

# Are You Ready For AP Calculus BC?

Show all work on a separate sheet of paper.

1.) Simplify: (a)  $\frac{x^3-9x}{x^2-7x+12}$  (b)  $\frac{x^2-2x-8}{x^3+x^2-2x}$  (c)  $\frac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$  (d)  $\frac{9-x^{-2}}{3+x^{-1}}$

2.) Rationalize the denominator: (a)  $\frac{2}{\sqrt{3}+\sqrt{2}}$  (b)  $\frac{4}{1-\sqrt{5}}$  (c)  $\frac{1}{1+\sqrt{3}-\sqrt{5}}$

3.) Write each of the following expression in the form  $ca^pb^q$  where  $c$ ,  $p$ , and  $q$  are numbers:

(a)  $\frac{(2a^2)^3}{b}$  (b)  $\sqrt{9ab^3}$  (c)  $\frac{a(\frac{2}{b})}{\frac{3}{a}}$  (d)  $\frac{ab-a}{b^2-b}$  (e)  $\frac{a^{-1}}{(b^{-1})\sqrt{a}}$  (f)  $\left(\frac{2}{a^3}\right)^2 \left(\frac{3}{b^2}\right)^{\frac{1}{a^2}}$

4.) Solve for  $x$  (do not use a calculator):

(a)  $5^{x+1} = 25$  (b)  $\frac{1}{3} = 3^{2x+2}$  (c)  $\log_2 x = 3$  (d)  $\log_3 x^2 = 2 \log_3 4 - 4 \log_3 5$

5.) Simplify: (a)  $\log_2 5 + \log_2(x^2 - 1) - \log_2(x - 1)$  (b)  $2 \log_4 9 - \log_2 3$  (c)  $3^{2 \log_3 5}$

6.) Simplify: (a)  $\log\left(10^{\frac{1}{2}}\right)$  (b)  $\log\left(\frac{1}{10^x}\right)$  (c)  $2 \log \sqrt{x} + 3 \log x^{\frac{1}{3}}$

7.) Solve the following equations for the indicated variables:

(a)  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ , for  $a$  (b)  $V = 2(ab + bc + ca)$ , for  $a$   
(c)  $A = 2\pi r^2 + 2\pi rh$ , for positive  $r$  (d)  $A = P + nrP$ , for  $P$   
(e)  $2x - 2yd = y + xd$ , for  $d$  (f)  $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$ , for  $x$

8.) For the equations (a)  $y = x^2 + 4x + 3$  (b)  $3x^2 + 3x + 2y = 0$  (c)  $9y^2 - 6y - 9 - x = 0$   
complete the square and reduce to one of the standard forms  $y = a(x - h)^2 + k$  or  $x = a(y - k)^2 + h$

9.) Find all real solutions to: (a)  $x^6 - 16x^4 = 0$  (b)  $4x^3 - 8x^2 - 25x + 50 = 0$  (c)  $8x^3 + 27 = 0$

10.) Solve for  $x$ : (a)  $3 \sin^2 x = \cos^2 x$ ;  $0 \leq x < 2\pi$  (b)  $\cos^2 x - \sin^2 x = \sin x$ ,  $-\pi < x \leq \pi$   
(c)  $\tan x + \sec x = 2 \cos x$ ;  $-\infty < x < \infty$

11.) Without using a calculator, evaluate the following:

(a)  $\cos 210^\circ$  (b)  $\sin \frac{5\pi}{4}$  (c)  $\tan^{-1}(-1)$  (d)  $\sin^{-1}(-1)$   
(e)  $\cos \frac{9\pi}{4}$  (f)  $\sin^{-1} \frac{\sqrt{3}}{2}$  (g)  $\tan \frac{7\pi}{6}$  (h)  $\cos^{-1}(-1)$

12.) Solve the equations: (a)  $4x^2 + 12x + 3 = 0$  (b)  $2x + 1 = \frac{5}{x+2}$  (c)  $\frac{x+1}{x} - \frac{x}{x+1} = 0$

13.) Solve the inequalities: (a)  $x^2 + 2x - 3 \leq 0$  (b)  $\frac{2x-1}{3x-2} \leq 1$  (c)  $x^2 + x + 1 > 0$

14.) Solve for  $x$ : (a)  $|-x + 4| \leq 1$  (b)  $|5x - 2| = 8$  (c)  $|2x + 1| = x + 3$

15.) (a) Find the domain of the function  $f(x) = \frac{3x+1}{\sqrt{x^2+x-2}}$ .

(b) Find the domain and range of the functions: i)  $f(x) = 7$  ii)  $g(x) = \frac{5x-3}{2x+1}$   
 iii)  $h(x) = e^{3x} - 4$  iv)  $j(x) = \log_4(2x - 3) + 1$  v)  $y = -2\sqrt{3-x} + 6$

16.) Simplify  $\frac{f(x+h)-f(x)}{h}$ , where (a)  $f(x) = 2x + 3$  (b)  $f(x) = \frac{1}{x+1}$  (c)  $f(x) = x^2$

17.) Find the inverse of the functions: (a)  $f(x) = 2x + 3$  (b)  $f(x) = \frac{x+2}{5x-1}$   
 (c)  $f(x) = x^2 + 2x - 1, x > 0$

18.) Find the derivative of each function.

(a)  $f(x) = 4x^3 - 7x + 3$  (b)  $f(x) = \sqrt{4x^2 + 1}$  (c)  $f(x) = 3 \cos^4(2x)$  (d)  $f(x) = e^{5x^2}$   
 (e)  $f(x) = 3x^2 \tan x$  (f)  $f(x) = \sin^{-1}(3x)$

19.) Find the indefinite integral of each function.

(a)  $\int (3x^5 + 7x - 4) dx$  (b)  $\int 4x\sqrt{3x^2 - 2} dx$  (c)  $\int \sec^2(5x) dx$  (d)  $\int 7xe^{3x^2} dx$   
 (e)  $\int \frac{dx}{x^2+6x+13}$  (f)  $\int \frac{x^3+5x-1}{x} dx$  (g)  $\int \frac{x^2-5x+2}{x-3} dx$  (h)  $\int \frac{x}{2x^2+3} dx$

20.) Evaluate each definite integral.

(a)  $\int_{-1}^2 (x^2 - 5x + 1) dx$  (b)  $\int_0^1 x \cdot \sqrt[3]{2x^2 - 1} dx$

21.) Find the derivative of the equation with respect to  $x$ .

(a)  $2xy + x^2 - 4 = 0$  (b)  $\cos xy = 5x - 3y + 9$

22.) Find the general solution for each differential equation.

(a)  $\frac{dy}{dx} = 2xy$  (b)  $\frac{dy}{dx} = \frac{3x^2}{y}$  (c)  $\frac{dy}{dx} = 5x - xy$

23.) Derive each relationship with respect to  $t$ .

(a)  $V = \pi r^2 h$  (b)  $x^2 y - 5x = 3$

24.) Graph the parametric or polar equation.

(a)  $x = 4t^2 - 1, y = -2t + 1$  (b)  $r = 3 \cos 2\theta$  (c)  $r = 2 - 4 \sin \theta$  (d)  $r = 2 \cos \theta$  (e)  $r = 3$

25.) You should know the following trigonometric identities and the unit circle!!!

Odd/Even

$\sin(-x) = -\sin x$        $\cos(-x) = \cos x$        $\tan(-x) = -\tan x$   
 $\csc(-x) = -\csc x$        $\sec(-x) = \sec x$        $\cot(-x) = -\cot x$

Reciprocal

$$\sin x = \frac{1}{\csc x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Quotient

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Pythagorean

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Sum and Difference

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

Double Angle

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 2x = 1 - 2\sin^2 x$$

Power-Reducing

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

26.) You should know the following logarithmic/exponential properties!!!

$$\log_a x + \log_a y = \log_a xy$$

$$\log_a x - \log_a y = \log_a \frac{x}{y}$$

$$p \log_a x = \log_a x^p$$

$$\frac{1}{p} \log_a x = \log_a x^{\frac{1}{p}}$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$