The following is extra practice for the final exam. Use your notes first to help with problems that are challenging. Answer key is also provided for you to check your work.

Exam Format:

Non-calc: 9 multiple choice and 9 open ended Calc: 5 open ended

NON-CALCULATOR PRACTICE:

Odd/Even/Symmetry

1. Determine if the equations have y-axis, origin or no symmetry. Then classify whether the function is even, odd, or neither.

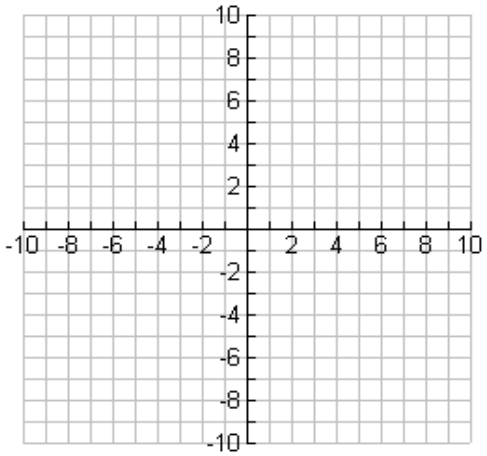
a. $y = x^6 - 4x^2$ b. $y = 3x^5 + 2x - 10$ c. $f(x) = \frac{x^3}{x^2-7}$ d. $f(x) = \frac{x}{x^5-x}$

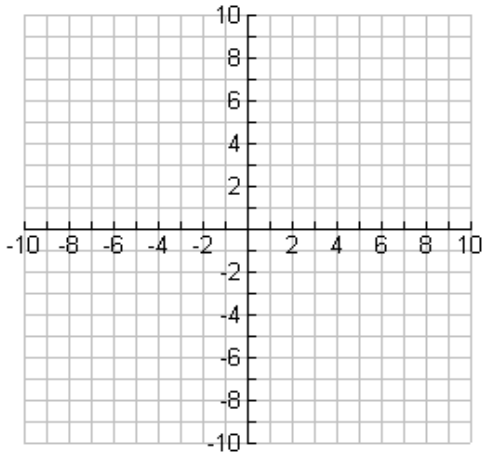
Polynomial and Rational Functions

2. Describe the end behavior of each function using limit notation.
   a. $h(x) = -4x^6 - 2x^2 + 1$ b. $h(x) = x^7(5-x)(2x+3)^3$

3. For the following functions determine the following:
   i. Determine the end behavior of the graph.
   ii. Determine the zeros and state the multiplicity of any repeated zeros
   iii. Use this information to sketch a graph of the function.

   a. $g(x) = -3x^2(x-3)^4(x+2)$ b. $f(x) = 2x^3(x+3)^5 - 6x^4(x+3)^4$





4. For the following functions,
a) Write each polynomial as the product of linear and irreducible quadratic factors.
b) Determine the rational and irrational zeros of the polynomial.

a. $f(x) = 2x^4 - x^3 - 17x^2 + 7x + 21$

b. $0 = x^3 + 2x^2 + 4x + 8$

Factored Form:
Rational Zero(s):
Irrational Zero(s):

Factored Form:
Rational Zero(s):
Irrational Zero(s):

5. Is $(x - 2)$ a factor of $x^3 + 3x^2 + 5x - 30$? If yes, factor the polynomial completely.

6. Solve the inequalities.
a. $6x^3 + 10x^2 + 15x < -25$

b. $(x - 4)^2 + 1 \geq 0$

c. $6x^4(x + 1)^2 \leq 3x^3(x + 1)^3$

7. Determine the domain.

a. $f(x) = \frac{x^2+5x+6}{x+7}$

b. $g(x) = \frac{x+3}{x^2+2x-3}$

c. $f(x) = \frac{2x^3+7x^2-4x}{x^3+2x^2-3x}$

8. Analyze the graphs of the following rational functions: Include domain, discontinuities, end behavior, asymptotes and intercepts. Then use this analysis to sketch the function.

a. $f(x) = \frac{3x}{x-2}$

Domain:

Discontinuities:
Hole (removable):

VA (infinite):

End Behavior:

HA:

OA:

x-intercept:
y-intercept:

b. $f(x) = \frac{x-2}{x^2-6x+8}$

Domain:

Discontinuities:
Hole (removable):

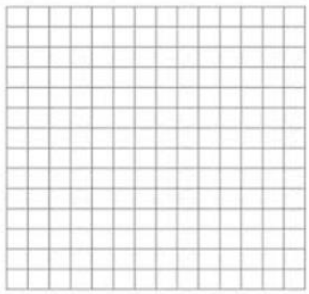
VA (infinite):

End Behavior:

HA:

OA:

x-intercept:
y-intercept:



c. $f(x) = \frac{x^2+2x-3}{x+2}$

Domain:

Discontinuities:
Hole (removable):

VA (infinite):

End Behavior:

HA:

OA:

x-intercept:
y-intercept:

9. Simplify the following rational expressions:

a. $\frac{2x^2-5x-3}{x^2-9}$

b. $\frac{4}{x-3} - \frac{2}{x+2}$

c. $\frac{\frac{5}{x-2}}{\frac{x}{x+1}+3}$

d. $\frac{\frac{2}{x}+\frac{x}{x-2}}{1-\frac{3}{x^2-2x}}$

10. Solve the equation.

a. $\frac{1}{x} + \frac{4x+12}{x^2-3x} = \frac{2}{x-3}$

b. $\frac{1}{x-2} + \frac{1}{x^2-7x+10} = \frac{6}{x-5}$

11. Solve the inequalities:

a. $\frac{(x+3)}{(x-5)} \leq 0$ b. $\frac{7}{x+3} < 1$

Composition and Inverse

12. Given $f(x) = \frac{5x+1}{2x-1}$ and $g(x) = \frac{3}{x-1}$, find$(g \circ f)(x)$. Then state the domain.

13. State whether the functions are one-to-one.

a. $f(x) = 3x^2 + x - 7$ b. $g(x) = x^3 - 8$ c. $h(x) = |x - 5| + 6$

14. Determine if the functions are invertible and restrict the domain if necessary. Then determine the inverse and it's domain.

a. $f(x) = (x-3)^2 + 12$ b. $f(x) = \frac{5}{2x-3}$ c. $y = |x+3| - 4$

Invertible? Yes No Invertible? Yes No Invertible? Yes No

Restricted domain (if necessary): Restricted domain (if necessary): Restricted domain (if necessary):

$f^{-1}(x) =$ $f^{-1}(x) =$ $f^{-1}(x) =$

Domain of $f^{-1}(x)$: Domain of $f^{-1}(x)$: Domain of $f^{-1}(x)$:

Exponential/Logarithmic Formulas Given: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ $A = Pe^{rt}$ $A(t) = A_0 e^{kt}$

15. NON- CALCULATOR: For the following functions determine the equation of the asymptote and end behavior using limits.

a. $f(x) = \left(\frac{1}{3}\right)^x - 7$ b. $f(x) = 3\ln(x+1)$ c. $f(x) = e^{x+4} - 3$

16. Condense the logarithmic expression. Simplify your result.

a. $2 \log x - \log 3$

b. $5 \ln x + \ln xy - 2 \ln 3x$

17. Expand the logarithmic expression completely.

a. $\log_9 \frac{x^2}{13y^5}$

b. $\ln\left(\frac{3x^3}{2\sqrt{x+1}\cdot(x)}\right)$

18. Evaluate the expression.

a. $\log_{16} \frac{1}{4}$

b. $\ln e^4 - \ln\left(\frac{1}{e^2}\right)$

c. $\log_3 81$

d. $\log_{\frac{1}{2}} 32$

19. Simplify each expression

a. e^{8lnx}

b. $\ln e^{(2x+3)}$

c. $e^{5 \ln x} + \ln e^{(12)}$

20. Solve. LEAVE EXACT, NON-CALCULATOR!

a. $8^{2x+3} = \left(\frac{1}{4}\right)^{x+1}$

b. $\log_9 4x = \log_9(2x+1) + \log_9(x)$

c. $\log_4(2x) + \log_4(x-2) = 2$

d. $2^x = 15$

e. $\ln(x+1) = 3$

f. $e^{2x+4} = 3$

21. A certain bacteria has a half-life of 39 days.
a) Determine the decay rate exactly and give both exact and decimal.

Rate (exact): _____

b) Determine how many bacteria are left after 100 days if the initial sample had 700 bacteria.

Solution (exact):_____

22. A certain bacteria has a half-life of 55 days.
a) Determine the decay rate exactly and give both exact and decimal.

Rate (exact): _____

b) Determine how many bacteria are left after 30 days if the initial sample had 806 bacteria.

Solution (exact):_____

Trigonometry Given Blank unit circle, sum and difference formulas, double angle formulas, Area formulas and Laws of Sines and Cosines.

23. If the point given is on the terminal side of θ, determine the exact value of all six trigonometric functions.

a. $(6, 8)$ b. $(2, -3)$

24. Name the quadrant in which the angle θ lies if:

a. $\cos\theta < 0,\ \csc\theta < 0$ _____

b. $\cot\theta < 0,\ \cos\theta > 0$ _____

c. $\sec\theta < 0,\ \tan\theta < 0$ _____

d. $\sin\theta > 0,\ \cos\theta > 0$ _____

25. What is the reference angle if $\theta = 247^0$

26. Give and expression for all angles that are coterminal with the given angle and then one positive and one negative coterminal angle.

a. $215°$ b. $\frac{7\pi}{15}$ c. $-\frac{4\pi}{7}$

All coterminal: _____ All coterminal: _____ All coterminal: _____
Positive: _____ Positive: _____ Positive: _____
Negative: _____ Negative: _____ Negative: _____

27. Find the exact value of the 5 remaining trig functions if $sec\ \theta = \frac{9}{8}$ and $\csc\theta < 0$.

28. Find the exact value of the 5 remaining trig functions if $\sin\theta = -\frac{2}{3}$ and $\cot\theta > 0$.

29. If $\sin\theta = -\frac{4}{5}$ and $\pi \leq \theta < \frac{3\pi}{2}$ what is $\cos(2\theta)$?

30. If $\cos\theta = 5/13$ and $\frac{3\pi}{2} \leq \theta < 2\pi$ what is $\sin(2\theta)$?

31. Find the exact value of the expression. NON-CALCULATOR

a. $\sec\left(\frac{4\pi}{3}\right)$ b. $\cot\left(-\frac{5\pi}{6}\right)$ c. $\tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)$ d. $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

e. $\sec^{-1}(-2)$ f. $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$ g. $\tan\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$ h. $\cos\left(\sin^{-1}\left(\frac{1}{4}\right)\right)$

i. $\sec^{-1}\left(\cot\left(\frac{3\pi}{4}\right)\right)$ j. $\cot(\pi) - \csc\left(\frac{\pi}{4}\right)$ k. $\cos^2\left(\frac{\pi}{6}\right)\sin\left(-\frac{5\pi}{3}\right)$ l. $\csc^2\left(\frac{5\pi}{6}\right) + \cot\left(\frac{5\pi}{4}\right)$

32. Given the functions below, find the amplitude, period, phase shift, and vertical shift.

a. $f(x) = 3\sin(3x - 8) + 4$ b. $f(x) = -\sin(x - 5)$ c. $f(x) = -\frac{2}{3}\sin\big(2(x + \pi)\big) - 3$

Domain: _____ Domain: _____ Interval of Continuity: _____

Range: _____ Range: _____ Range: _____

Amplitude: _____ Amplitude: _____ Amplitude: _____

Period: _____ Period: _____ Period: _____

Phase Shift: _____ Phase Shift: _____ Phase Shift: _____

Vertical Shift: _____ Midline: _____ Vertical Shift: _____

33. Find the exact value of cos 75°.

34. Find the exact value of tan 15°.

35. Find the exact value of $\cos(2\theta)$ and $\sin(2\theta)$, if $\cos\theta = \frac{2}{5}$ and $\csc\theta < 0$.

36. Find the exact value of $\cos(A + B)$ and $\sin(A - B)$, if $\tan A = \frac{3}{2}$ and A is in Q1. CosB=-2/5 and is in Q2.

37. Simplify the expressions. Reduce to a single trigonometric function that is not rational.

a. $1 - \frac{sin^2\theta}{1+\cos(-\theta)}$ b. $\sin x\cos^2 x + \sin^3 x$ c. $\frac{1-\sec^2\theta}{\sin^2\theta}$

38. Verify the identity

a. $\cos(\frac{\pi}{2} + \theta) = -\sin\theta$

b. $\sin^2 x \tan^2 x \csc^2 x + \cos^2 x \tan^2 x \csc^2 x = \sec^2 x$

c. $\sec\theta = \sin\theta(\tan\theta + \cot\theta)$

d. $\frac{\csc^2\theta - \cot^2\theta}{1 - \sin^2\theta} = \sec^2\theta$

e. $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} = \frac{2\cos(\theta)}{1-\sin\theta}$

f. $\frac{\sin\theta}{\csc\theta - 1} - \frac{\sin\theta}{\csc\theta + 1} = \sin(2\theta)\sec\theta\tan^2\theta$

39. Solve in the interval $[0, 2\pi)$

a. $6\cos x - 3 = 0$

b. $(\cot\theta + 1)(\csc\theta - \frac{1}{2}) = 0$

c. $3\tan^2 x + 2 = 5$

d. $\cos x \sin x = 3\cos x$

e. $5 + 7\sin^2\theta = 8 + 3\sin^2\theta$

e. $\cos^2 x = 2\cos(x)$

Vectors and Parametrics:

40. If $\mathbf{r} = \langle 3, 9\rangle$ and $\mathbf{s} = \langle -3, 6\rangle$, determine the following:

a. $5\mathbf{r} - 2\mathbf{s}$ b. $2r + \frac{1}{2}s$ c. $3(r - 2s)$

41. Let $\overrightarrow{AB}$ be the vector with given initial point A and terminal point B. Write $\overrightarrow{AB}$ as a linear combination of the vectors **i** and **j**.

a. $A(10, -4)$ and $B(-1, -3)$ b. $A(3, 0)$ and $B(2, -7)$

43. Find the magnitude and direction angle of the following vectors.

a. Exact: $\langle 1, -\sqrt{3}\rangle$ b. Calculator for angle (degrees): $-i - 5j$

43. Find the component form of v with the given magnitude and direction angle. EXACT

a. $|\mathrm{v}| = 18, \theta = 240°$ b. $|\mathrm{v}| = 5, \theta = 135°$

44. Find the direction angle of the vector $p = -4\mathrm{i} + 4\mathrm{j}$

45. Write the following parametric equations in rectangular form:

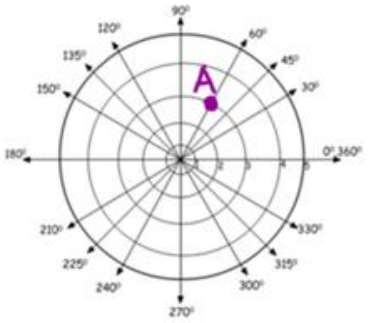
a. $x = 3t - 1;\ y = 2t^2 + 6$ b. $x = 4\cos\theta;\ y = 2\sin\theta$ c. $x(t) = 5t;\ y(t) = 3\sqrt{t - 1}$

Polar and Complex: No additional formulas given! Should know/come up with:
$x = r\cos\theta\,, y = r\sin\theta\,,\ x^2 + y^2 = r^2\ \ and \tan\theta = \frac{y}{x}$

46. Find the rectangular coordinates of:
a. (4, 120°) b. (-2, $3\pi/4$)

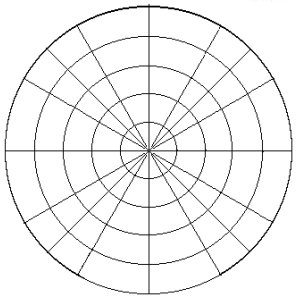
47. Find two polar coordinates for the rectangular point (-1, -1) if $-2\pi \le \theta \le 2\pi$.

48. Identify four polar coordinates for point A if $-2\pi \le \theta \le 2\pi$.



49. Plot the following polar points on the polar paper below.

$A\left(2, \frac{3\pi}{2}\right)$
$B\left(-3, \frac{5\pi}{6}\right)$
$C\left(0, \frac{\pi}{12}\right)$
$D\left(-3, -\frac{3\pi}{4}\right)$



50. Write the polar equations in rectangular form:
a. r = -6sinθ b. r = 2cosθ c. r = 5

51. Write the polar equation as a vector.
a. $r = 10$ b. $r = 2\cos\theta$ c. $r = -4\sin\theta$

Limits and Continuity

52. Determine whether each function is continuous at the given x-value(s). If discontinuous, identify the type of discontinuity.

a. $f(x) = \dfrac{x-2}{x+4}$; at $x = -4$

b. $f(x) = \dfrac{x+1}{x^2+3x+2}$; at $x = -1$ and $x = -2$

53. Evaluate each limit, if it exists. Justify any limits that do not exist.

a. $\lim\limits_{x \to 0^+} \left(4 - \sqrt{x}\right)$

b. $\lim\limits_{x \to 4} \dfrac{x^2 - 16}{x - 4}$

c. $\lim\limits_{x \to -1^+} \dfrac{1}{x+1}$

d. $\lim\limits_{x \to 3} \left(x^2 + 3x - 8\right)$

e. $\lim\limits_{x \to -6} \dfrac{1}{x+6}$

f. $\lim\limits_{\theta \to \frac{\pi}{2}^-} \tan\theta$

g. $\lim\limits_{x \to -2} f(x)$ *if* $f(x) = \begin{cases} x - 1 & x < -2 \\ x^2 & x \geq -2 \end{cases}$

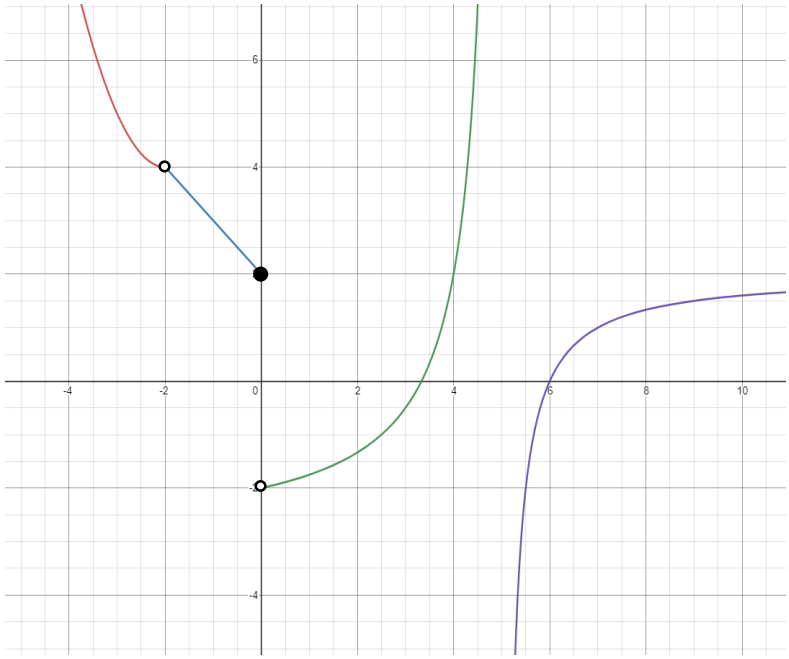
h. $\lim\limits_{x \to 5} f(x)$ *if* $f(x) = \begin{cases} x - 1 & x < 5 \\ 3 & x = 5 \\ x^2 - 4x - 1 & x > 5 \end{cases}$

i. $\lim\limits_{x \to \infty} x^4 - 3x^2 + 7$

j. $\lim\limits_{x \to -\infty} \dfrac{3x^2+7}{x^2-1}$

k. $\lim\limits_{x \to \infty} \ln(x)$

54. Given the graph of $f(x)$ at the right, determine the following or justify why they do not exist.



a) $\lim_{x\to -\infty} f(x)$=_____

b) $\lim_{x\to \infty} f(x)$=_____

c) $\lim_{x\to -2^-} f(x)$=_____

d) $\lim_{x\to -2^+} f(x)$=_____

e) $\lim_{x\to 0^-} f(x)$=_____

f) $\lim_{x\to 0^+} f(x)$=_____

g) $\lim_{x\to 0} f(x)$=_____

h) $\lim_{x\to 5^-} f(x)$=_____

i) $\lim_{x\to 5^+} f(x)$=_____

j) $\lim_{x\to 5} f(x)$=_____

k) Determine the name and location of any discontinuities on the graph of $f(x)$. (3 pts)

Location:	Name of Discontinuity:
x = _____	_____
_____	_____
_____	_____
_____	_____

Calculator Practice:

Exponential/Logarithmic Formulas Given: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ $A = Pe^{rt}$ $A(t) = A_0 e^{kt}$

1. Suppose $1000 is invested in an account that compounds continuously at a rate of 4.25%. Determine how long it will take to double. Round to the nearest year.

2. A scientist has 37 grams of a radioactive substance that decays 30% continuously. How many grams of radioactive substance remain after 9 years? Round to nearest tenth.

3. A certain radioactive substance has a half-life of 2488 years. Find the decay rate in EXACT FORM. Then, if there was 100g initially, find the amount of substance left after 1000 years. Round to nearest tenth.

Trigonometry Given Blank unit circle, sum and difference formulas, double angle formulas, Area formulas and Laws of Sines and Cosines.

4. Two observes simultaneously measure the angle of elevation of a helicopter. One angle measured is A: 25^0 and the other is B: 40^0. If the observers are 100 feet apart and the helicopter lies over the line joining them. How far away from the helicopter are the observers A and B?

5. Solve the following triangles. Round to the nearest hundredth.

a. *a = 11cm, b = 6 cm, A = 22°* b. *a = 13 m, b = 12 m, c = 8m* c. *a = 9 cm, b = 10 cm, C = 42°*

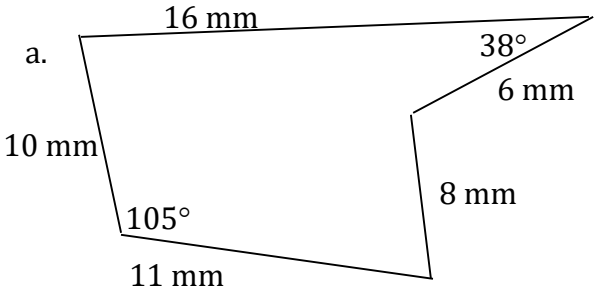
d. $a = 5\ cm, A = 36^o, B = 42^o$ e. $a = 25in, c = 18in, C = 63^\circ,$ f. $B = 20^o, a = 6mm, b = 4mm$

6. Determine the area of each triangle to the nearest tenth.

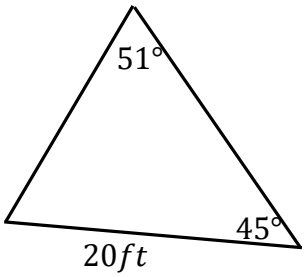
a. $A = 95°,\ b = 12m, c = 18\ m$ b. $a = 44, b = 47, c = 53$

7. Determine the area of the shape below with given dimensions.

a.



b.



<u>Vectors and Parametrics:</u>

8. Find the direction angle. Round to the nearest degree, when necessary.

a. $\langle -2, 3 \rangle$ b. $3\mathbf{i} - 3\mathbf{j}$ c. $\langle -1, -5 \rangle$ d. i