Calculus I Summer Work Solutions

"Four Ways to Represent a Function"

1) $f(x) = x + \sqrt{2-x}$

$g(u) = u + \sqrt{2-u}$

$\boxed{\text{yes}}$ $f = g$

3) a) $f(1) = 3$

b) $f(-1) \approx 0.75$

c) $f(x) = 1$ when $x=0$ or $x=3$

d) $f(x) = 0$ when $x \approx -0.75$

e) domain: $[-2,4]$

range: $[-1,3]$

f) increasing $(-2,1)$

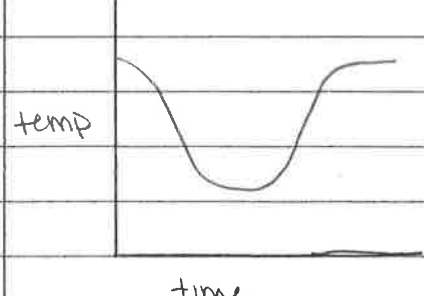
7) not a function

9) yes function

domain: $[-3,2]$

range: $[-3,-2) \cup [-1,3]$

13) When ice cubes initially get put in the temperature will begin to drop, eventually level off, then begin to rise to room temperature again



15) a) 6AM → 500 megawatts

6PM → about 725 megawatts

b) lowest between 2 and 5AM makes sense b/c people are asleep

highest between 11AM and 4PM makes sense people are up, working, and it is the hottest part of the day.

23) a) see graph paper

b) 2001 $\Rightarrow$ about 125 million

2005 $\rightarrow$ about 205 million

25) $f(x) = 3x^2 - x + 2$

$f(2) = 3(4) - (2) + 2 = 12$

$f(-2) = 3(4) - (-2) + 2 = 16$

$f(a) = 3a^2 - a + 2$

$f(-a) = 3a^2 + a + 2$

$f(a+1) = 3(a^2 + 2a + 1) - (a+1) + 2$

$= 3a^2 + 5a + 4$

$2f(a) = 2(3a^2 - a + 2)$

$= 6a^2 - 2a + 4$

$f(2a) = 3(4a^2) - 2a + 2$

$= 12a^2 - 2a + 2$

$f(a^2) = 3a^4 - a^2 + 2$

$[f(a)]^2 = (3a^2 - a + 2)^2$

$= 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4$

$= 9a^4 - 6a^3 + 13a^2 - 4a + 4$

$f(a+h) = 3(a^2 + 2ah + h^2) - (a+h) + 2$

$3a^2 + 6ah + 3h^2 - a - h + 2$

27) $f(x) = 4 + 3x - x^2$

$$\frac{f(3+h) - f(3)}{h}$$

$$= \frac{(4 + 9 + 3h - 9 - 6h - h^2) - 4}{h}$$

$$= \frac{-3h - h^2}{h} = \boxed{-3 - h}$$

29) $f(x) = \frac{1}{x}$ $\quad \frac{f(x) - f(a)}{x - a}$

$$= \frac{\frac{1}{x} - \frac{1}{a}}{x - a} = \frac{\frac{a - x}{ax}}{\frac{x - a}{1}}$$

$$= \frac{-(x-a)}{ax} \cdot \frac{1}{x - a} = \boxed{\frac{-1}{ax}}$$

31) $f(x) = \frac{x + 4}{x^2 - 9}$

domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

33) $f(t) = \sqrt[3]{2t - 1}$

since cuberoot

domain $(-\infty, \infty)$

35) $h(x) = \frac{1}{\sqrt[4]{x^2-5x}}$

$x^2-5x>0$

$x(x-5)>0$

$\begin{array}{ccccc} & 0 & & 5 & \\ + & | & - & | & + \\ ✓ & & x & & ✓ \end{array}$ domain: $(-\infty,0)\cup(5,\infty)$

37) $F(p)=\sqrt{2-\sqrt{p}}$ $(p\geq 0)$

$2-\sqrt{p}\geq 0$

$2\geq\sqrt{p}$

$4\geq p$

domain: $[0,4]$

49) see graph paper

53) $x+(y-1)^2=0$

$(y-1)^2=-x$

$y-1=-\sqrt{-x}$

$\boxed{y=1-\sqrt{-x}}$

63) $V(x)=x(12-2x)(20-2x)$

$=x(240-64x+4x^2)$

$=240x-64x^2+4x^3$

73) $f(x)=\frac{x}{x^2+1}$

$f(-x)=\frac{-x}{(-x)^2+1}=\frac{-x}{x^2+1}$

$f(-x)=f(x)\rightarrow$ EVEN

75) $f(x)=\frac{x}{x+1}$

$f(-x)=\frac{-x}{-x+1}$

NEITHER

77) $f(x)=1+3x^2-x^4$

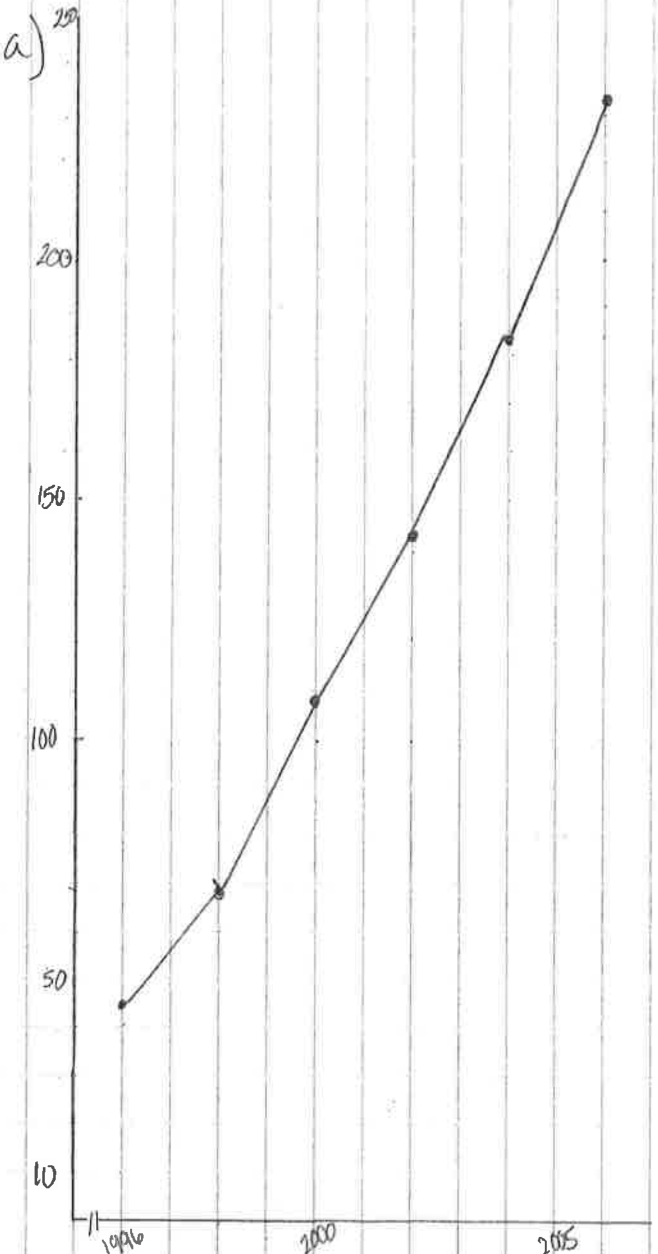
$f(-x)=1+3(-x)^2-(-x)^4$

$=1+3x^2-x^4$

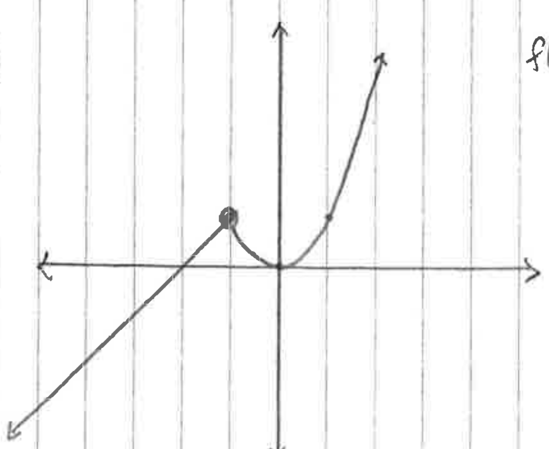
$f(-x)=f(x)\rightarrow$ EVEN

Four ways to Represent a Function

23) a)



49)



$$f(x) = \begin{cases} x+2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$$

"Mathematical Models"

1) a) logarithmic
b) root
c) rational
d) polynomial (degree 2)
e) exponential
f) trigonometric

3) a) $y = x^2 \rightarrow h$
b) $y = x^3 \rightarrow f$
c) $y = x^8 \rightarrow g$

9) $f(x) = ax^3 + bx^2 + cx + d$
roots $\rightarrow x = -1, x = 0, x = 2$
so $f(x) = x(x+1)(x+2) + d$
$= x^3 + 3x^2 + 2x + d$
$f(1) = 1 + 3 + 2 + d = 6$
$6 + d = 6$
$d = 0$
$f(x) = x^3 + 3x^2 + 2x$

11) $c = 0.0417D(a+1)$ $D = 200$
a) $c = 0.0417(200)(a+1)$
$c = 8.34a + 8.34$
$m = 8.34$ means for each year older the child is the dosage increases 8.34 mg
b) newborn $\rightarrow$ 8.34 mg

15) (113 chirps, 70°F)
(173 chirps, 80°F)
a) $\frac{80-70}{173-113} = \frac{1}{6}$
$T - 70 = \frac{1}{6}(N - 113)$
$T = \frac{N}{6} + \frac{307}{6}$

b) $\frac{1}{6}$ for each additional chirp per min the temp. increases 1/6°F
c) $T = \frac{150}{6} + \frac{307}{6}$

$T \approx 76.2°F$

19) a) due to the oscillating of the curve (data pts) a Trig function makes most sense
b) The data looks fairly linear (with a negative slope)

"New Functions from Old Functions"

1) a) $y=f(x)+3$
b) $y=f(x)-3$
c) $y=f(x-3)$
d) $y=f(x+3)$
e) $y=-f(x)$
f) $y=f(-x)$
g) $y=3f(x)$
h) $y=\frac{1}{3}f(x)$

3) a) $y=f(x-4) \rightarrow 3$
b) $y=f(x)+3 \rightarrow 1$
c) $y=\frac{1}{3}f(x) \rightarrow 4$
d) $y=-f(x+4) \rightarrow 5$
e) $y=2f(x+6) \rightarrow 2$

5) use graph paper

7) moved left 4
flipped over x-axis
moved down 1
$y=-f(x+4)-1$
$y=-\sqrt{3x+12-x^2-8x-16}-1$
$=-\sqrt{-x^2-5x-4}-1$

#9-23 see graph paper

29) $f(x)=x^3+2x^2 \quad g(x)=3x^2-1$
a) $(f+g)(x)=x^3+5x^2-1$
domain: $(-\infty,\infty)$
b) $(f-g)(x)=x^3-x^2+1$
domain: $(-\infty,\infty)$
c) $(fg)(x)=3x^5+6x^4-x^3-2x^2$
domain: $(-\infty,\infty)$
d) $(f/g)(x)=\frac{x^3+2x^2}{3x^2-1}$
domain: $(-\infty,-\frac{\sqrt{3}}{3})\cup(-\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3})\cup(\frac{\sqrt{3}}{3},\infty)$

31) $f(x)=x^2-1 \quad g(x)=2x+1$
a) $(f\circ g)(x)=f(2x+1)$
$=4x^2+4x$ domain: $(-\infty,\infty)$
b) $(g\circ f)(x)=g(x^2-1)$
$=2x^2-1$ domain: $(-\infty,\infty)$
c) $(f\circ f)(x)=f(x^2-1)$
$=x^4-2x^2$ domain: $(-\infty,\infty)$
d) $(g\circ g)(x)=g(2x+1)$
$=4x+3$ domain: $(-\infty,\infty)$

33) $f(x)=1-3x \quad g(x)=\cos x$

a) $(f\circ g)(x)=f(\cos x)$

$=1-3\cos x$ domain: $(-\infty,\infty)$

b) $(g\circ f)(x)=g(1-3x)$

$=\cos(1-3x)$ domain: $(-\infty,\infty)$

c) $(f\circ f)(x)=f(1-3x)$

$=9x-2$ domain: $(-\infty,\infty)$

d) $(g\circ g)(x)=g(\cos x)$

$=\cos(\cos x)$ domain: $(-\infty,\infty)$

35) $f(x)=x+\frac{1}{x} \quad g(x)=\frac{x+1}{x+2}$

a) $(f\circ g)(x)=f\left(\frac{x+1}{x+2}\right)=\frac{x+1}{x+2}+\frac{1}{\frac{x+1}{x+2}}$

$=\frac{x^2+2x+1}{(x+1)(x+2)}+\frac{x^2+4x+4}{(x+1)(x+2)}$

$=\frac{2x^2+6x+5}{x^2+3x+2}$

domain: $(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)$

b) $(g\circ f)(x)=g\left(x+\frac{1}{x}\right)$

$=\frac{x+\frac{1}{x}+1}{x+\frac{1}{x}+2}=\frac{x^2+x+1}{x^2+2x+1}$

domain: $(-\infty,-1)\cup(-1,0)\cup(0,\infty)$

c) $(f\circ f)(x)=f\left(x+\frac{1}{x}\right)$

$=x+\frac{1}{x}+\frac{1}{x+\frac{1}{x}}=x+\frac{1}{x}+\frac{x}{x^2+1}$

$=\frac{x^4+x^2+x^2+1+x^2}{x^3+x}=\frac{x^4+3x^2+1}{x^3+x}$

domain: $(-\infty,0)\cup(0,\infty)$

d) $(g\circ g)(x)=g\left(\frac{x+1}{x+2}\right)=\frac{\frac{x+1}{x+2}+1}{\frac{x+1}{x+2}+2}$

$=\frac{x+1+x+2}{x+1+2x+4}=\frac{2x+3}{3x+5}$

domain: $(-\infty,-2)\cup(-2,-5/3)\cup(-\frac{5}{3},\infty)$

37) $f(x)=3x-2 \quad g(x)=\sin x \quad h(x)=x^2$

$(f\circ g\circ h)(x)=f(g(x^2))$

$=f(\sin x^2)$

$=3(\sin x^2)-2$

39) $f(x)=\sqrt{x-3} \quad g(x)=x^2$

$h(x)=x^3+2$

$(f\circ g\circ h)(x)=f(g(x^3+2))$

$=f(x^6+4x^3+4)$

$=\sqrt{x^6+4x^3+1}$

41) $F(x)=(2x+x^2)^4$

$f\circ g \rightarrow f(x)=x^4$

$g(x)=2x+x^2$

43) $F(x)=\dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}}$

$f(x)=\dfrac{x}{1+x}$

$g(x)=\sqrt[3]{x}$

45) $v(t)=\sec(t^2)\tan(t^2)$

$f(t)=\sec(t)\tan(t)$

$g(t)=t^2$

47) $R(x)=\sqrt{\sqrt{x}-1}$

$f\circ g\circ h \rightarrow f(x)=\sqrt{x}$

$g(x)=x-1$

$h(x)=\sqrt{x}$

49) $H(x)=\sec^4(\sqrt{x})$

$f(x)=x^4$

$g(x)=\sec x$

$h(x)=\sqrt{x}$

51) a) $f(g(2))=f(5)=4$

b) $g(f(0))=g(0)=3$

c) $(f\circ g)(0)=f(3)=0$

d) $(g\circ f)(6)=g(6)=$ DNE

e) $(g\circ g)(-2)=g(1)=4$

f) $(f\circ f)(4)=f(2)=-2$

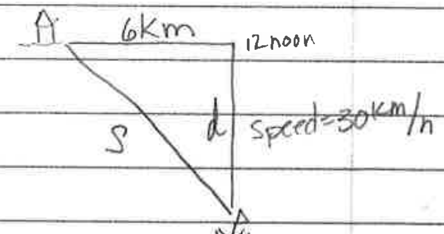
53) a) $r=60t$

b) $A=\pi r^2$

$A=\pi(60t)^2$

$=3600t^2\pi$

55)



a) $s^2=d^2+36$

$s=\sqrt{d^2+36}$

b) $d=30t$

c) $s=\sqrt{900t^2+36}$ the distance to between ship & lighthouse t hours after noon.

61) a) $g(x)=2x+1 \quad h(x)=4x^2+4x+7$

$h(x)=(4x^2+4x+1)+6$
$=(2x+1)^2+6$

So here $f(x)=x^2+6$

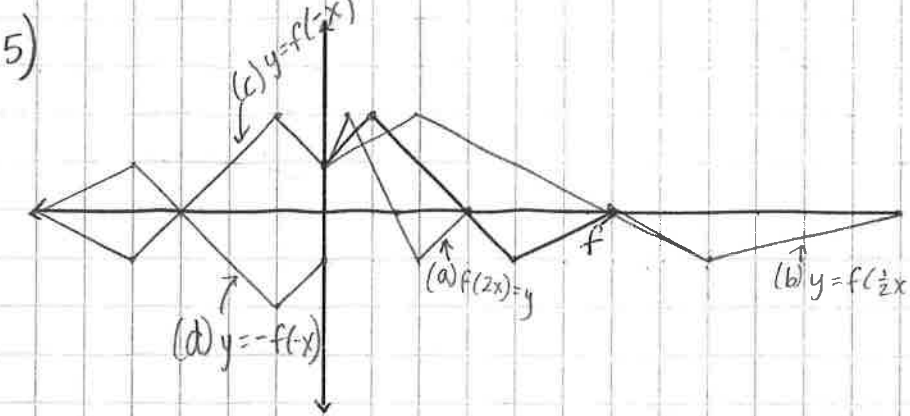
b) $f(x)=3x+5 \quad h(x)=3x^2+3x+2$

$h(x)=3(x^2+x-1)+5$

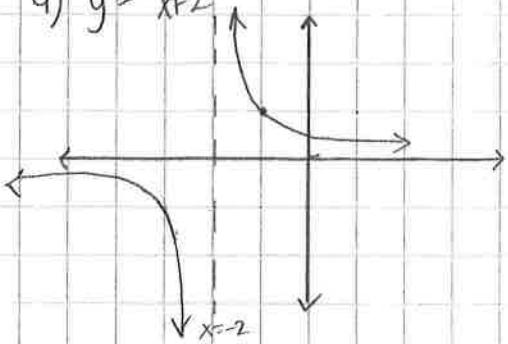
So here $g(x)=x^2+x-1$

"New Functions from Old Functions"

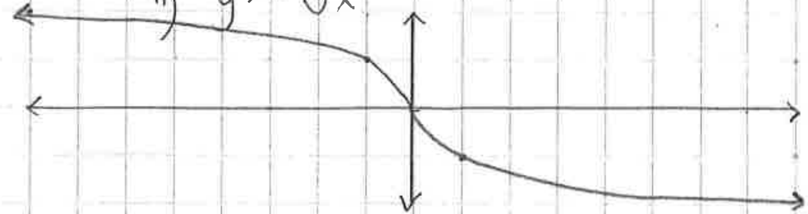
5)



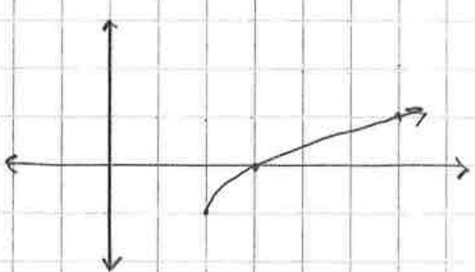
9) $y=\frac{1}{x+2}$



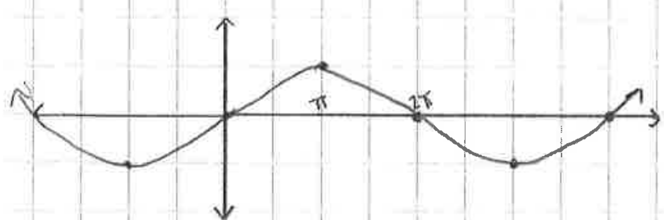
11) $y=-\sqrt[3]{x}$



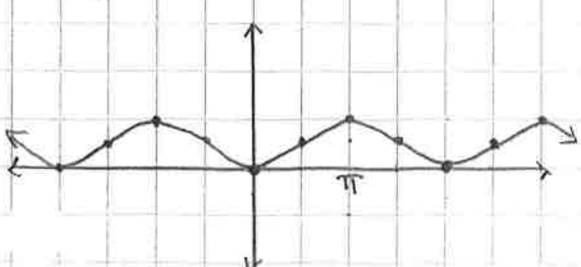
13) $y=\sqrt{x-2}-1$



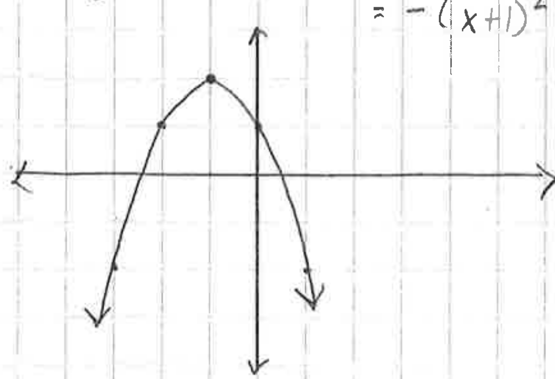
15) $y=\sin(\frac{1}{2}x)$



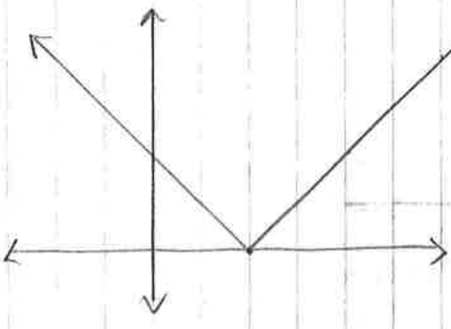
17) $y=\frac{1}{2}(1-\cos x)=\frac{1}{2}-\frac{1}{2}\cos x$



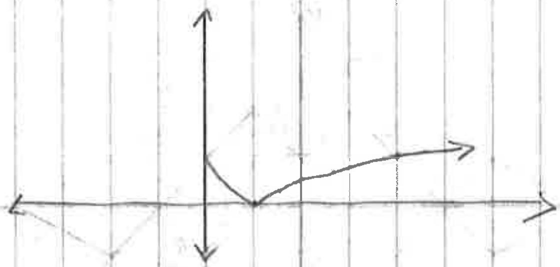
19) $y=1-2x-x^2=-(x^2+2x+1)+1+1$
$=-(x+1)^2+2$



21) $y = |x-2|$



23) $y = |\sqrt{x} - 1|$



"Limit of a Function"

5) a) $\lim_{x\to 1} f(x)=2$

b) $\lim_{x\to 3^-} f(x)=1$

c) $\lim_{x\to 3^+} f(x)=4$

d) $\lim_{x\to 3} f(x)=DNE$
b/c $\lim_{x\to 3^-} f(x)\neq \lim_{x\to 3^+} f(x)$

e) $f(3)=3$

7) a) $\lim_{t\to 0^-} g(t)=-1$

b) $\lim_{t\to 0^+} g(t)=-2$

c) $\lim_{t\to 0} g(t)=DNE$ b/c $\lim_{t\to 0^-} g(t)\neq \lim_{t\to 0^+} g(t)$

d) $\lim_{t\to 2^-} g(t)=2$

e) $\lim_{t\to 2^+} g(t)=0$

f) $\lim_{t\to 2} g(t)=DNE$ b/c $\lim_{t\to 2^-} g(t)\neq \lim_{t\to 2^+} g(t)$

g) $g(2)=1$

h) $\lim_{t\to 4} g(t)=3$

9) a) $\lim_{x\to -7} f(x)=-\infty$

b) $\lim_{x\to -3} f(x)=\infty$

c) $\lim_{x\to 0} f(x)=\infty$

d) $\lim_{x\to 6^-} f(x)=-\infty$

e) $\lim_{x\to 6^+} f(x)=\infty$

f) $x=-7, x=-3, x=0, x=6$

11) see graph paper

13) graphed on calculator

$$f(x)=\frac{1}{1+2^{1/x}}$$

a) $\lim_{x\to 0^-} f(x)=1$

b) $\lim_{x\to 0^+} f(x)=0$

c) $\lim_{x\to 0} f(x)=DNE$

15) see graph paper

25) used table on calculator

$$\lim_{x\to 1} \frac{x^6-1}{x^{10}-1}\approx .6$$

29) $\lim_{x \to -3^+} \frac{x+2}{x+3} = -\infty$

31) $\lim_{x \to 1} \frac{2-x}{(x-1)^2} = \infty$

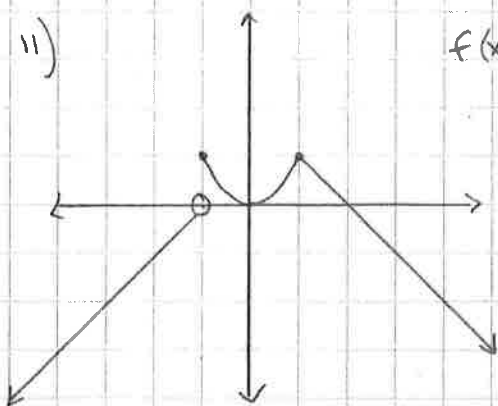
33) $\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)} = -\infty$

35) $\lim_{x \to 2\pi} x\csc x = -\infty$

37) $\lim_{x \to 2^+} \frac{x^2-2x-8}{x^2-5x+6} = -\infty$

"Limit of a Function"

11)

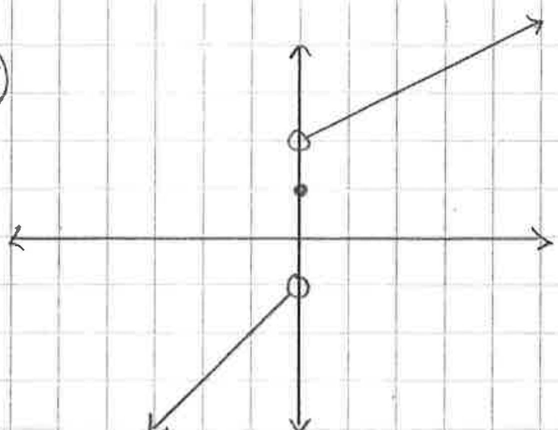


$$f(x)=\begin{cases}1+x & \text{if } x<-1\\ x^2 & \text{if } -1\le x<1\\ 2-x & \text{if } x\ge 1\end{cases}$$

$\lim_{x\to -1} f(x) = \text{DNE}$

$\lim_{x\to 1} f(x) = 1$

15)



$\lim_{x\to 0^-} f(x) = -1$ $\lim_{x\to 0^+} f(x) = 2$ $f(0) = 1$

(curve may vary but pts on y-axis should be same)

"Limit Laws"

1) $\lim_{x\to 2} f(x)=4 \quad \lim_{x\to 2} g(x)=-2$

$\lim_{x\to 2} h(x)=0$

a) $\lim_{x\to 2}[f(x)+5g(x)]$

$=\lim_{x\to 2} f(x)+\lim_{x\to 2} 5g(x)$

$= 4+5\lim_{x\to 2} g(x)$

$= 4+5(-2) = -6$

b) $\lim_{x\to 2}[g(x)]^3=[\lim_{x\to 2} g(x)]^3$

$=-8$

c) $\lim_{x\to 2}\sqrt{f(x)}=\sqrt{\lim_{x\to 2} f(x)}=2$

d) $\lim_{x\to 2}\frac{3f(x)}{g(x)}=\frac{3(\lim_{x\to 2} f(x))}{\lim_{x\to 2} g(x)}$

$=\frac{3(4)}{-2}=-6$

e) $\lim_{x\to 2}\frac{g(x)}{h(x)}=\frac{\lim_{x\to 2} g(x)}{\lim_{x\to 2} h(x)}=$ DNE

since $\lim_{x\to 2} h(x)=0$

f) $\lim_{x\to 2}\frac{g(x)h(x)}{f(x)}=\frac{(\lim_{x\to 2} g(x))(\lim_{x\to 2} h(x))}{\lim_{x\to 2} f(x)}$

$=\frac{-2(0)}{4}=0$

3) $\lim_{x\to 3}(5x^3-3x^2+x-6)$

$=\lim_{x\to 3} 5x^3-\lim_{x\to 3} 3x^2+\lim_{x\to 3} x-\lim_{x\to 3} 6$

$=5\lim_{x\to 3} x^3-3\lim_{x\to 3} x^2+\lim_{x\to 3} x-\lim_{x\to 3} 6$

$=5(\lim_{x\to 3} x)^3-3(\lim_{x\to 3} x)^2+\lim_{x\to 3} x-\lim_{x\to 3} 6$

$=5(3^3)-3(3^2)+3-6$

$=105$

5) $\lim_{t\to -2}\frac{t^4-2}{2t^2-3t+2}$

$=\frac{\lim_{t\to -2}(t^4-2)}{\lim_{t\to -2}(2t^2-3t+2)}$

$=\frac{\lim_{t\to -2} t^4-\lim_{t\to -2} 2}{\lim_{t\to -2} 2t^2-\lim_{t\to -2} 3t+\lim_{t\to -2} 2}$

$=\frac{(\lim_{t\to -2} t)^4-\lim_{t\to -2} 2}{2(\lim_{t\to -2} t)^2-3(\lim_{t\to -2} t)+\lim_{t\to -2} 2}$

$=\frac{(-2)^4-2}{2(2)^2-3(-2)+2}=\frac{14}{16}=\frac{7}{8}$

7) $\lim_{x \to 8} (1+\sqrt[3]{x})(2-6x^2+x^3)$

$= \left[\lim_{x\to 8}(1+\sqrt[3]{x})\right]\left[\lim_{x\to 8}(2-6x^2+x^3)\right]$

$= \left[\lim_{x\to 8} 1 + \lim_{x\to 8}\sqrt[3]{x}\right]\left[\lim_{x\to 8} 2 - \lim_{x\to 8} 6x^2 + \lim_{x\to 8} x^3\right]$

$= \left[\lim_{x\to 8} 1 + \sqrt[3]{\lim_{x\to 8} x}\right]\left[\lim_{x\to 8} 2 - 6(\lim_{x\to 8} x)^2 + (\lim_{x\to 8} x)^3\right]$

$= (1+\sqrt[3]{8})\left[2-6(8)^2+(8)^3\right]$

$= (1+2)(2-384+512)$

$= 3(130)$

$= 390$

9) $\lim_{x\to 2} \sqrt{\dfrac{2x^2+1}{3x-2}}$

$= \sqrt{\lim_{x\to 2}\left(\dfrac{2x^2+1}{3x-2}\right)}$

$= \sqrt{\dfrac{\lim_{x\to 2}(2x^2+1)}{\lim_{x\to 2}(3x-2)}}$

$= \sqrt{\dfrac{\lim_{x\to 2} 2x^2 + \lim_{x\to 2} 1}{\lim_{x\to 2} 3x - \lim_{x\to 2} 2}}$

$= \sqrt{\dfrac{2(\lim_{x\to 2} x)^2 + \lim_{x\to 2} 1}{3(\lim_{x\to 2} x) - \lim_{x\to 2} 2}}$

$= \sqrt{\dfrac{2(2)^2+1}{3(2)-2}} = \sqrt{\dfrac{8+1}{6-2}} = \sqrt{\dfrac{9}{4}} = \dfrac{3}{2}$

13) $\lim_{x\to 5} \dfrac{x^2-5x+6}{x-5} = \text{DNE}$

17) $\lim_{h\to 0} \dfrac{(-5+h)^2-25}{h}$

$= \lim_{h\to 0} \dfrac{25-10h+h^2-25}{h}$

$= \lim_{h\to 0} -10+h = -10$

23) $\lim_{x\to -4} \dfrac{\frac{1}{4}+\frac{1}{x}}{4+x}$

$= \lim_{x\to -4} \dfrac{\frac{x+4}{4x}}{\frac{4+x}{1}}$

$= \lim_{x\to -4} \left[\dfrac{x+4}{4x}\cdot\dfrac{1}{4+x}\right]$

$= \lim_{x\to -4} \dfrac{1}{4x} = -1/16$

25) $\lim_{t\to 0} \left(\dfrac{\sqrt{1+t}-\sqrt{1-t}}{t}\right)\left(\dfrac{\sqrt{1+t}+\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}\right)$

$= \lim_{t\to 0} \dfrac{1+t-(1-t)}{t(\sqrt{1+t}+\sqrt{1-t})}$

$= \lim_{t\to 0} \dfrac{2t}{t(\sqrt{1+t}+\sqrt{1-t})} = \lim_{t\to 0} \dfrac{2}{\sqrt{1+t}+\sqrt{1-t}}$

$= \dfrac{2}{1+1} = 1$

31) $\lim_{h\to 0} \dfrac{(x+h)^3-x^3}{h} = \lim_{h\to 0} \dfrac{x^3+3x^2h+3xh^2+h^3-x^3}{h} = \lim_{h\to 0} 3x^2+3xh+h^2$

$= 3x^2$

"Review"

1) a) a relation mapping each input to exactly one out put
domain: inputs
range: outputs
b) visual depiction in co-ordinate plane
c) Vertical Line test

3) a) even: $f(-x) = f(x)$
from graph → symmetric about y-axis
examples: $y = x^2$
$y = \cos x$
$y = \sqrt{1-x^2}$
b) odd: $f(-x) = -f(x)$
from graph → symmetric about origin
examples: $y = 1/x$
$y = x^3$
$y = \sin x$

4) as x increases so does $f(x)$

8) see graph paper

9) a) $A \cap B$ (intersection)
b) $A \cap B$
c) $A \cap B$ where $g(x) \neq 0$

11) a) $y = f(x) + 2$ b) $y = f(x) - 2$
c) $y = f(x-2)$ d) $y = f(x+2)$
e) $y = -f(x)$ f) $y = f(-x)$
g) $y = 2f(x)$ h) $y = \frac{1}{2} f(x)$
i) $y = f(\frac{1}{2}x)$ j) $y = f(2x)$

1) a) $f(2) \approx 3.8$
b) $f(x) = 3$ when $x \approx 2.25$
$x \approx 5.5$
c) $[-6, 6]$
d) $[-4, 4]$
e) increasing $(-4, 4)$
f) odd → symmetric about origin

2) a) not function
b) yes function
domain: $[-3, 3]$
range: $[-2, 3]$

3) $f(x) = x^2 - 2x + 3$

$\frac{f(a+h) - f(a)}{h}$

$= \frac{a^2 + 2ah + h^2 - 2a - 2h + 3 - (a^2 - 2a + 3)}{h}$

$= \frac{2ah + h^2 - 2h}{h} = 2a + h - 2$

5) $f(x) = \frac{2}{3x-1}$

domain: $(-\infty, 1/3) \cup (\frac{1}{3}, \infty)$

7) $y = 1 + \sin x$

domain: $(-\infty, \infty)$

#11-15) see graph paper

9) a) $y = f(x) + 8$ moved up 8

b) $y = f(x+8)$ moved left 8

c) $y = 1 + 2f(x)$ stretched vertically facter of 2 moved up 1

d) $y = f(x-2) - 2$ moved right 2 and down 2

e) $y = -f(x)$ reflected over x-axis

f(x) $y = 3 - f(x)$ reflected over x-axis moved up 3

17) a) $f(x) = 2x^3 - 3x^2 + 2$

$f(-x) = -2x^3 - 3x^2 + 2$

Neither

b) $f(x) = x^3 - x^7$

$f(-x) = -x^3 + x^7 = -(x^3 - x^7)$

odd

c) $f(x) = \cos(x^2)$

$f(-x) = \cos[(-x)^2] = \cos(x^2)$

even

d) $f(x) = 1 + \sin x$

$f(-x) = 1 + \sin(-x) = 1 - \sin x$

neither

19) $f(x) = \sqrt{x}$ $g(x) = \sin x$

a) $(f \circ g)(x) = \sqrt{\sin x}$

domain: $\ldots(-4\pi, -3\pi) \cup (-2\pi, -\pi) \cup (0, \pi) \cup (2\pi, 3\pi) \cup \ldots$

b) $(g \circ f)(x) = \sin\sqrt{x}$

domain: $[0, \infty)$

c) $(f \circ f)(x) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$

domain: $[0, \infty)$

d) $(g \circ g)(x) = \sin(\sin x)$

domain: $(-\infty, \infty)$

23) a) (i) $\lim_{x \to 2^+} f(x) = 3$

(ii) $\lim_{x \to -3^-} f(x) = -2$

(iii) $\lim_{x \to -3} f(x) = \text{DNE}$

from left & Rt different

(iv) $\lim_{x \to 4} f(x) = 2$

(v) $\lim_{x \to 0} f(x) = \infty$

(vi) $\lim_{x \to 2^-} f(x) = -\infty$

b) $x = 0$

c) $x = -3$ (jumps)

$x = 0$ (vert. asym)

$x = 2$ (jumps)

$x = 4$ (hole)

25) $\lim_{x \to 0} \cos(x + \sin x)$

$= 1$

27) $\lim_{x \to -3} \dfrac{x^2 - 9}{x^2 + 2x - 3} = 0$

29) $\lim_{h \to 0} \dfrac{(h-1)^3 + 1}{h}$

$= \lim_{h \to 0} \dfrac{h^3 - 3h^2 + 3h - 1 + 1}{h}$

$= \lim_{h \to 0} h^2 - 3h + 3 = 3$

31) $\lim_{r \to 9} \dfrac{\sqrt{r}}{(r-9)^4} = \infty$

used graph

33) $\lim_{u \to 1} \dfrac{u^4 - 1}{u^3 + 5u^2 - 6u} = \lim_{u \to 1} \dfrac{(u-1)(u^3 + u^2 + u + 1)}{u(u-1)(u+6)}$

$= \lim_{u \to 1} \dfrac{u^3 + u^2 + u + 1}{u(u+6)} = \dfrac{4}{7}$

35) $\lim_{s \to 16} \frac{4 - \sqrt{s}}{s - 16}$

$= \lim_{s \to 16} \frac{-(\sqrt{s} - 4)}{(\sqrt{s} - 4)(\sqrt{s} + 4)}$

$= \lim_{s \to 16} \frac{-1}{\sqrt{s} + 4} = \frac{-1}{8}$

37) $\lim_{x \to 0} \left[\left(\frac{1 - \sqrt{1 - x^2}}{x} \right) \left(\frac{1 + \sqrt{1 - x^2}}{1 + \sqrt{1 - x^2}} \right) \right]$

$= \lim_{x \to 0} \frac{1 - 1 + x^2}{x(1 + \sqrt{1 - x^2})} = \lim_{x \to 0} \frac{x}{1 + \sqrt{1 - x^2}} = 0$

45) a) used graph to find

(i) $\lim_{x \to 0^+} f(x) = 3$ (ii) $\lim_{x \to 0^-} f(x) = 0$

(iii) $\lim_{x \to 0} f(x) = \text{DNE}$ (iv) $\lim_{x \to 3^+} f(x) = 0$

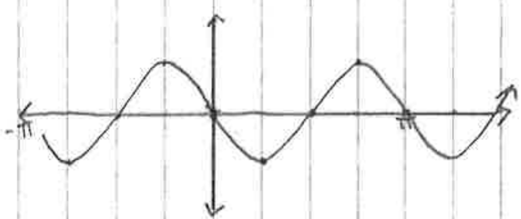
(v) $\lim_{x \to 3^-} f(x) = 0$ (vi) $\lim_{x \to 3} f(x) = 0$

b) $x = 0$

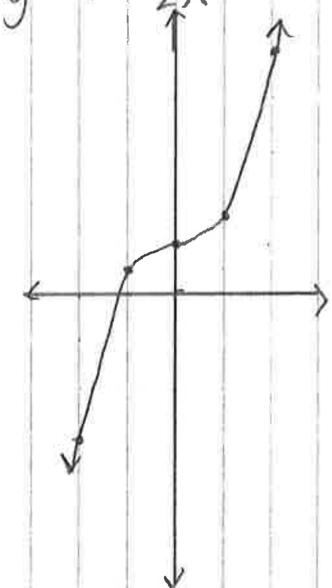
c) see graph paper

"Review"

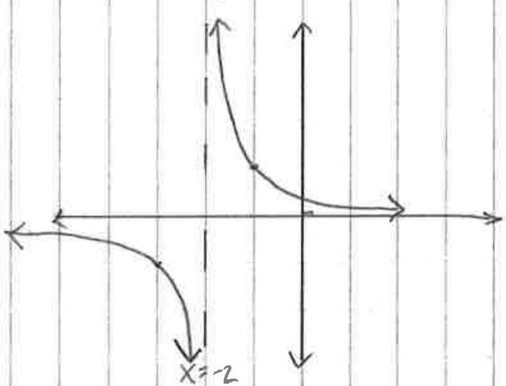
11) $y = -\sin 2x$



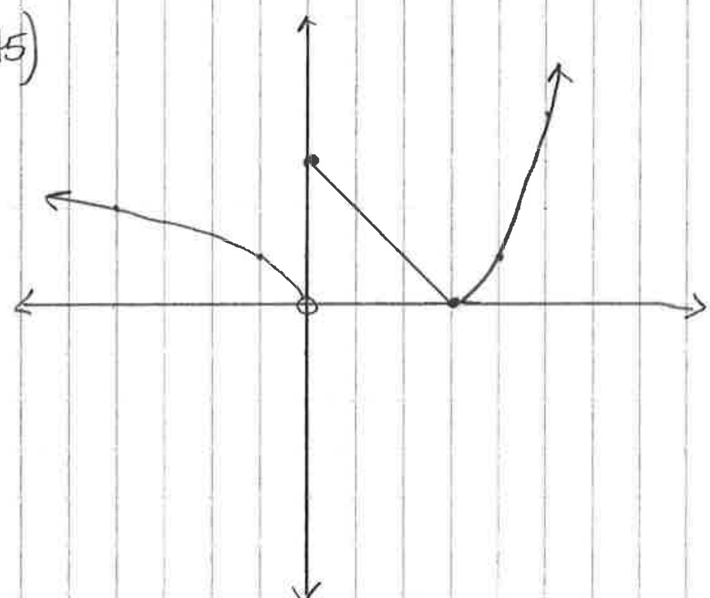
13) $y = 1 + \frac{1}{2}x^3$



15) $f(x) = \frac{1}{x+2}$



45)



"Diagnostic Tests"

"Test A"

1) a) $(-3)^4 = 81$

b) $-3^4 = -81$

c) $3^{-4} = 1/81$

d) $5^{23}/5^{21} = 25$

e) $(2/3)^{-2} = 9/4$

f) $16^{-1/4} = 1/2$

2) a) $\sqrt{200} - \sqrt{32} = 10\sqrt{2} - 4\sqrt{2} = 6\sqrt{2}$

b) $(3a^3b^3)(4ab^2)^2 = 48a^5b^7$

c) $\left(\frac{3x^{3/2}y^3}{x^2y^{1/2}}\right)^{-2} = \left(\frac{3y^{5/2}}{x^{1/2}}\right)^{-2} = \frac{x}{9y^5}$

3) a) $3(x+6) + 4(2x-5)$
$= 11x - 2$

b) $(x+3)(4x-5)$
$= 4x^2 + 7x - 15$

c) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$
$= a - b$

d) $(2x+3)^2 = 4x^2 + 12x + 9$

e) $(x+2)^3 = x^3 + 6x^2 + 12x + 8$

4) a) $4x^2 - 25 = (2x+5)(2x-5)$

b) $2x^2 + 5x - 12 = (2x-3)(x+4)$

c) $x^3 - 3x^2 - 4x + 12$
$= (x+2)(x-2)(x-3)$

d) $x^4 + 27x = x(x+3)(x^2 - 3x + 9)$

e) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2} = 3x^{-1/2}(x-2)(x-1)$

f) $x^3y - 4xy = xy(x-2)(x+2)$

5) a) $\frac{x^2+3x+2}{x^2-x-2} = \frac{x+2}{x-2}$

b) $\frac{2x^2-x-1}{x^2-9} \cdot \frac{x+3}{2x+1} = \frac{x-1}{x+3}$

c) $\frac{x^2}{x^2-4} - \frac{x+1}{x+2} = \frac{1}{x-2}$

d) $\frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}} = -(y+x)$

6) a) $\frac{\sqrt{10}}{\sqrt{5} - 2} = 5\sqrt{2} + 2\sqrt{10}$

b) $\frac{\sqrt{4+h} - 2}{h}$ = Rationalized

7) a) x^2+x+1

$=(x^2+x+1/4)+3/4$

$=(x+1/2)^2+3/4$

b) $2x^2-12x+11$

$=2(x^2-6x+9)+11-18$

$=2(x-3)^2-7$

8) a) $x+5=14-\frac{1}{2}x$

$x=6$

b) $\frac{2x}{x+1}=\frac{2x-1}{x}$ $x=1$

c) $x^2-x-12=0$

$x=4$ $x=-3$

d) $2x^2+4x+1=0$

$x=\frac{-2\pm\sqrt{2}}{2}$

e) $x^4-3x^2+2=0$

$x=\pm 1$ $x=\pm\sqrt{2}$

f) $3|x-4|=10$ $x=22/3$ $x=2/3$

g) $2x(4-x)^{-1/2}-3\sqrt{4-x}=0$

$x=12/5$

9) a) $-4<5-3x\leq 17$

$[-4,3)$

b) $x^2<2x+8$

$(-2,4)$

c) $x(x-1)(x+2)>0$

$(-2,0)\cup(1,\infty)$

d) $|x-4|<3$

$(1,7)$

e) $\frac{2x-3}{x+1}\leq 1$

$(-1,4]$

10) a) false

b) true

c) false

d) false

e) false

f) true

"Test B"

1) a) $y+5=-3(x-2)$
b) $y=-5$
c) $x=2$
d) $y+5=\frac{1}{2}(x-2)$

2) radius $=\sqrt{(4+2)^2+(-1-3)^2}$
$=2\sqrt{13}$
$(x+1)^2+(y-4)^2=52$

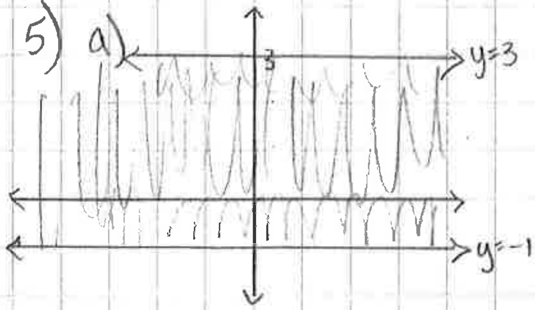
3) $x^2+y^2-6x+10y+9=0$
$(x^2-6x+9)+(y^2+10y+25)=-9+9+25$
$(x-3)^2+(y+5)^2=25$
center $(3,-5)$ $r=5$

4) a) $m=\frac{4+12}{-7-5}=\frac{16}{-12}=-4/3$
b) $y-4=-4/3(x-5)$
$(0, 32/3)$ $(8,0)$
c) $\left(\frac{-7+5}{2}, \frac{4-12}{2}\right)=(-1,-4)$
d) $AB=\sqrt{(4+12)^2+(-7-5)^2}$
$=20$
e) $y+4=3/4(x+1)$
f) $(x+1)^2+(y+4)^2=100$

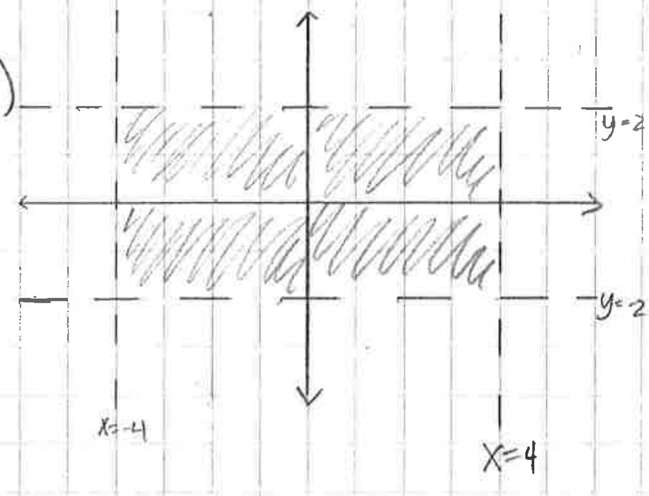
5) see graph paper

"Test B"

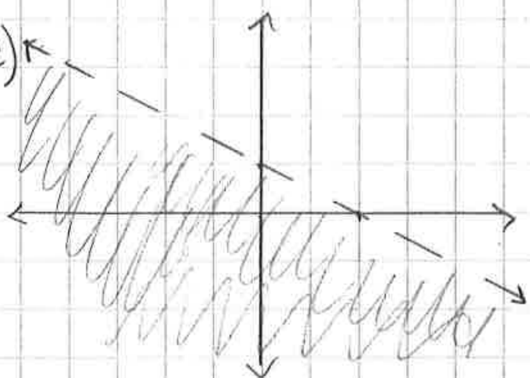
5) a)



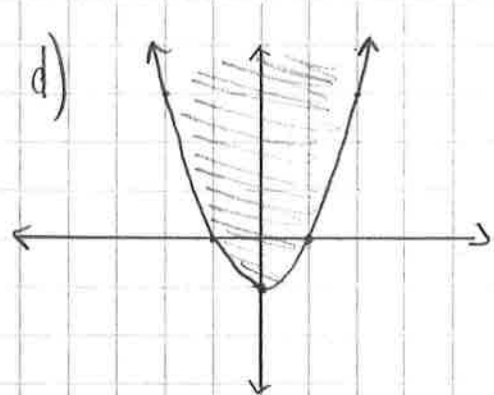
b)



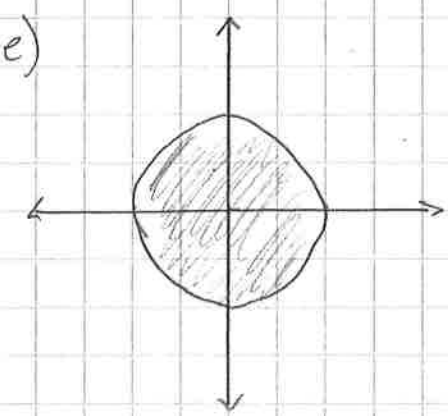
c)



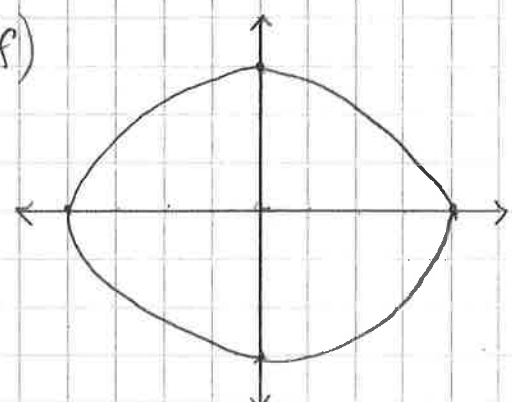
d)



e)



(f)



"Test C"

1) a) $f(-1) = -2$
b) $f(2) \approx 2.9$
c) $f(x) = 2$ when $x = -2$
$x = 1$
d) $f(x) = 0$ when $x \approx -2.5$
$x \approx 0.4$
e) domain: $[-3, 3]$
range: $[-2, 3]$

2) $f(x) = x^3$ $\quad \frac{f(2+h) - f(2)}{h}$

$$\frac{(2+h)^3 - 8}{h} = \frac{8 + 12h + 6h^2 + h^3 - 8}{h}$$

$$= 12 + 6h + h^2$$

3) a) $f(x) = \frac{2x+1}{x^2+x-2}$
domain: $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$
b) $g(x) = \frac{\sqrt[3]{x}}{x^2+1}$ domain $(-\infty, \infty)$
c) $h(x) = \sqrt{4-x} + \sqrt{x^2-1}$
domain: $(-\infty, -1]$

4) a) $y = -f(x)$ reflected over x-axis
b) $y = 2f(x) - 1$ vertically stretched by factor of 2 moved down 1
c) $y = f(x-3) + 2$ moved right 3 up 2

5) see graph paper

6) $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

a) $f(-2) = 1 - 4 = -3$
$f(1) = 2 + 1 = 3$
b) see graph paper

7) $f(x) = x^2 + 2x - 1$ $\quad g(x) = 2x - 3$
a) $(f \circ g)(x) = f(2x-3)$
$= 4x^2 - 12x + 9 + 4x - 6 - 1$
$= 4x^2 - 8x + 2$
b) $(g \circ f)(x) = g(x^2 + 2x - 1)$
$= 2x^2 + 4x - 2 - 3$
$= 2x^2 + 4x - 5$

7c) $(g \circ g \circ g)(x) = g(g(2x-3))$
$= g(4x - 6 - 3)$
$= g(4x - 9)$
$= 8x - 18 - 3$
$= 8x - 21$

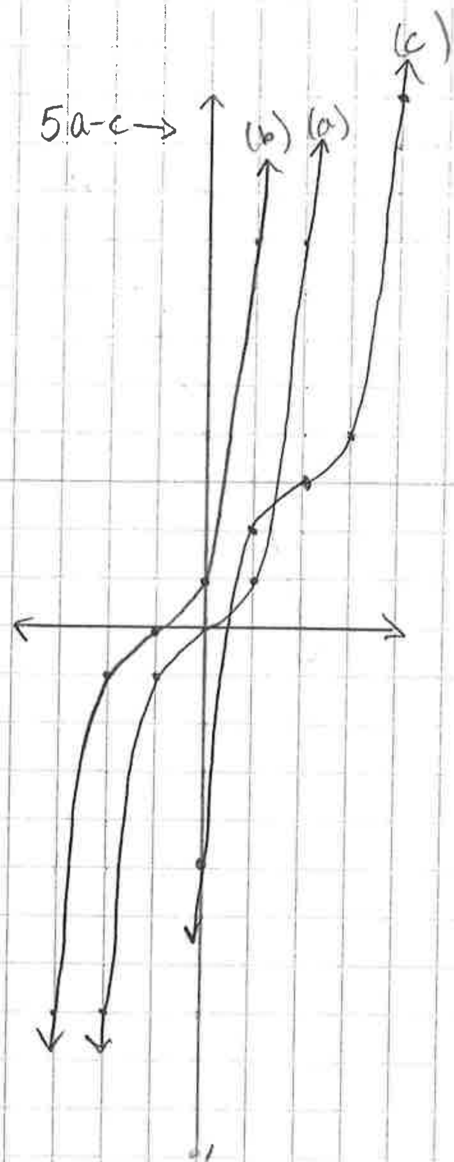
"Test C"

5) a) $y = x^3$

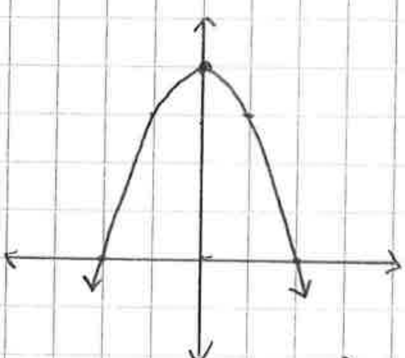
b) $y = (x+1)^3$

c) $y = (x-2)^3 + 3$

5a-c →

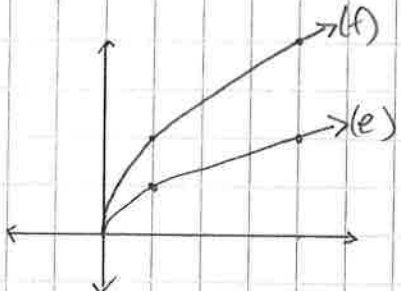


d) $y = 4 - x^2$

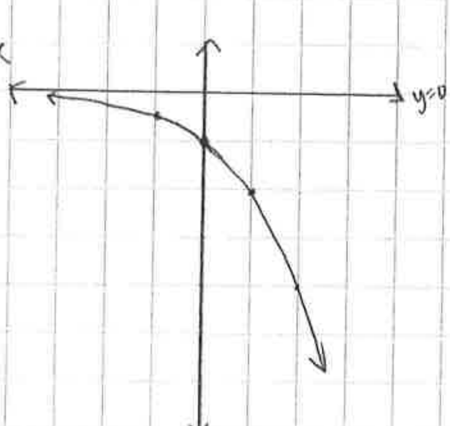


e) $y = \sqrt{x}$

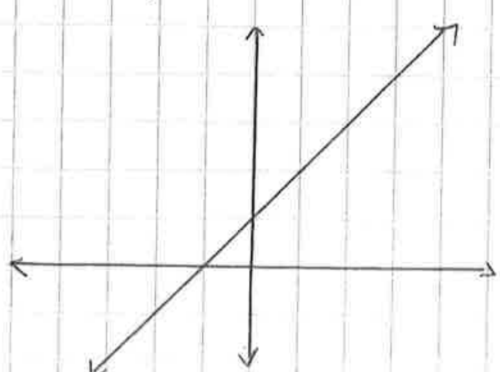
f) $y = 2\sqrt{x}$



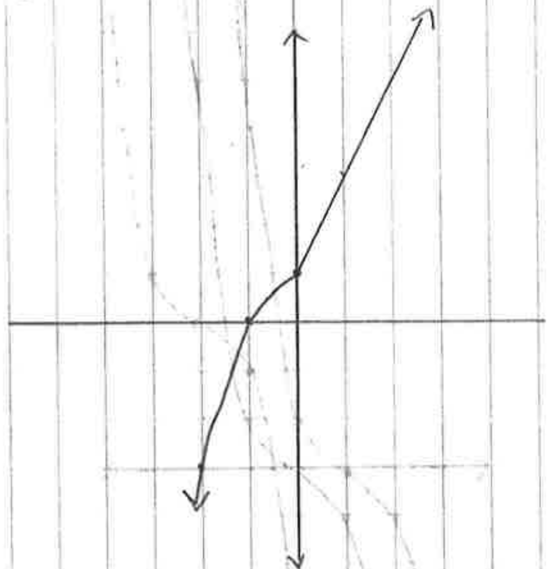
(g) $y = -2^x$



(h) $y = 1 + x$



6) b) $f(x) = \begin{cases} 1 - x^2 & \text{if } x \le 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$



"Test D"

1) a) $300° \cdot \frac{\pi}{180°} = \frac{5\pi}{3}$

b) $-18° \cdot \frac{\pi}{180°} = \frac{-\pi}{10}$

2) a) $\frac{5\pi}{6} \cdot \frac{180°}{\pi} = 150°$

b) $2 \cdot \frac{180°}{\pi} = \frac{360°}{\pi} = 114.59°$

3) $S = \theta r$

$S = \left(30° \cdot \frac{\pi}{180°}\right)(12) = 2\pi$

4) a) $\tan \pi/3 = \sqrt{3}$

b) $\sin 7\pi/6 = -\sin \pi/6 = -1/2$

c) $\sec 5\pi/3 = \sec \pi/3 = 2$

5) $\sin\theta = \frac{a}{24}$ $\quad \cos\theta = \frac{b}{24}$

$a = 24\sin\theta$ $\quad b = 24\cos\theta$

6) $\sin x = \frac{1}{3}$ $\quad \sec y = 5/4$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$= \left(\frac{1}{3}\right)(4/5) + \left(\frac{2\sqrt{2}}{3}\right)(3/5)$

$= 4/15 + \frac{6\sqrt{2}}{15} = \frac{2}{15}(2+3\sqrt{2})$

7) a) $\tan\theta \sin\theta + \cos\theta = \sec\theta$

$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta} = \sec\theta$

$\frac{1}{\cos\theta} = \sec\theta$

b) $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$

$\frac{2\frac{\sin x}{\cos x}}{1+\frac{\sin^2 x}{\cos^2 x}} =$

$\frac{2\sin x \cos x}{\cos^2 x + \sin^2 x} =$

$2\sin x \cos x = \sin 2x$

8) $\sin 2x = \sin x$

$2\sin x \cos x = \sin x$

$2\sin x \cos x - \sin x = 0$

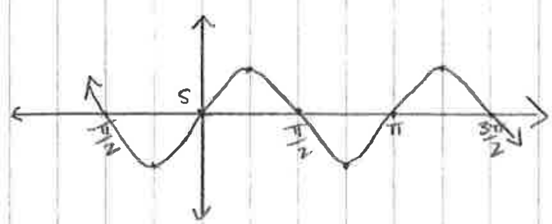
$\sin x(2\cos x - 1) = 0$

$\sin x = 0$ $\quad \cos x = 1/2$

$x = 0, \pi, 2\pi$ $\quad x = \frac{\pi}{3}, \frac{5\pi}{3}$

9) see graph paper

9) $y = 1 + \sin 2x$



$a = 1$
$b = 2$ period $= \pi$ Qtr $= \pi/4$
$c = 0$ start $x = 0$
$d = 1$

Start: $(0, 0)$
Q1: $(\pi/4, 1)$
Q2: $(\pi/2, 0)$
Q3: $(3\pi/4, -1)$
Q4: $(\pi, 0)$

Conic Sections: Parabolas

16. $y^2 - 4y - 4x = 0$
$y^2 - 4y + 4 = 4x + 4$
$(y-2)^2 = 4(x+1)$
$\frac{1}{4}(y-2)^2 = x+1$
$\frac{1}{4}(y-2)^2 - 1 = x$

vertex $(-1, 2)$
focus $(0, 2)$
directrix $x = -2$

22. $x^2 - 2x + 8y + 9 = 0$
$x^2 - 2x + 1 = -8y - 8$
$(x-1)^2 = -8(y+1)$
$-\frac{1}{4(2)}(x-1)^2 - 1 = y$

vertex $(1, -1)$
focus $(1, -3)$
directrix $y = 1$

38. $x = -\frac{1}{4p}(y-3)^2 + 5$
$4.5 = -\frac{1}{4p}(4-3)^2 + 5$
$-\frac{1}{2} = -\frac{1}{4p}$

$2 = 4p$
$p = 1/2$

$\boxed{x = -\frac{1}{2}(y-3)^2 + 5}$ or
$-2(x-5) = (y-3)^2$

40. vertex $(3, -3)$ focus $(3, -\frac{9}{4})$
$p = \left|-3 + \frac{9}{4}\right| = \frac{3}{4}$

$\boxed{y = \frac{1}{3}(x-3)^2 - 3}$ or

$3(y+3) = (x-3)^2$

42. vertex $(-1, 2)$ focus $(-1, 0)$
$p = -2$

$\boxed{y = -\frac{1}{8}(x+1)^2 + 2}$ or

$-8(y-2) = (x+1)^2$

44. vertex $(-2, 1)$ directrix $x = 1$

$\boxed{x = -\frac{1}{12}(y-1)^2 - 2}$ or

$-12(x+2) = (y-1)^2$

46. focus $(0, 0)$ directrix $y = 4$
$p = -2$ vertex $(0, 2)$
$\boxed{y = -\frac{1}{8}x^2 + 2}$ or
$-8(y-2) = x^2$

56. $y = -\frac{1}{4p}x^2$

a)

$-0.4 = -\frac{1}{4p}(16)^2$

$-\frac{2}{5} = -\frac{1}{4p}(256)$

$\frac{1}{5} = \frac{32}{p}$

$p = 160$

$\boxed{y = -\frac{1}{640}x^2}$ or $-640y = x^2$

b) $-0.1 = -\frac{1}{640}x^2$

$64 = x^2$

$x = \pm 8$

$\boxed{\text{8 ft from center}}$

Conic Sections: Ellipses

12. $\frac{(x+2)^2}{1} + \frac{(y+4)^2}{1/4} = 1$

center $(-2,-4)$

vertices $(-1,-4)$ and $(-3,-4)$

foci $(-2 \pm \sqrt{3}/2, -4)$

$c^2 = 1^2 - (\frac{1}{2})^2$

$c^2 = 3/4$

$c = \pm \sqrt{3}/2$

18. $x^2 + 4y^2 - 6x + 20y - 2 = 0$

a) $x^2 - 6x + 9 + 4(y^2 + 5y + \frac{25}{4}) = 2 + 9 + 25$

$(x-3)^2 + 4(y + 5/2)^2 = 36$

$\boxed{\frac{(x-3)^2}{36} + \frac{(y+5/2)^2}{9} = 1}$

b) center $(3, -5/2)$

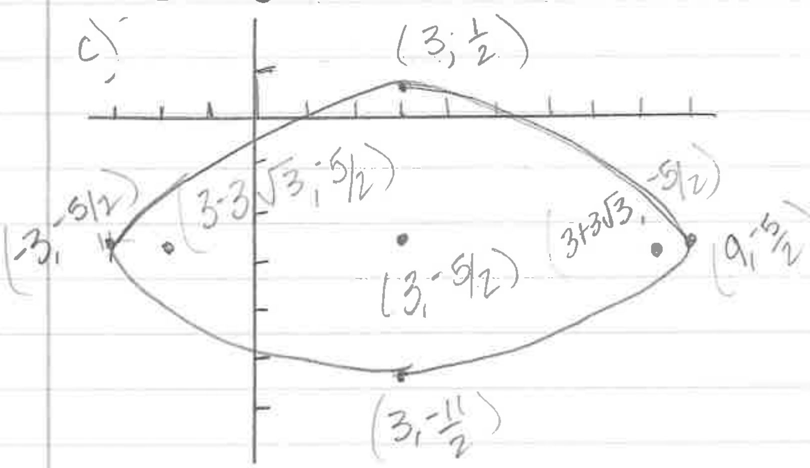
vertices $(9, -5/2)$ and $(-3, -5/2)$

foci $(3 \pm 3\sqrt{3}, -5/2)$

$c^2 = 36 - 9 = 27$

$c = \pm 3\sqrt{3}$

c)



16. a) $9x^2 + 4y^2 - 54x + 40y + 37 = 0$

$9(x^2 - 6x + 9) + 4(y^2 + 10y + 25) =$

$-37 + 81 + 10$

$9(x-3)^2 + 4(y+5)^2 = 144$

$\boxed{\frac{(x-3)^2}{16} + \frac{(y+5)^2}{36} = 1}$

b) center $(3,-5)$

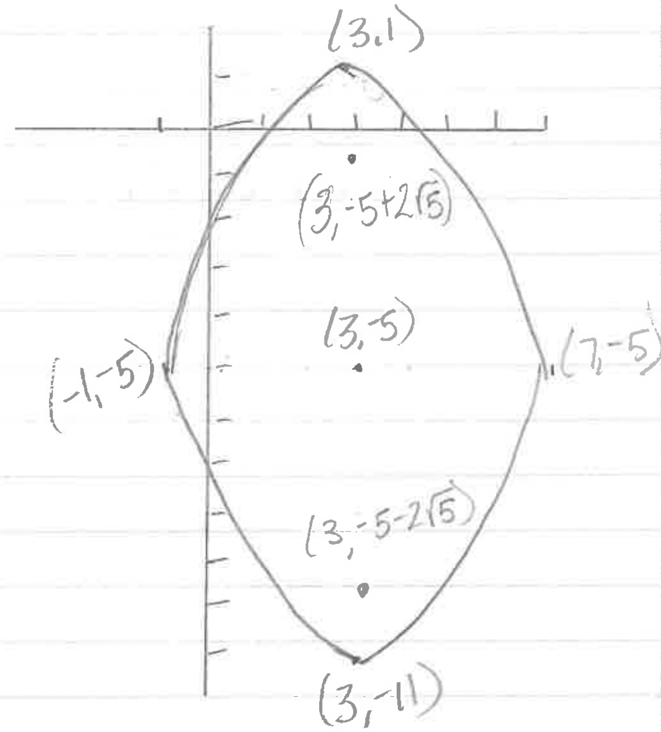
vertices $(3, 1)$ and $(3, -11)$

foci $(3, -5 \pm 2\sqrt{5})$

$c^2 = 36 - 16 = 20$

$c = \pm 2\sqrt{5}$

c)



32. center $(2,-1)$

$a=2 \quad b=1$

$$\boxed{\frac{(x-2)^2}{4}+\frac{(y+1)^2}{1}=1}$$

34. Foci $(0,0)(4,0) \rightarrow c=2$

major axis length $=8 \rightarrow a=4$

$4=16-b^2$

$b^2=12$

center $(2,0)$

$$\boxed{\frac{(x-2)^2}{16}+\frac{y^2}{12}=1}$$

36. center $(2,-1)$

vertex $(2,1/2) \rightarrow a=3/2$

minor axis $\rightarrow 2 \rightarrow b=1$

$$\boxed{(x-2)^2+\frac{(y+1)^2}{9/4}=1} \text{ or}$$

$$(x-2)^2+\frac{4(y+1)^2}{9}=1$$

38. Center $(3,2)$

$a=3c \quad$ foci $(1,2)\ (5,2)\ c=2$

$a=6 \quad 4=36-b^2$

$b^2=32$

$$\boxed{\frac{(x-3)^2}{36}+\frac{(y-2)^2}{32}=1}$$

40. Vertices $(5,0)\ (5,12) \rightarrow a=6$

endpts minor $(0,6)\ (10,6)$

center $(5,6) \qquad \hookrightarrow b=5$

$$\boxed{\frac{(x-5)^2}{25}+\frac{(y-6)^2}{36}=1}$$

44. $a=3 \quad b=2$

$c^2=9-4$

$c=\pm\sqrt{5}$

$\boxed{(-\sqrt{5},0) \text{ and } (\sqrt{5},0)}$

string would be $2a=\boxed{6\text{ft}}$

46. $e=0.97$

length major axis $=35.88$

$a=17.94 \quad a^2=321.8436$

$e=\frac{c}{a}=0.97=\frac{c}{17.94}$

$c=17.4018 \quad c^2=302.8226432$

$302.8226432=321.8436-b^2$

$b^2=19.0209568$

$$\boxed{\frac{x^2}{321.84}+\frac{y^2}{19.02}=1}$$

Conic Sections: Hyperbolas

16. a) $4x^2 - 25y^2 = 100$

$$\boxed{\frac{x^2}{25} - \frac{y^2}{4} = 1}$$

b) center $(0,0)$

vertices $(5,0)$ $(-5,0)$

$c^2 = a^2 + b^2$

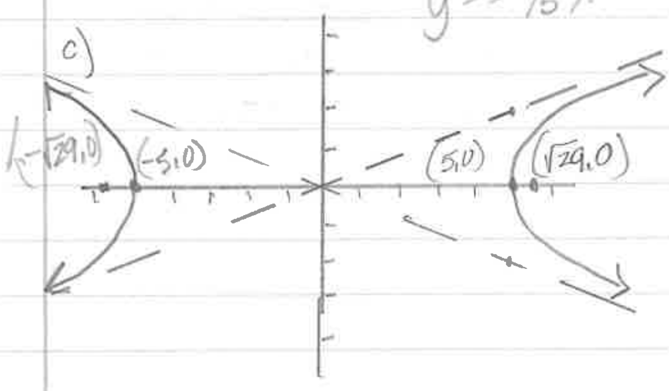
$c^2 = 25 + 4 \rightarrow c = \pm\sqrt{29}$

Foci $(\pm\sqrt{29}, 0)$

asymptotes $y = \pm \frac{b}{a} x$

$y = \pm \frac{2}{5} x$

c)



22 a) $16y^2 + 64y - x^2 + 2x = -63$

$16(y^2 + 4y + 4) - (x^2 - 2x + 1) = -63 + 64 - 1$

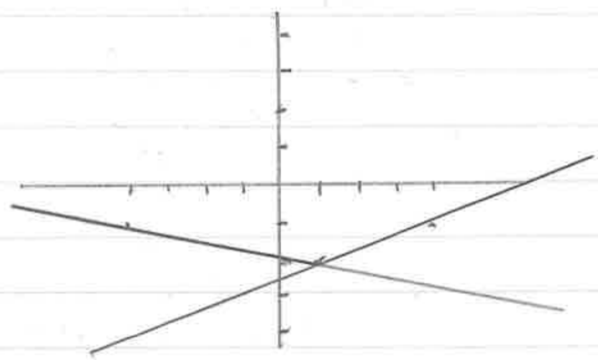
$16(y+2)^2 - (x-1)^2 = 0$

$16(y+2)^2 = (x-1)^2$

$(y+2)^2 = \frac{1}{16}(x-1)^2$

$y + 2 = \pm\frac{1}{4}(x-1)$

$\therefore$ just two lines



24. a) $9x^2 + 54x - y^2 + 10y = -55$

$9(x^2 + 6x + 9) - (y^2 - 10y + 25) = -55 + 81 - 25$

$9(x+3)^2 - (y-5)^2 = 1$

$\frac{(x+3)^2}{1/9} - (y-5)^2 = 1$

$a = 1/3 \quad b = 1 \qquad c^2 = a^2 + b^2$

$= 1/9 + 1$

$= 10/9$

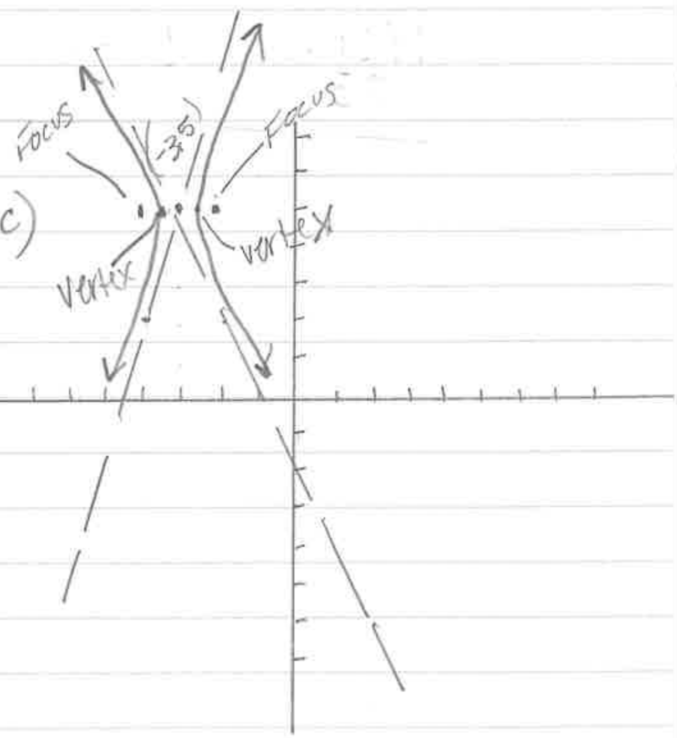
b) center: $(-3, 5)$

vertices: $(-\frac{10}{3}, 5)$ $(-\frac{8}{3}, 5)$

Foci: $(-3 \pm \sqrt{10}/3, 5)$

asymptotes $y - 5 = \pm 3(x+3)$

c)



30. Foci $(\pm 10,0)$ $c=10$

asymbtotes $y=\pm\frac{3}{4}x$

$\frac{b}{a}=\frac{3}{4} \rightarrow b=\frac{3}{4}a$

$100=a^2+\frac{9}{16}a^2$

$100=\frac{25}{16}a^2$

$64=a^2$ $a=8$

$b=6$

center $(0,0)$

$$\boxed{\frac{x^2}{64}-\frac{y^2}{36}=1}$$

38. vertices $(1,2)$ $(1,-2)$ $a=2$

center $(1,0)$

passes through $(0,\sqrt{5})$

$\frac{y^2}{4}-\frac{(x-1)^2}{b^2}=1$

$\frac{5}{4}-\frac{1}{b^2}=1$

$5b^2-4=4b^2$

$b^2=4$

$$\boxed{\frac{y^2}{4}-\frac{(x-1)^2}{4}=1}$$

40. vertices $(3,0)$ $(3,-6)$

$a=3$ center $(3,-3)$

asymptotes $y=x-6$ $y=-x$

$\frac{a}{b}=1$ $b=3$

$$\boxed{\frac{(y+3)^2}{9}-\frac{(x-3)^2}{9}=1}$$