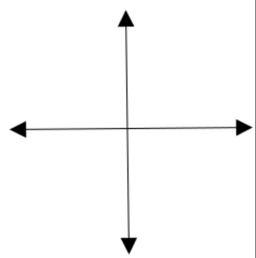
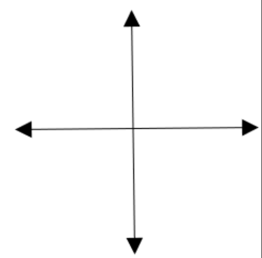
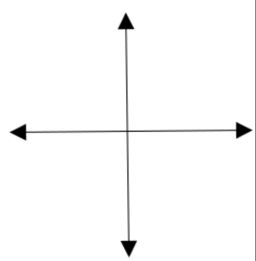
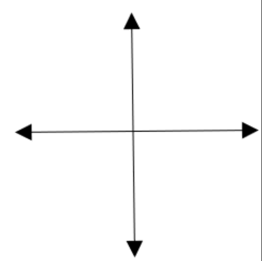
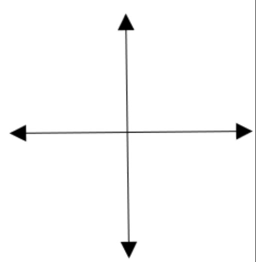
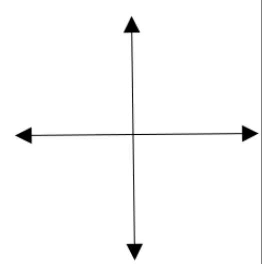
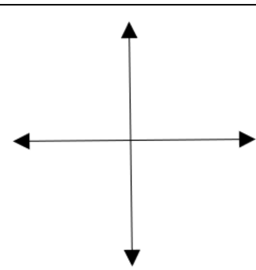
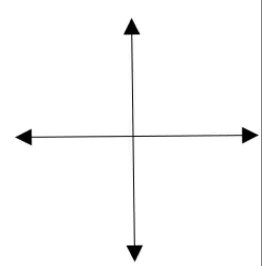
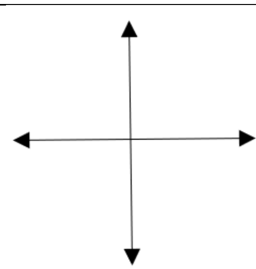
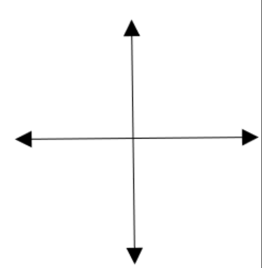
$\sin^2 x + \cos^2 x = 1$ $\qquad \tan^2 x + 1 = \sec^2 x$ $\qquad 1 + \cot^2 x = \csc^2 x$

$\sin(2x) = 2 \sin x \cos x$ $\qquad \cos(2x) = \cos^2 x - \sin^2 x$ **or** $1 - 2\sin^2 x$ **or** $2\cos^2 x - 1$

Whenever you see a video icon, click it to watch a short video about the content. For example, this video link will help you with the next page.

You are expected to know the general shape, domain, and range of each parent function in the table.

"Parent" functions mean no transformations have been applied. Transformation (shifting, stretching, compressing, or reflecting) may change the domain or range.

Linear functions (x) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$ 	**Quadratic** functions (x^2) **D:** $(-\infty, \infty)$ **R:** $[\mathbf{0}, \infty)$ 
Odd-degree polynomials (x^n for **odd** $\boldsymbol{n}$) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$ 	**Even-degree** polynomials (x^n for **even** $\boldsymbol{n}$) **D:** $(-\infty, \infty)$ **R:** $[\mathbf{0}, \infty)$ 
Exponential functions (a^x, e^x) **D:** $(-\infty, \infty)$ **R:** $(\mathbf{0}, \infty)$ Horizontal asymptote: $y = 0$ 	**Logarithmic** functions $(\log_a x, \ln x)$ **D:** $(\mathbf{0}, \infty)$ **R:** $(-\infty, \infty)$ Vertical asymptote: $x = 0$ 
Sinusoidal functions $(a \sin x, a \cos x)$ **D:** $(-\infty, \infty)$ **R:** $[-\boldsymbol{a}, \boldsymbol{a}]$ 	**Tangent** function $(\tan x)$ **D:** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{(2n+1)\pi}{2}, \frac{(2n+3)\pi}{2}\right)$ **R:** $(-\infty, \infty)$ Vertical asymptotes: $x = \frac{(2n+1)\pi}{2}$ 
Reciprocal functions $\left(\frac{1}{x}\right)$ **D:** $(-\infty, \mathbf{0}) \cup (\mathbf{0}, \infty)$ **R:** $(-\infty, \mathbf{0}) \cup (\mathbf{0}, \infty)$ Vertical asymptote: $x = 0$ Horizontal asymptote: $y = 0$ 	**Absolute value** functions $(\|x\|)$ **D:** $(-\infty, \infty)$ **R:** $[\mathbf{0}, \infty)$ 
Even root functions ($\sqrt{x}$, $\sqrt[n]{x}$ for even n) **D:** $[\mathbf{0}, \infty)$ **R:** $[\mathbf{0}, \infty)$ 	**Odd root** functions ($\sqrt[3]{x}$, $\sqrt[n]{x}$ for odd $n > 1$) **D:** $(-\infty, \infty)$ **R:** $(-\infty, \infty)$ 

For #1-8, write an equation for each line in point-slope form.

1. Containing $(4,-1)$ with a slope of $\frac{1}{2}$

____

2. Crossing the x-axis at $x=-3$ and the y-axis at $y=6$

____

3. Containing the points $(-6,-1)$ and $(3,2)$

____

4. Write an equation of a line passing through $(5,-3)$ with an undefined slope.

____

5. Write an equation of a line passing through $(-4,2)$ with a slope of 0.

____

6. Write an equation of a line passing through $(2,8)$ that is parallel to $y=\frac{5}{6}x-1$.

____

7. Write an equation of a line passing through $(4,7)$ that is perpendicular to the y-axis.

____

8. Write an equation of a line passing through $(6,-7)$ that is perpendicular to $y=-2x-5$.

____

For #9-16, solve each equation for x. Note that some equations with have a specific value, but most will have a solution in terms of other variables. (For example: $x=\frac{a+b}{c}$ may be a solution.)

9. $x^2+3x=8x-6$

____

10. $\frac{2x-5}{x+y}=3-y$

____

11. $3xy + 6x - xz = 12$

12. $A = ax + bx$

13. $cx = vx$

14. $r = t - x(z - y)$

15. $\frac{3+x}{5-x} = 6 + y$

16. $\frac{y+2}{4-x} = 4(2 - z)$

For #17-22, solve each quadratic by factoring.

This video demonstrates factoring. For the exercises below, you must factor and then solve.

17. $x^2 - 4x - 12 = 0$

18. $x^2 - 6x + 9 = 0$

19. $x^2 - 9x + 14 = 0$

20. $x^2 - 36 = 0$

____

21. $9x^2 - 1 = 0$

____

22. $4x^2 + 4x + 1 = 0$

____

For #23-27, evaluate the following knowing that $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$.

23. $f\left(\frac{1}{2}\right) =$ ______________

24. $g(-2) =$ ______________

25. $f(1) + g(0) =$ ______________

26. $f(0) \cdot g(0) =$ ______________

27. $\frac{g(-6)}{f(-6)} =$ ______________

For #28-35, use $f(x) = x^2 - 1$, $g(x) = 3x$, and $h(x) = 5 - x$ to find each composite function.

28. $f(g(x)) =$

____

29. $g(f(x)) =$

____

30. $f(f(4)) =$

____

31. $g(h(-4)) =$

____

32. $f\left(g(h(1))\right) =$

____

33. $f\big(g(x-1)\big) =$

____

34. $g\big(f(x^3)\big) =$

____

35. $\frac{f(x+h)-f(x)}{h} =$ 

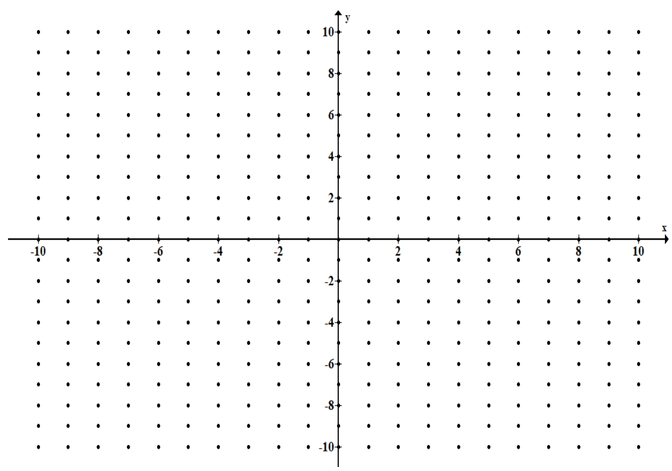
____

This is an important for calculus.
What is the name of this expression?

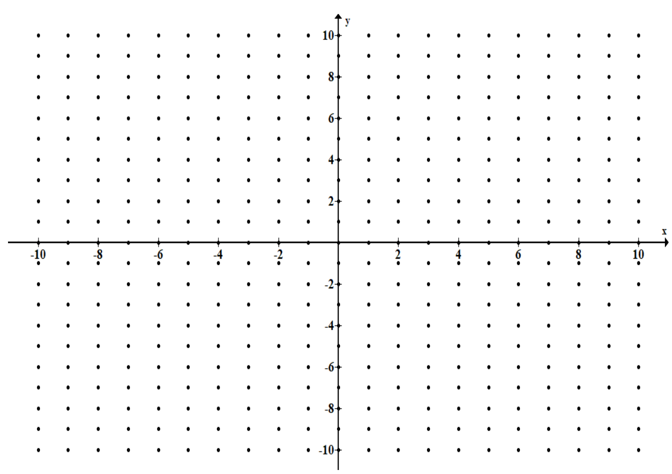
____

For #36-38, graph each piecewise function.

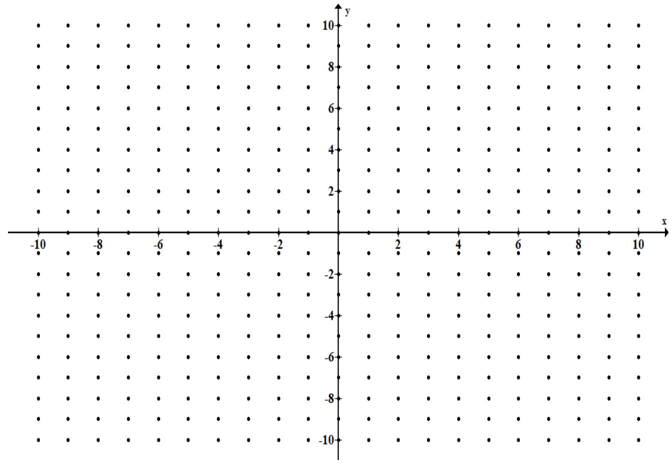
36. $f(x) = \begin{cases} x+3 & ; \ x < 0 \\ -2x+5 & ; \ x \geq 0 \end{cases}$



37. $g(x) = \begin{cases} \frac{1}{2}x & ; \ -4 \leq x \leq 2 \\ 2x-3 & ; \ x > 2 \end{cases}$



38. $h(x) = \begin{cases} |x| & ; \ x \le 1 \\ 2 - |x-2| & ; \ x > 1 \end{cases}$



For #39-43, solve each exponential equation and round answers to the nearest thousandth. Some equations can be solved by writing each side as the same base while others will require a logarithm.

39. $5^x = \frac{1}{5}$

40. $6^x = 1296$

41. $6^{2x-7} = 216$

42. $5^{3x-1} = 49$

43. $10^{x+5} = 125$

For #44-47, simplify each expression without the use of a calculator. The exponential properties on page 2 of this packet will help.

44. $e^{\ln 4} =$

45. $e^{2\ln 3} =$

46. $\ln e^9 =$

47. $5\ln e^3 =$

For #48-53, solve each exponential or logarithmic equation by hand. Round answers to the nearest thousandth.

48. $e^{x} = 34$ ____________

49. $3e^{x} = 120$ ____________

50. $e^{x} - 8 = 51$ ____________

51. $\ln x = 2.5$ ____________

52. $\ln(3x - 2) = 2.8$ ____________

53. $2\ln(e^{x}) = 5$ ____________

For #54-66, find the exact value of the expression using the Unit Circle. To be clear, "exact" answer means no decimals!

54. $\sin 120° =$ ________

55. $\cos\frac{11\pi}{6} =$ ________

56. $\tan 225° =$ ________

57. $\sin\left(-\frac{2\pi}{3}\right) =$ ________

58. $\sin 150° =$ ________

59. $\tan\frac{7\pi}{4} =$ ________

60. $\csc\left(\frac{\pi}{4}\right) =$ ________

61. $\sec(-210°) =$ ________

62. $\cot\left(\frac{5\pi}{4}\right) =$ ________

63. $\sin\left(\frac{9\pi}{4}\right) =$ ________

64. $\sec\left(-\frac{\pi}{4}\right) =$ ________

65. $\tan\left(-\frac{4\pi}{3}\right) =$ ________

66. $\cos\left(\frac{8\pi}{3}\right) =$ ________

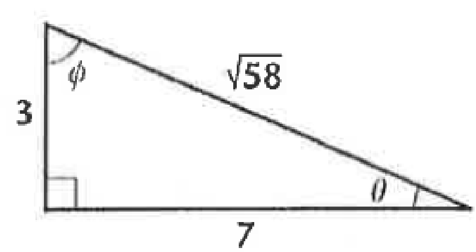
For #67-70, evaluate each trigonometric expression using the right triangle provided. You do NOT need to rationalize the denominator.

67. $\sin\theta =$ ________

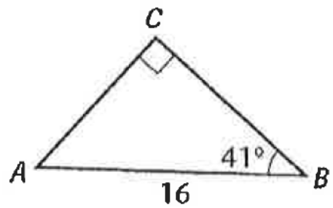
68. $\cos\theta =$ ________

69. $\tan\phi =$ ________

70. $\sec\phi =$ ________



71. Solve the triangle, rounding all angles and sides to the nearest thousandth. ("Solving a triangle" means to find all missing sides and angles.)



$m\angle A =$ ___________

$AC =$ ___________

$CB =$ ___________

For #72-79, evaluate each inverse trigonometric function using the Unit Circle. Write all answer in radians, not degrees. Do not use a calculator.

72. $\sin^{-1}\left(\frac{1}{2}\right) =$ _______

73. $\sin^{-1}(-1) =$ _______

74. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ _______

75. $\tan^{-1}(\sqrt{3}) =$ _______

76. $\tan^{-1}(-1) =$ _______

77. $\tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) =$ _______

78. $\sin\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right) =$ ______

79. $\sin^{-1}(\cos(0)) =$ ________

80. Explain how the graph of $f(x)$ and its inverse, $f^{-1}(x)$, compare.

____

____

For #81-83, find the inverse of each function.

81. $g(x) = \frac{5}{x-2}$

$g^{-1}(x) =$ ____________________

82. $f(x) = \frac{x^2}{3}$

$f^{-1}(x) =$ ____________________

83. $y = \sqrt{4-x} + 1$

$y^{-1} =$ ____________________

84. If the graph of $f(x)$ has the point $(2,7)$, then what is one point on the graph of $f^{-1}(x)$? ____________

For #85-89, write each inequality in interval notation. For example, $x > 3$ becomes $(3, \infty)$.

85. $1 < x \leq 10$ ____________________

86. $x < 0$ **or** $x \geq 4$ ____________________

87. $x \geq -2$ ____________________

88. $x \geq 4$ **and** $x > 10$ ____________________

89. $x > 5$ **or** $x < 7$ ____________________

For #90-99, find the domain and range of each function. Write answers in interval notation. Confirm your answer by graphing the function on your calculator. The parent functions on page 3 of this packet will help.

90. $f(x) = \sqrt{x+5}$

D: ____________________ R: ____________________

91. $g(x) = x^2 - 5$

D: ____________________ R: ____________________

92. $y(t) = \frac{1}{t+7}$

D: ____________________ R: ____________________

93. $h(x) = \frac{5}{x^2+1}$

D: ____________________ R: ____________________

94. $f(x) = \sqrt{x^2+5}$

D: ____________________ R: ____________________

95. $g(t) = t^3 + 2t - 7$

D: ____________________ R: ____________________

96. $h(x) = 3\sin(\pi x) - 1$

D: ____________________ R: ____________________

97. $y(x) = \sqrt[5]{2x+3}$

D: ____________________ R: ____________________

98. $f(x) = -3e^{2x} + 5$

D: ____________________ R: ____________________

99. $g(t) = \log_4(x-2) + 1$

D: ____________________ R: ____________________

For #100-102, find the difference quotient of each function. (Refer back to #35 if needed.)

100. $g(x) = x^2 - 3x$

____

101. $f(x) = \frac{2}{x+1}$

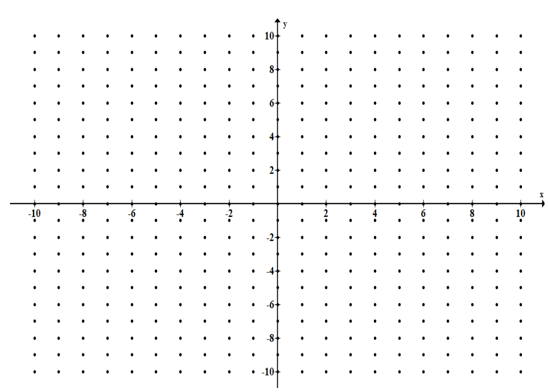
____

102. $h(x) = \sqrt{x-3}$

____

The remaining exercises are more challenging and specifically from concepts and skills covered in Pre-Calculus. You must show all work to earn credit.

103. State the domain and range of $f(x) = \frac{2x^2-6x-20}{x^3-2x^2-15x}$



D: ____________________ R: ____________________

104. Consider the function $f(x) = \frac{e^x}{\log x - x^3}$.

a. Use your calculator to find the relative maximum and minimum y-value of $f(x)$.

min = ______________ max = ________________

b. State the domain of $f(x)$ in interval notation.

D: ____________________________

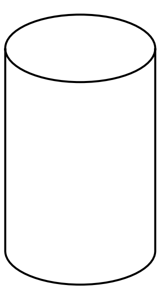
c. State when the function is increasing and decreasing. Write in interval notation.

increasing: __________________________________ decreasing: ______________________________

105. A rectangular sheet of tin measures 20 inches by 12 inches. Suppose you cut a square out of each corner and fold up the sides to make an open-topped box. What size square should you cut out in order to maximize the volume of the box? Show all work to earn credit.

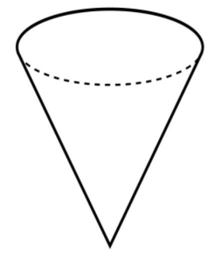
____

106. You have been asked to design a cylindrical can that will hold 1000 cubic centimeters. What dimensions (height and radius) will use the least amount of material?



$r =$ _____________ $h =$ ______________

107. An inverted conical reservoir has a height of 10 inches and a base diameter of 12 inches. It is slowly being filled with water. Write an expression for the volume of the water in terms of its…



Use similar triangles

a. radius

$V(r) =$ ____________________

b. height

$V(h) =$ ____________________

108. Evaluate the following limits algebraically.

a. $\lim\limits_{x \to 1} e^{x^3 - x} =$ ________

b. $\lim\limits_{x \to -3} \dfrac{x^2 - 9}{x^2 + 2x - 3} =$ ________

c. $\lim\limits_{x \to 5^+} \dfrac{x + 5}{x - 5} =$ ________

d. $\lim\limits_{h \to 0} \dfrac{(h - 1)^3 + 1}{h} =$ ________

e. $\lim\limits_{x \to 0} \dfrac{\sqrt{4 + x} - 2}{x} =$ ________

f. For constant c, $\lim\limits_{x \to c} x =$ ________

g. For constants a and c, $\lim_{x \to a} c =$ ________

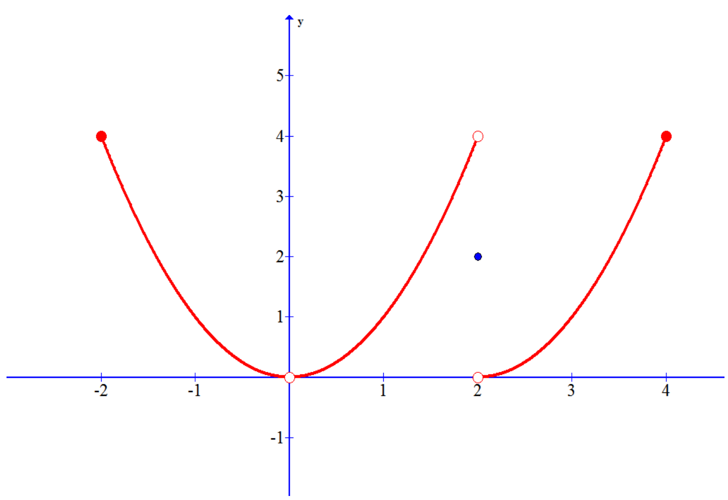
h. $\lim_{h \to 0} \dfrac{h}{\sqrt{x+h} - \sqrt{x}} =$ ________

i. $\lim_{x \to \infty} \dfrac{3x^3 + 5x^2 - 7x}{8x^3 - 13} =$ ________

j. $\lim_{v \to 4^+} \dfrac{4 - v}{|4 - v|} =$ ________

k. $\lim_{x \to \infty} \dfrac{\sqrt{x^2 - 9}}{2x - 6} =$ ________

109. Use the graph of the function to answer the following questions. Be as specific as possible



a. $f(2) =$ ________

b. $\lim_{x \to 2^+} f(x) =$ ________

c. $\lim_{x \to 2^-} f(x) =$ ________

d. $\lim_{x \to 2} f(x) =$ ________

e. $f(0) =$ ________

f. $\lim_{x \to 0} f(x) =$ ________

110. Find the instantaneous rate of change at any point x for the function $f(x) = 2x^2 - x$ using the definition of the derivative below.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

111. Estimate the area under the curve $y = x^2 + 1$ on the interval $[0,2]$ using Riemann sums with four equally-spaced subdivisions with heights determined by using:

a. **L**RAM (**left endpoint**)

b. **R**RAM (**right endpoint**)

c. **M**RAM (**midpoint**)