SUMMER WORK PRIOR TO AP CALCULUS BC 2020- 2021

Dear Students,

Going into AP Calculus, there are certain skills that have been taught to you over the previous years that I assume you have. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

Being successful in AP Calculus BC will also require you to have specific study skills as well. Keeping an organized notebook, completing all given assignments with intent of mastery, being an active learner during class, seeking help outside of class during tutoring hours, and being a respectful member of our class are a few examples. Most of the world's problems are solved by groups of people so it is important that we work together throughout the year. While working together is key, we also must keep in mind that we must pull our own weight, and not depend on others to pick up our slack. Managing your time in your junior/senior year can prove to be challenging depending on the other courses you are taking and extracurricular activities you participate in. Please think about your role as a member of this class and if you are willing to commit for the full year. This course may be one of the most demanding you've ever experienced. It is also one of the most rewarding.

This summer work packet has been designed to provide a review of algebra, pre-calculus, and trigonometry skills that are essential for student success. It is a mistake to work this entire packet at the beginning of the summer or at the end of the summer. Give yourself some time! I recommend starting at the beginning of July and gradually working so that you finish the week before school begins. These topics will not be retaught during the class time. I am always happy to help you brush up on topics during extra help hours. Our class minutes will be designated to mastering the calculus objectives that will be tested in May of 2021.

All work must be shown neatly to receive full credit. It should be done in pencil. The assignment is meant to be done in the order of its presentation. I encourage students to work together on this assignment. Please keep in mind that working together is much different than copying. You can benefit greatly from working with your peers whether you're the person giving help or receiving help. Please be mindful of the honor policy of Porter-Gaud and be sure you are not breaking that trust while you work together on this assignment if you choose to do so. This assignment will be graded for completion and it is due on the first day of class. You may not use a calculator on any of the following problems.

If you have any questions please don't hesitate to e-mail me at vkuchler@portergaud.edu. I will be checking my e-mail periodically throughout the summer. I hope you have a great break and I look forward to seeing you in the fall!

Sincerely,

Ms. Kuchler

Review of Algebra/Pre-Calculus/Trigonometry

1 – 6] Are the following statements true? If not, explain in words why not?

1) $\frac{2k}{2x+h} = \frac{k}{x+h}$

2) $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$

3) $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$

4) $3\frac{a}{b} = \frac{3a}{3b}$

5) $3\frac{a}{b} = \frac{3a}{b}$

6) $3\frac{a+b}{c} = \frac{3a+b}{c}$

7 – 12] Factor each of the following completely.

7) $a^2 - b^2$

8) $a^3 - b^3$

9) $8x^3 + y^3$

10) $4x^2 - 21x - 18$

11) $2x^2 + x - 3$

12) $3x^2 + 6x^3 - 9x$

13-17] Simplify.

13) $\dfrac{\frac{x}{2}}{\frac{x}{4}}$

14) $h \div \dfrac{x+h}{h}$

15) $\dfrac{\sqrt{x-2}+\frac{5}{\sqrt{x-2}}}{x-2}$

16) $\dfrac{x(5x+1)-3(x^2+1)}{(x-1)^2}$

17) $\dfrac{2x(x+1)^2-3(x+1)^3}{8x^3+30x^2+18x}$

18] Solve the equation.

a) $4x^2 - 21x - 18 = 0$

b) $x^3 + 3x^2 - 5x - 15 = 0$

c) $x^4 - 9x^2 + 8 = 0$

d) $\log_8(7x - 13) = 3$

e) ln(x) + ln(8) - ln(3) = -2

f) $\sqrt{x^2-30}=\sqrt{x}$

g) $\frac{3x}{x+1}=\frac{12}{x^2-1}+2$

f) $\sin^2(x) = \sin(x)$ where $0 \le x < 2\pi$.

h) $\sqrt{3}+2\cos(4x)=0$ where $0 \le x < 2\pi$.

i) $2\tan^4(x) - \tan^2(x) - 15 = 0$ where $0 \le x < 2\pi$

19) Find the equation of the line that passes through the point (2,4) and is parallel to the line

$2x + 3y - 8 = 0$.

20) Find the equation of the line that is perpendicular to the line $2x + 3y - 8 = 0$ at the point (1,2).

21) The line with the slope of 5 that passes through the point (-1,3) intersects the x-axis at a point. What are the coordinates of this point?

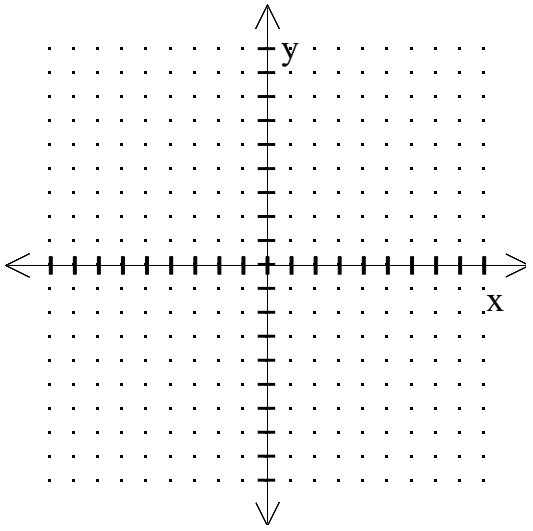
22) What are the coordinates of the point at which the line passing through the points (1,-3) and (-2,3) intersects the y-axis?

23) Find all points of intersection of the graphs of $x^2 + 3x - y = 3$ and $x + y = 2$.

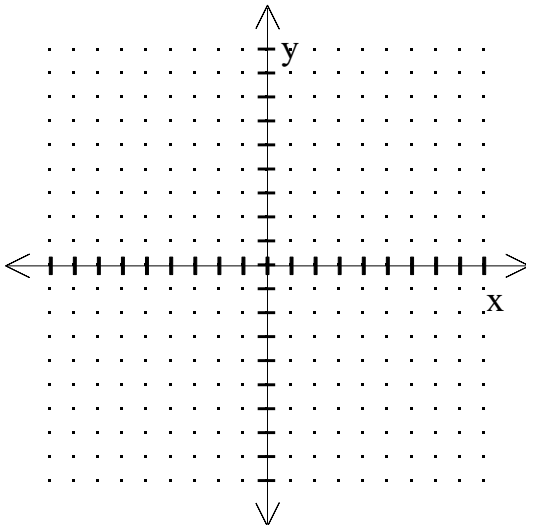
24) If the point (-1,1) lies on the graph of the equation $kx^2 - xy + y^2 = 5$, find the value of k.

25-26] Sketch the graph of each function.

25) $f(x)=\begin{cases}1 & x\le 0\\ -1 & x>0\end{cases}$



26) $g(x)=\begin{cases}2x, & x<-1\\ 2x^2, & -1\le x<2\\ -x+3, & x>2\end{cases}$



27 - 29] Given $f(x)=\dfrac{1}{x-5}$ and g(x) = x² – 5 complete the following.

27) f(g(7))

28) g(f(x))

29) g(g(x))

30) A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x.

31) Graph the parametrized curve described by $x = 2\sin(t)$, $y = -3\cos(t)$, $0 \le t \le \pi$. Indicate the direction in which the curve is traced.

a) Find a Cartesian equation for the parametrized curve. What portion of the graph of the Cartesian equation is traced by the parametrized curve?

32) An angle measuring $\frac{3\pi}{8}$ radians has its vertex at the center of a circle whose radius is 4 feet. Find the length of the subtended arc.

33) Let $y = 3\sin(2x - \pi) + 2$. Determine the period, domain, and range of the function. Then graph the function for $0 \le x \le 2\pi$.

34) Circle the two expressions that are equivalent.

a) $\cos^2(x)$ b) $(\cos x)^2$ c) $\cos x^2$

35) Circle the two expressions that are equivalent.

a) $(\sin x)^{-1}$ b) $\arcsin(x)$ c) $\sin x^{-1}$ d) $1/\sin(x)$

36 - 53] Evaluate the limit.

36) $\lim_{x\to 2} 3x^2+5$

37) $\lim_{x\to -1} \frac{x^4+x^3}{x+1}$

38) $\lim_{x\to 25} \frac{\sqrt{x}-5}{x-25}$

39) $\lim_{x\to -2} \frac{x-4}{x^2-2x-8}$

40) $\lim_{x\to -2} \frac{x^3+8}{x+2}$

41) $\lim_{x\to 5} \frac{x-5}{|x-5|}$

42) $\lim_{x\to 1} f(x)$ *if* $f(x)=\begin{cases} 3-x, & x\neq 1 \\ 1, & x=1 \end{cases}$

43) $\lim_{x\to -1} \frac{x^2+3x+2}{x^2+1}$

44) $\lim_{x\to 2} \sqrt{x^2-4}$

45) *If* $\lim_{x\to c} f(x)=-\frac{1}{2}$, *and* $\lim_{x\to c} g(x)=\frac{2}{3}$, *find* $\lim_{x\to c} \frac{f(x)}{g(x)}$

46) $\lim_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}$

47) $\lim_{x\to 2}\sec\left(\dfrac{\pi x}{3}\right)$

48) $\lim_{x\to 0}\dfrac{x}{\tan x}$

49) $\lim_{x\to 3^+}\sqrt{2x-5}$

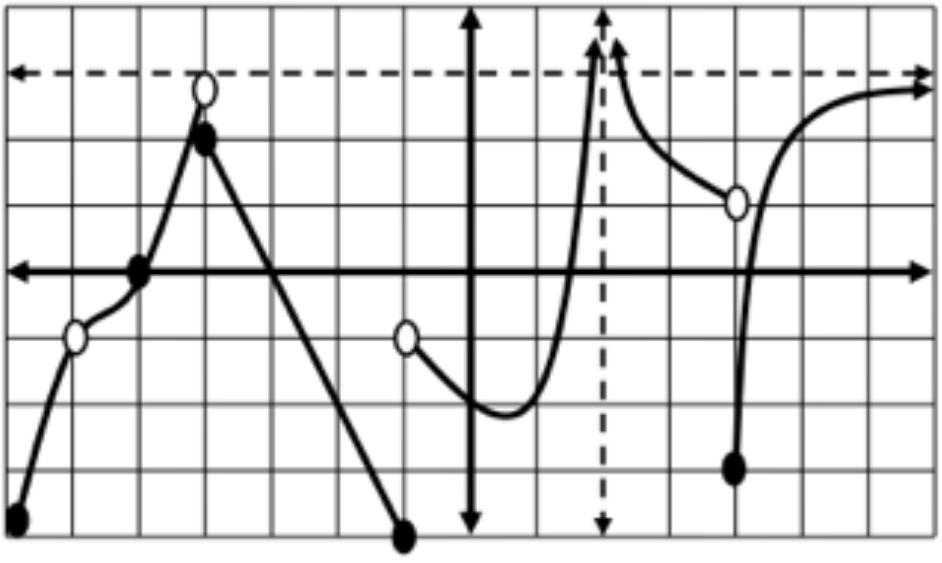
50) $\lim_{x\to 2^-}\dfrac{1}{x-2}$

51) $\lim_{x\to 2}\dfrac{1}{(x-2)^2}$

52) $\lim_{x\to 0}\left(2+\dfrac{5}{x^2}\right)$

53) $\lim_{x\to\infty}\dfrac{x^2-x-6}{5x^2}$

54) Use the graph of f(x) below to answer the following questions.



a) $\lim_{x\to -4} f(x) =$

b) $\lim_{x\to 2} f(x) =$

c) $\lim_{x\to\infty} f(x) =$

d) $\lim_{x\to 4^+} f(x) =$

e) $\lim_{x\to 6} f(x) =$

f) $\lim_{x\to 5} f(x) =$

55) Determine the value of c such that f(x) is continuous on the real line.

$$f(x)=\begin{cases} x-2, & x\le 5 \\ cx-3, & x>5 \end{cases}$$

<u>Review of Calculus Topics Covered in Pre-Calculus</u>

56) If $f(x) = x^3(x - 2)$, then $f'(5) =$

57) Find an equation of the line tangent to the graph of $g(x)=\dfrac{\tan(x)}{x}$ at $x=\dfrac{\pi}{4}$.

58) Let f be the function given by $f(x)=\dfrac{x-2}{x^2-4}$.

a) Find the domain of f(x).

b) Find $f'(x)$.