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Entering

AP Calculus AB

Summer Math Packet

AP Calculus AB **Summer Packet 2021**

Please read <u>all</u> of the following directions carefully.

- All of the problems in this packet should be solved algebraically **<u>without a calculator</u>**. If you need to leave an answer in logarithmic form such as $x = \ln 5$ that's fine. I would prefer this over a rounded decimal answer anyway.

- All work must be shown and all <u>answers should be fully simplified</u>. Some problems might not require a great deal of work in order to arrive at an answer; that's okay. Just show whatever you need to solve the problem. But remember, **NO CALCULATORS**.

- All intervals must be written in <u>interval notation</u>. For example, if your answer is $x \geq 2$, you need to write $[2, \infty)$.

- All problems should be completed on <u>separate paper (plain white or notebook)</u> and not written on this printout. You may use graph paper for the graphs if you wish but it's not required. Make sure all problems are labeled and easy to find.

- Please write down on a separate sheet of paper any problems you had trouble with because you will have two days in class to ask any questions related to this project (not to work on it, just to go over any material you need help with) and then you will be tested on this material. Any problem in this packet is subject to being on the first test.

- Don't forget that even though you will not have your textbooks over the summer, the internet has some great resources. A few good sites are listed below.
  http://www.khanacademy.com http://www.purplemath.com
  http://www.coolmath.com/algebra http://www.brightstorm.com

- A good site for learning about piecewise functions is:
  http://www.analyzemath.com/Graphing/piecewise_functions.html

- If there is a topic that you just need help with, try googling the topic. For example, if you don't remember how to solve an exponential equation, try googling "solving exponential equations" and a bunch of sites will probably pop right up with help and examples.

- Understand that everything in this packet is expected to be review. You can receive help with small questions from past topics, but you will not be re-taught entire units such as logarithms or trigonometry. You need to have some mastery of these topics before entering the class.

- Don't procrastinate! There are about two problems for each day of summer so pace yourself.

- **<u>There will be a test on the contents of this packet within the first week of school.</u>**

1) Find the *x* and *y*-intercepts for each of the following:

a) $y = x^2 + 4x - 2$

b) $y = (x-2)\sqrt{4-x^2}$

c) $y = \frac{x^2+2x}{(2x+1)^2}$

d) $x^2y - x^2 + 9y = 0$

2) Find all points of intersection of each of the following:

a) $2x - 3y = 20$ and $5x + 3y = 1$

b) $x^2 + y^2 = 20$ and $2x + y = 10$

3) Write the equation of the line with the following characteristics

a) passes through (3, -4) and (5, 4)

b) is a horizontal line with a *y*-intercept at -3

c) is a vertical line that passes through (6, -9)

d) has an *x*-intercept at 5 and a *y*-intercept at -3

e) is parallel to the line $5x + 4y = 7$, passes through the point (-6, 4) and is written in point-slope form

f) is perpendicular to the line $6x - 3y = 0$, passes through the point $\left(\frac{3}{4}, \frac{7}{8}\right)$ and is written in point-slope form

4) For the function $f(x) = x^2 - 6x$, find each of the following:

a) $f(q)$

b) $f(t+4)$

5) Find the value of $\frac{f(x)-f(3)}{x-3}$ for each of the following functions:

a) $f(x) = 4x + 6$

b) $f(x) = 4x^2 - x + 3$

c) $f(x) = \frac{3}{x}$

6) Find the value of $\frac{f(x+h)-f(x)}{h}$ for each of the following functions:

a) $f(x) = 4x + 6$

b) $f(x) = 4x^2 - x + 3$

c) $f(x) = \frac{3}{x}$

7) For the piecewise function $p(x) = \begin{cases} \sqrt{x+4}, x \leq 5 \\ (x-5)^2, x > 5 \end{cases}$ find each of the following:

a) $p(-2)$ b) $p(5)$ c) $p(11)$

8) Write the following as piecewise functions:

a) $y = |2x + 3|$ b) $y = |5 - 8x| + 7$

9) If $f(x) = \frac{2}{x}$ and $g(x) = x^2 - 1$, find a) $f(g(x))$ and b) $g(f(x))$ and state the domain of each.

10) Determine the domain for each of the following:

a) $g(x) = \frac{3x^2+2x-8}{2x^2-7x+6}$ b) $h(x) = \sqrt{x} + \sqrt{2-x}$

11) Find the inverse of each of the following:

a) $f(x) = 5x - 4$ b) $g(x) = \frac{3x+4}{x-5}$ c) $h(x) = x^3 + 2$

12) For each of the following, find $f(-x)$ and use it to determine if the function is odd, even, or neither:

a) $f(x) = x^2(4 - x^2)$ b) $f(x) = \sqrt[3]{x}$ c) $f(x) = x \sin x$

13) Graph each of the following:

a) $f(x) = -3x + 3$ b) $f(x) = 2 - x^2$ c) $f(x) = \begin{cases} 2x - 3, & x \leq 3 \\ (x-6)^2 - 2, x > 3 \end{cases}$

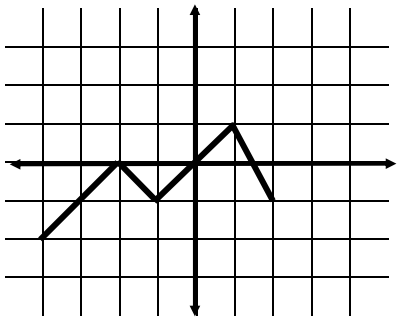
e) $f(x) = |\, 5 - x \,|$ f) $f(x) = \sqrt{4 - x^2}$ g) $f(x) = \ln(x - 2)$

h) $f(x) = e^x + 2$ i) $f(x) = -\ln x - 3$ j) $f(x) = -2e^x$

14) a – j) State the domain, range, end behavior, and intervals of increase and decrease of each function graphed in problem #13

15) If the graph of $f(x)$ is shown below, graph each of the following:

a) $f(-x)$ b) $-f(x)$ c) $|f(x)|$ d) $f(x)+2$ e) $f(x+2)$



16) State the value of all 6 trig functions for each of the following angles:

a) $\frac{3\pi}{4}$ b) $\frac{11\pi}{3}$ c) $\frac{3\pi}{2}$

17) State the amplitude, period, phase shift and vertical shift for each of the following:

a) $y = 2 + \cos\left[4\left(x - \frac{\pi}{3}\right)\right]$ b) $y = 1 - 2\sin(3x + \pi)$

18) Simplify the following:

a) $\sin x \cos x \tan x \sec x \cot x$ b) $\frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x}$

19) Solve the following (use identities where necessary) given that $0 \le x < 2\pi$

a) $\cos x \tan x = \cos x$ b) $\sin x = \cos 2x$ c) $\sin 3x = \frac{1}{2}$

d) $\cot^2 x - \csc x = 1$ e) $\sec^2 x - 2\sec x = 0$

20) Simplify the following given that all angles are between 0 and π:

a) $\cos\left(2\arcsin\frac{\sqrt{2}}{2}\right)$ b) $\sin\left(\arccos 0 + \arcsin\frac{1}{2}\right)$

21) Rewrite each of the following as an algebraic expression with no trig functions involved. (Hint: draw triangles and use Pythagorean Theorem.)

a) $\cos(\arcsin 2x)$ b) $\tan(\arccos x)$ c) $\sin(\operatorname{arccot} 3x)$

22) State horizontal asymptote(s), vertical asymptote(s) and hole(s) for each of the following:

a) $y = \frac{2}{x}$

b) $y = \frac{3x^2-10x-8}{6x^2+7x+2}$

c) $y = \frac{4x^2+12x}{x^3-3x^2-18x}$

23) Solve the following:

a) $\frac{3x^2-10x-8}{6x^2+7x+2} = 0$

b) $\frac{3x^2-10x-8}{6x^2+7x+2} < 0$

c) $\frac{5x^2+20x}{x^3-3x^2-28x} \geq 0$

d) $\sqrt{2x^2-7x-15} > 0$

24) Solve the following:

a) $\log_{27}\sqrt{3} = x$

b) $\log_x 27^{\frac{1}{3}} = \frac{1}{2}$

c) $9 = 4 + \log_2(x+4)$

d) $\frac{1}{3}\ln x = \ln 7$

e) $6\ln(x+3) = 12$

f) $e^{3x} = 7$

g) $6^{(x-3)} = 100$

h) $\log_b 8 = \log_b x + \log_b(x-2)$

25) Simplify:

a) $\frac{15x^2}{5\sqrt{x}}$

b) $\frac{x^3-x+1}{\sqrt{x}}$

26) Evaluate:

a) $32^{-\frac{2}{5}}$

b) $\ln 1$

c) $\ln e^7$

d) $\ln e$

e) e^0

f) $e^{\ln x}$

27) "BA" (BAD ALGEBRA) SECTION – The solution to each of the following equations contains at least one step (and possibly more) that involves bad algebra. Your job is to find the bad algebra, explain (very briefly) why it is bad algebra, and re-solve the problem correctly. (All BA's in ap calculus result in full credit lost for any problem, every time they occur.)

a) $10x^2 + 7x = 12$
$x(10x + 7) = 12$
$x = 12$ and $10x + 7 = 12$
$x = 12, x = \frac{1}{2}$

b) $(x - 5)^2 = 16$
$x^2 - 25 = 16$
$x^2 = 41$
$x = \pm\sqrt{41}$

c) $x^2 > 9$
$x > \pm 3$

d) $\sin 2x = \sin x, 0 \le x < 2\pi$
$2\sin x = \sin x$
$\sin x = 0$
$x = 0$

e) $x^3 = x^2$
divide both sides by x^2
$x = 1$

f) $e^{3\ln x} = 27$
$3x = 27$
$x = 9$

g) $\ln(x - 3) = 2$
$\ln x - \ln 3 = 2$
$\ln x = 2 + \ln 3$
$x = e^{2+\ln 3}$
$x = e^2 \cdot e^{\ln 3}$
$x = 3e^2$

h) $5x + 2x^{-1} = -11$
$\frac{5x+2}{x} = -11$
$5x + 2 = -11x$
$16x = -2$
$x = -\frac{1}{8}$

i) $2x^{-1} = 4$
$\frac{1}{2x} = 4$
$8x = 1$
$x = \frac{1}{8}$

j) $2(x - 1)^2 - x^2 = 14$
$(2x - 2)^2 - x^2 = 14$
$4x^2 - 4 - x^2 = 14$
$3x^2 = 18$
$x^2 = 6$
$x = \sqrt{6}$

k) $\sqrt{x^2 - 16} = 3$
$x - 4 = 3$
$x = 7$

l) $\frac{1}{x} + \frac{1}{2} = \frac{1}{3}$
$x + 2 = 3$
$x = 1$

m) $x = \sqrt{4}$
$x = \pm 2$

n) $\sqrt{x} = 9$
$x = 3$