

# St. Anne's-Belfield School

Dear Students,

Welcome to AP Calculus! We are excited to work with you next year, but before we can begin that journey, we should go over the summer work expectations for these courses. In order to ensure that students are prepared to dig deeply into the wonderful world of calculus, we have created the following set of problems that focuses on essential algebraic skills. You should complete these problems to the best of your ability (please show your work in an organized manner on your own paper) and be ready to turn in your solutions at the start of our first class in August. Additionally, you will have your first quiz on the second day of class, which will focus directly on the content covered in this summer work assignment.

This assignment is collaborative in nature, so feel free to connect with your friends and talk through these key topics and problems. You may also consult external resources of your choosing. Popular examples include [Khan Academy](#) (example problems and instructional videos) and [Open Stax](#) (free online textbooks).

Additionally, students enrolled in AP BC Calculus will need to complete a second assignment that fully overviews limits. This assignment will come with its own instructions and guidelines.

We hope you have a wonderful summer and look forward to seeing you in August.

Kindly,

Mr. Lippmann, Mr. Remchuk, and Ms. Scheel

# St. Anne's-Belfield School

## Topic 1: Tools of Algebra

### Exponent Rules

**Question 1:** Simplify fully and write with positive exponents.

$$\frac{1}{4}(16x^2)^{-\frac{3}{4}}(32x)$$

**Question 2:** Simplify fully and write with positive exponents.

$$x\left(x^{\frac{1}{2}} - x\right)^{-2}$$

### Fraction Operations

**Question 3:** Combine, and simplify fully.

$$\frac{2x-1}{x-1} - \frac{3x}{2x+1}$$

**Question 4:** Combine, and simplify fully.

$$\frac{5}{2x} + \frac{3}{3x^2} - \frac{x}{x^2-1}$$

### Complex Fractions

*Definition: A complex fraction is a fraction that includes fractions in its numerator and/or denominator.*

**Question 5:** Eliminate the complex fractions and simplify fully.

$$\frac{x^{-1}+y^{-1}}{x+y}$$

**Question 6:** Eliminate the complex fractions and simplify fully.

$$\frac{10x^2(5x-1)^{\frac{1}{2}}-2x(5x-1)^{\frac{1}{2}}}{5x-1}$$

### Logarithms

**Question 7:** Simplify fully using properties of logarithms.

$$9 + 2 + 8$$

**Question 8:** Simplify fully using properties of logarithms.

$$\left(\sqrt{3}^5\right) - \left(\sqrt{3}\right)^5$$

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## Topic 2: General Functions

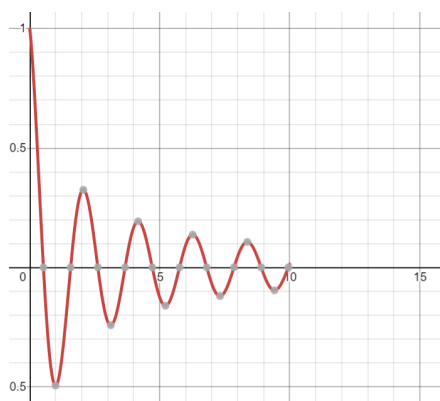
### Functions

**Question 9:** If  $f(x) = 4x - x^2$ , find  $f(f(3a))$ .

**Question 10:** If  $f(x) = 4x - x^2$ , find  $\frac{f(x+h)-f(x-h)}{2h}$  and simplify fully.

### Domain and Range

**Question 11:** Identify the domain and range of the function below. Write your answer in *interval notation*.



**Question 12:** Identify the domain and range of the function below. Write your answers in *inequality notation*.

$$\sqrt{x - x^2 + 2}$$

### Inverses

**Question 13:** Find the inverse of the function below.

$$f(x) = \frac{2x+1}{3-2x}$$

**Question 14:** Find the inverse of the function below, and then verify they are inverses by showing that

$$g(g^{-1}(x)) = x$$

$$g(x) = \frac{x^2}{x^2+1}$$

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## Topic 3: Parent Functions and Transformations

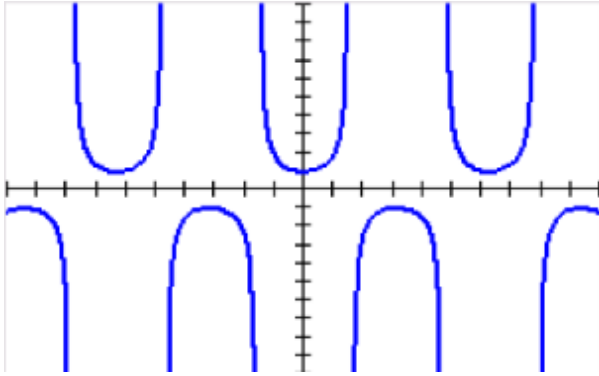
### Parent Functions

**Question 15:** The graphs on the following two pages represent common parent functions and are lettered A – P. Write the appropriate letter in each blank to match each function name/equation with its corresponding graph. Then identify the domain and range of each of these parent functions.

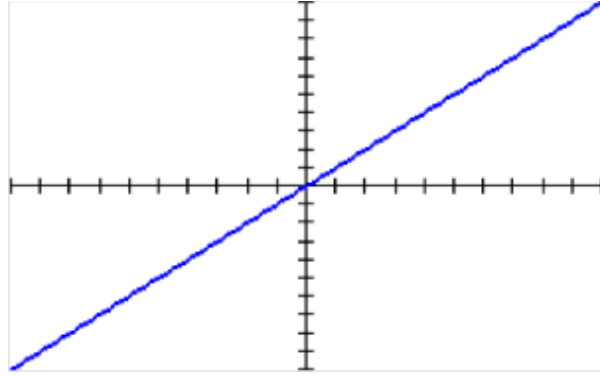
Letter	Equation	Domain	Range
	$y = \ln \ln (x)$		
	$y = \sin \sin (x)$		
	$y = \sec \sec (x)$		
	$y = e^{-x}$		
	$y =  x $		
	$y = \frac{1}{x}$		
	$y = e^x$		
	$y = \sqrt{x}$		
	$y = a, a \text{ is a constant}$		
	$y = \csc \csc (x)$		
	$y = x^3$		
	$y = \cos \cos (x)$		
	$y = x^2$		
	$y = \tan \tan (x)$		
	$y = \sqrt[3]{x}$		
	$y = x$		

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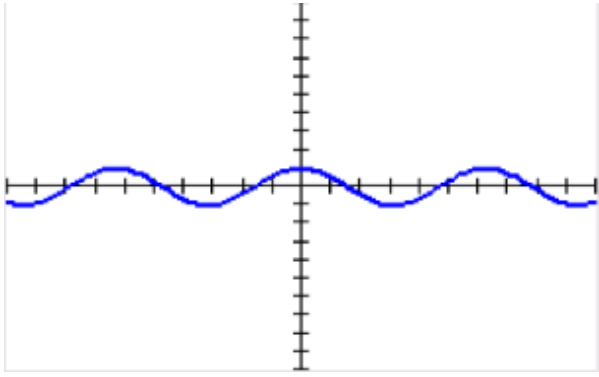
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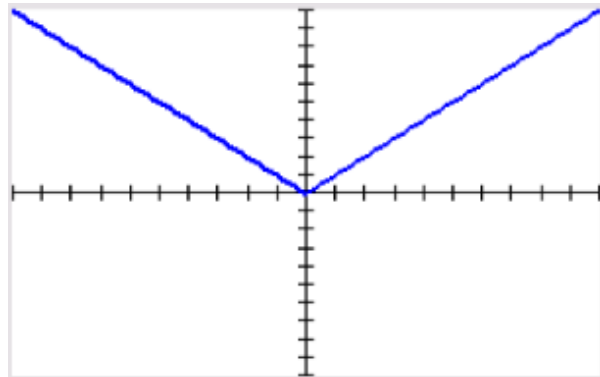
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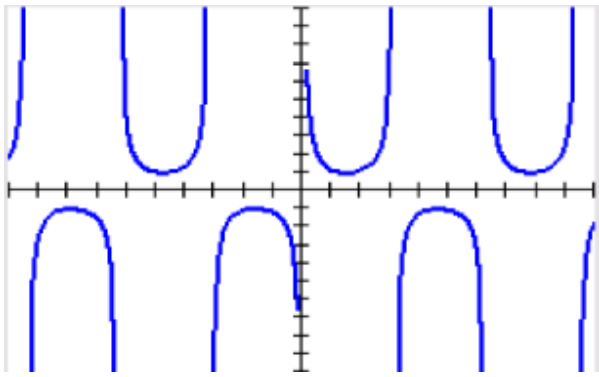
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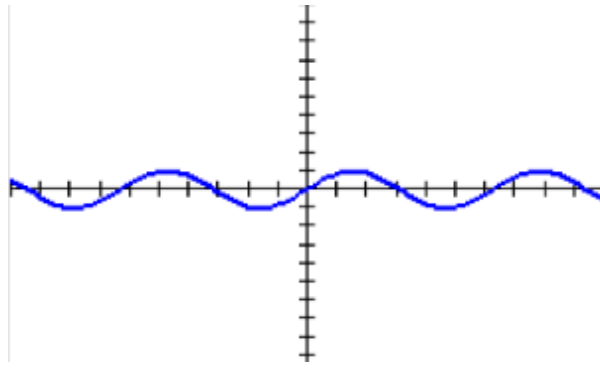
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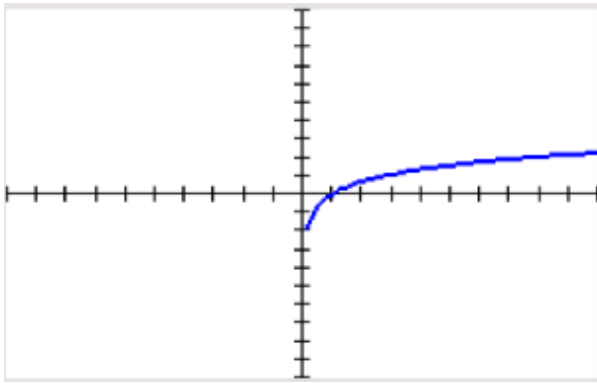


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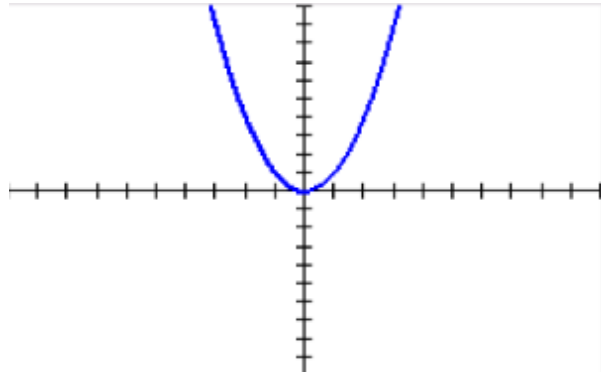


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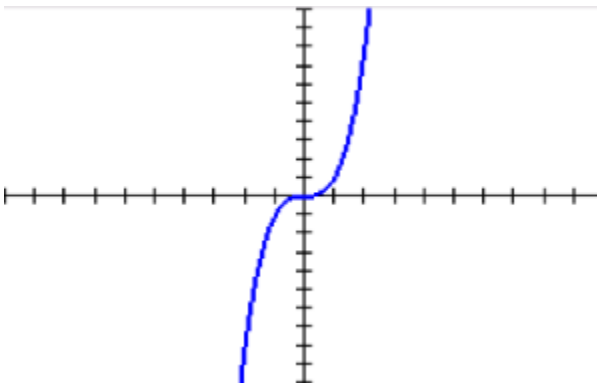
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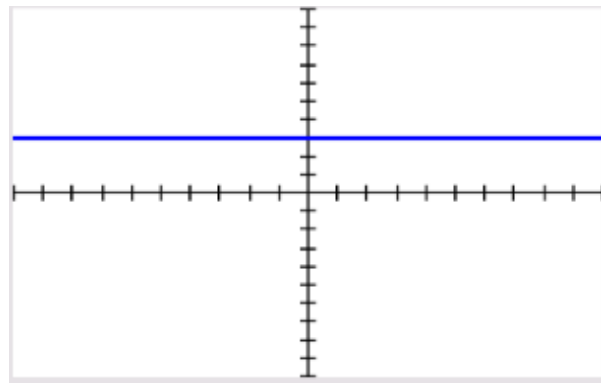
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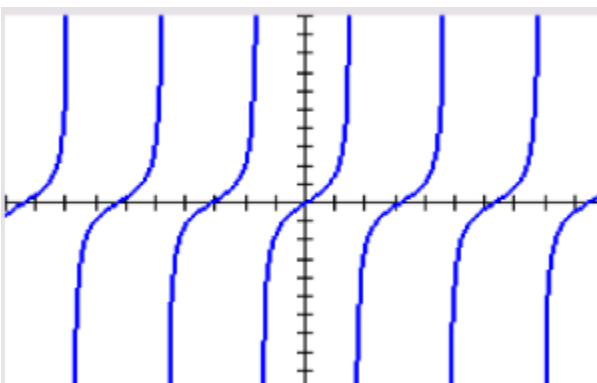
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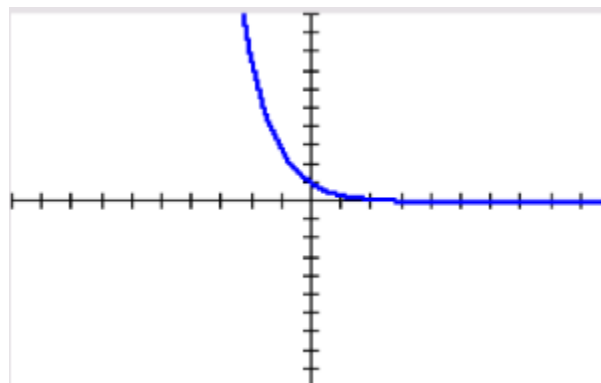
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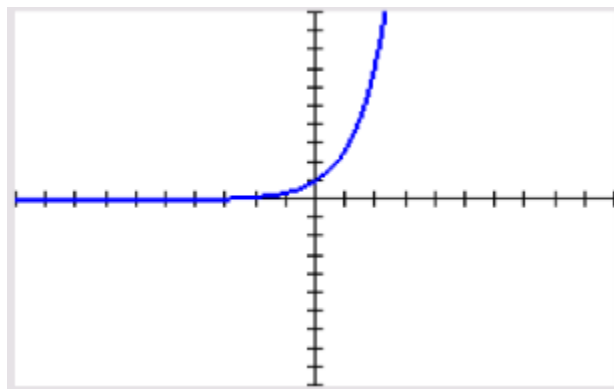
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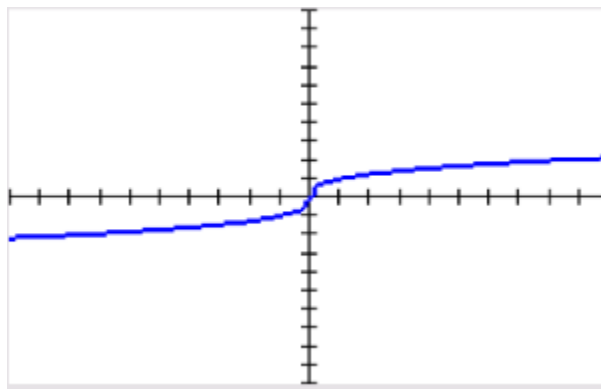
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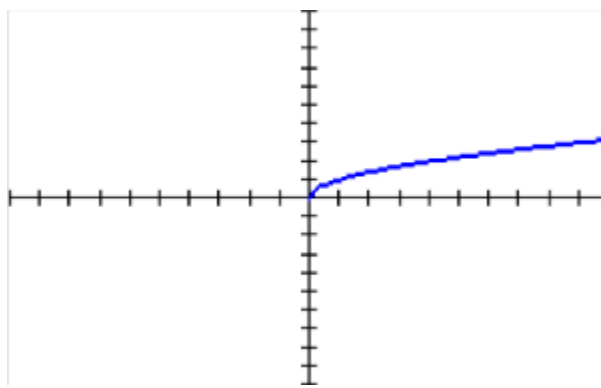
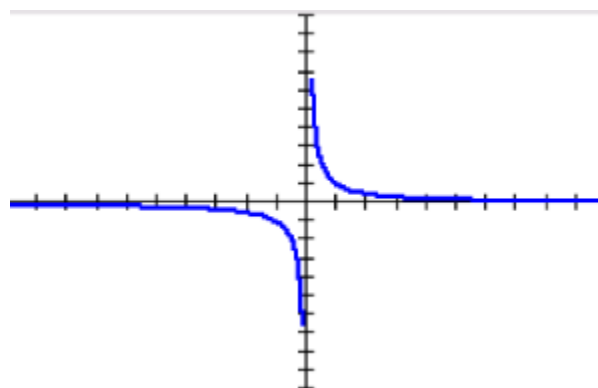
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P



## Transformations

**Question 16:** Sketch a graph of function  $f(x)$  listed below

$$f(x) = -2|x - 1| + 4$$

**Question 17:** Identify all transformations present in equation  $g(x)$  below. Then identify the domain and range of  $g(x)$ .

$$g(x) = -2^{x+2} - 3$$



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## Topic 4: Linear and Polynomial Functions

### Linear Functions

**Question 18:** Write the equation of the line that passes through the points  $(2, -8)$  and  $(\frac{1}{2}, 1)$ .

**Question 19:** Identify the equation of the line that passes through the point  $(5, -3)$  and is parallel to the line  $x + y = 4$ .

### Factoring Methods

**Question 20:** Factor Fully

$$4x^4 + 7x^2 - 36$$

**Question 21:** Factor Fully

$$x^5 + x^3 + x^2 + 1$$

**Question 22:** Factor Fully

$$250x^3 - 128$$

**Question 23:** Factor Fully

$$9a^4 - a^2b^2$$

### Quadratics

**Question 24:** Find the domain of  $y = \frac{2x-1}{6x^2-5x-6}$ .

**Question 25:** For what value(s) of  $m$  does the quadratic  $y = x^2 + mx - m$  have exactly two real solutions?

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## Topic 5: Solutions to Equations and Inequalities

*General Note: Equations can have extraneous solutions, so be careful.*

**Question 26:** Solve  $3x - 4x^2 = -5$

**Question 27:** Solve  $x^3 - 5x^2 + 5x - 25 = 0$

**Question 28:** Solve  $|1 - 7x| = 13$

**Question 29:** Solve  $|4x - 5| + 5x + 2 = 0$

**Question 30:** Solve  $\frac{2}{3} - \frac{1}{x} = \frac{5}{6}$

**Question 31:** Solve  $\frac{2x-1}{x-1} - \frac{3x}{2x+1} = \frac{x^2+11}{2x^2-x-1}$

**Question 32:** Solve  $\log \log ((2x - 3)^3) = 300$

**Question 33:** Solve  $2^{8x+5} = 8^{x^2-2}$

**Question 34:** Write the solution to the inequality below in *interval notation*.

$$(x + 2)^2 < 25$$

**Question 35:** Write the solution to the inequality below in *inequality notation*.

$$\frac{5}{x-6} \geq \frac{1}{x+2}$$

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## Topic 6: Trigonometry

### Unit Circle

**Question 36:** Evaluate the following trigonometric values.

- $\cos \cos (0)$
- $\cot \cot \left(\frac{\pi}{6}\right)$
- $\sin \sin \left(\frac{\pi}{4}\right)$
- $\cos \cos \left(\frac{\pi}{3}\right)$
- $\tan \tan \left(\frac{\pi}{2}\right)$

**Question 37:** Evaluate and simplify fully.

$$\left(\cos \cos \frac{2\pi}{3} - \tan \tan \frac{3\pi}{4}\right)^2$$

**Question 38:** Consider the point  $P = (-2\sqrt{5}, -\sqrt{5})$  that lies on the terminal side of angle  $\theta$ . Find the 6 trig functions of  $\theta$  ( $\sin \sin (\theta)$ ,  $\cos \cos (\theta)$ , etc.).

**Question 39:** If  $\tan \tan (\theta) = \frac{3}{4}$ , find  $\sin \sin (\theta)$ .

### Trigonometric Identities

There are no required problems for this topic, but students should be familiar with the following definitions and identities.

$$\begin{aligned} \csc \csc x &= \frac{1}{\sin \sin x} & \sec \sec x &= \frac{1}{\cos \cos x} & \tan \tan x &= \frac{\sin \sin x}{\cos \cos x} \\ \cot \cot x &= \frac{1}{\tan \tan x} = \frac{\cos \cos x}{\sin \sin x} \end{aligned}$$

$$x + x = 1$$

$$1 + x = x$$

$$1 + x = x$$

### Trigonometric Equations

**Question 40:** Solve for  $x$  on  $[0, 2\pi)$ .

$$\sin \sin (3x) = \frac{1}{\sqrt{2}}$$

**Question 41:** Solve for  $x$  on  $[0, 2\pi)$ .

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$$3\tan^2(x) - 1 = 0$$

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## Topic 7: Calculator Skills

Students will need a graphing calculator for AP Calculus. In fact, two of the four parts of the AP Calculus AP Exam require the use of a graphing calculator. We encourage you to have a TI83, TI83+, TI84, TI84+, or TI84CE calculator, though you may use any calculator on College Board's approved calculator list for this course. Please talk to your teacher if you have any questions about approved calculators or need help attaining one for class.

While there are no required problems for this topic, students should be familiar with the following features and tools on their calculator. Additional calculus-specific features will be overviewed during class next year.

- Key buttons: Ans, Ins, Entry, Sto→,  $\sqrt{\quad}$ ,  $\sqrt[3]{\quad}$ , and other features needed to evaluate expressions
- Fraction Bar (only available on TI84 and newer models)
- Key Skills: Graph a function, and set the window to an appropriate region for viewing a function's key properties. Identify the roots, maximum, and minimum values of a function using the 2<sup>nd</sup> Calc Menu. Identify intersections of pairs of functions, and use this technique to solve equations.

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## Topic 8: Limits

A traditional Calculus course will begin with an overview of limits, limit notation, and limiting arguments. Some of these topics appear in Integrated Mathematics curriculum, while others will be covered directly in class. The questions below represent what we will consider a baseline for students in AP Calculus.

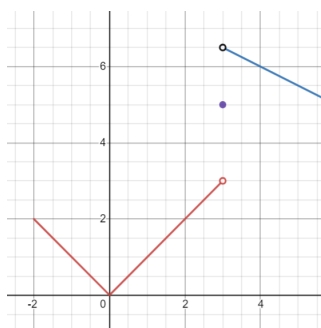
**Question 42:** Evaluate the limit shown below:

$$\frac{10x-5}{3x^2+2x}$$

**Question 43:** Evaluate the limit shown below:

$$\frac{x^3-8}{x^2+x-6}$$

**Question 44:** Identify the value of  $f(x)$ , given the graph of  $f(x)$  shown below:



**Question 45:** Identify the end behavior of  $y = \frac{x^2-5}{2x^2-12}$  and write your answer(s) in proper limit notation.