

**DUE: August 9, 2021**

This assignment is for students who have completed Advanced Math or Advanced Math Honors and are taking Calculus CP in the 2021-2022 school year.

Did you read the instructions? \_\_\_\_\_

What math are you taking in the 2021-2022 school year? \_\_\_\_\_

The expectation of the Math Department at Archbishop Hannan High School is that its students become Tenacious Problem Solvers! Thus, as you work on these problems be sure and document your strategies, your mathematical explanations, any drawings, tables or graphs that you use, and the best, complete answer you can find. We hope that you are challenged by these problems and enjoy them. We look forward to the discussion of these problems that we will have in the first weeks of school. Come prepared to defend your solution!

Use of a graphing calculator is expected to solve these problems (unless stated otherwise). However, you must document your work in writing. For example, the solution to question 1a is found by performing the following calculation:  $h(0) = 400 - 16(0)^2 = 400$ . You must write that whole equation, not just 400 feet. Round to the nearest hundredth when necessary and always include units of measure when appropriate.

1. Write the equation of the line that goes through the points  $(\frac{5}{4}, 3)$  and  $(7, \frac{1}{3})$ .

2. After being dropped from the top of a tall building, the height of an object is described by

$$h(t) = 400 - 16t^2, \text{ where } h \text{ is measured in feet and } t \text{ is measured in seconds.}$$

a) How tall is the building?

b) How long does it take to reach the ground?

c) How high is the object when  $t = 2$  seconds?

3. a. If  $P(x) = 3(x + 1)(x - 2)(2x - 5)$ , then what are the  $x$ -intercept(s) and  $y$ -intercept(s) of its graph?

b. Write an equation of a function whose graph intercepts the  $x$ -axis only at  $-2$ ,  $7$ , and  $8$ . What is its  $y$ -intercept?

4. Given a function  $f$ , each solution to the equation  $f(x) = 0$  is called a zero of  $f$ . Without using a calculator, find the zeros of the following:

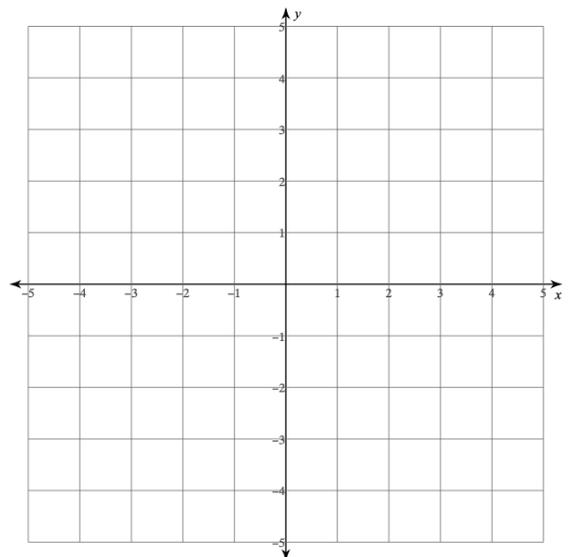
(a)  $r(x) = \sqrt{2x + 5}$

(b)  $p(x) = x^3 - 4x$

(c)  $s(x) = \sin 3x$

(d)  $L(x) = \log[5(x - 3)]$

4. Graph  $f(x) = \frac{x^2}{x^2 - 1}$ . What are the domain and range of  $f$ ?



5. Using a sketch of a unit circle and geometric reasoning, explain (with great detail) how you could determine the following:

a)  $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

b)  $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

c)  $\tan \frac{\pi}{4} = 1$

6. Solve each of the following equations by hand.

(a)  $8^x = 32$       (b)  $27^x = 243$       (c)  $1000^x = 100000$

7. Without a calculator evaluate the following.

(a)  $\log_5 125$       (b)  $\log_{0.125} 64$       (c)  $\log_4 8$       (d)  $\frac{\log 125}{\log 5}$

8. It is true that  $(5/6)^n < 0.0001$  for all sufficiently large values of  $n$ . How large is “sufficiently large”?

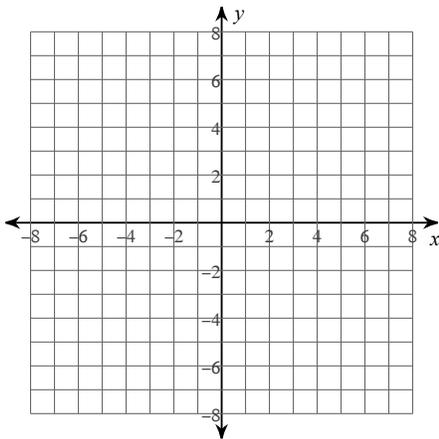
9. Farmer Brown wants to make a rectangular enclosure for his goats that will maximize their grazing area. He has 100 feet of fencing that he is going to use. He will build the pen so that it backs up to his barn wall so that he only has to enclose 3 sides with fencing.
- a) Draw the situation below and label the length of the pen  $x$  and the width of the pen  $y$ .
- b) Write an equation for the total length of fencing in terms of  $x$  and  $y$ . Solve it for  $y$ .
- c) Now write an equation for the area of the pen and substitute your expression for  $y$  in part b into the area formula so that area is expressed only in terms of  $x$ .
- d) Graph the equation in part c in your calculator. What dimensions of the pen will maximize area? What will that maximum area be? (Hint: 2<sup>nd</sup> trace (calculate) #4 will find a maximum for you!)

## Essential Skills

The following problems represent the essential skills you need to be successful in Calculus.

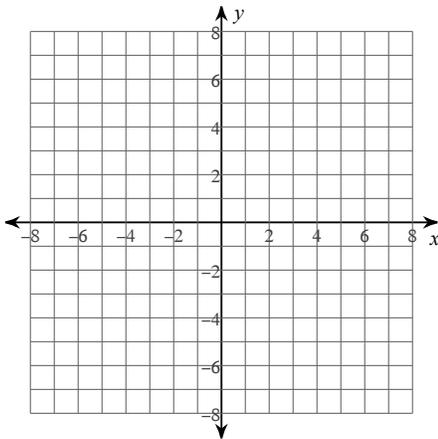
- 1) a) Factor the following polynomial.
- b) Identify if the polynomial is even or odd.
- c) Identify the zeroes of the function, and for each zero, whether or not it "bounces" or "crosses" the x axis.
- d) Sketch a graph of the polynomial.

$$y = x^3 + 5x^2 + 6x$$



Identify the holes, vertical asymptotes, x-intercepts, y-intercepts, and horizontal asymptote of the rational function. Then sketch the graph.

$$2) f(x) = \frac{-x^2 - 4x}{x^2 - x - 6}$$



**Solve each equation.**

3)  $2^{2x} = 2^{-x-2}$

4)  $-10 \cdot 10^{-5.5x} = -12$

5)  $\log_{17}(-2x - 2) = \log_{17}(x + 4)$

6)  $\log_9 5 + \log_9 -4x = 2$

7)  $-32 = -(16a)^{\frac{5}{4}}$

8)  $m^2 + 56 = 15m$

**Find all solutions to each equation in radians.**

9)  $-3\cos \theta = -3$

10)  $-8\cos 4\theta = 4$

**Expand each logarithm.**

11)  $\log_3 \frac{x^2}{y^5}$

**Condense each expression to a single logarithm.**

12)  $30\log_6 u + 5\log_6 v$

**Rewrite each equation in exponential form.**

13)  $\log_{289} 17 = \frac{1}{2}$

**Convert each degree measure into radians and each radian measure into degrees.**

14)  $295^\circ$

15)  $\frac{17\pi}{12}$