Mathematics Standards of Learning

for
Virginia
Public Schools

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Preface

In 1995, the Virginia Board of Education published Standards of Learning in English, mathematics, science, and history and social science for kindergarten through grade 12. Subsequently, Standards of Learning were developed for all academic content areas. The Standards of Learning provide a framework for instructional programs designed to raise the academic achievement of all students in Virginia and prepare them for college and careers.

The Standards of Learning are recognized as a model for other states. Pursuant to legislation from the 2000 Virginia General Assembly, the Board of Education established a seven-year cycle for review of the Standards of Learning. As a result, the 1995 *Mathematics Standards of Learning* were reviewed in 2001, 2009, and 2016, the results of which are contained in this document. The standards were revised with input from parents, teachers, administrators, representatives from higher education, and the business community. The standards set clear, concise, and measurable academic expectations for students. Parents and guardians are encouraged to work with their children, their children's teachers, and their children's schools to help them achieve these academic standards.

Introduction

Preparing Virginia's students to pursue higher education, to compete in a global workforce, and to be informed citizens requires rigorous mathematical knowledge and skills. Students must gain an understanding of fundamental ideas in number sense, computation, measurement, geometry, probability, data analysis and statistics, and algebra and functions, and they must develop proficiency in mathematical skills.

The 2016 *Mathematics Standards of Learning* identify academic content for essential components of the mathematics curriculum at different grade levels for Virginia's public schools. Information from the College Board, ACT, the National Assessment of Educational Progress (NAEP) Framework, the *Curriculum Focal Points* from the National Council of Teachers of Mathematics (NCTM), *Principles and Standards for School Mathematics* from NCTM, *Focus in High School Mathematics: Reasoning and Sense Making* from NCTM, the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report from the American Statistical Association, and the Report of the President's National Mathematics Advisory Panel were considered in identifying mathematics content necessary for success for all students in postsecondary pursuits.

Standards are identified for kindergarten through grade eight and for a core set of high school courses. Throughout a student's mathematics schooling from kindergarten through grade eight, specific content strands are included. These content strands are Number and Number Sense; Computation and Estimation; Measurement and Geometry; Probability and Statistics; and Patterns, Functions, and Algebra. The Standards of Learning within each strand progress in complexity throughout the grade levels and into high school course content. While the standards are organized by strand and identified numerically, local curricula and pacing guides should determine the instructional sequence of the content.

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the standards and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problem types. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students to clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "… the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation.

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the Mathematics Standards of Learning, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 Mathematics Standards of Learning form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Kindergarten

The kindergarten standards place emphasis on developing the concept of number by counting; combining, sorting, and comparing sets of objects; recognizing, describing, and creating simple repeating patterns; and recognizing shapes and sizes of figures and objects. Students will investigate measurement through direct comparisons, collect data, and create graphs. The concept of fractions is introduced through sharing experiences.

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The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

K.1   The student will
a) tell how many are in a given set of 20 or fewer objects by counting orally; and
b) read, write, and represent numbers from 0 through 20.

K.2   The student, given no more than three sets, each set containing 10 or fewer concrete objects, will
a) compare and describe one set as having more, fewer, or the same number of objects as the other set(s); and
b) compare and order sets from least to greatest and greatest to least.

K.3   The student will
a) count forward orally by ones from 0 to 100;
b) count backward orally by ones when given any number between 1 and 10;
c) identify the number after, without counting, when given any number between 0 and100 and identify the number before, without counting, when given any number between 1 and 10; and
d) count forward by tens to determine the total number of objects to 100.

K.4   The student will
a) recognize and describe with fluency part-whole relationships for numbers up to 5; and
b) investigate and describe part-whole relationships for numbers up to 10.

K.5   The student will investigate fractions by representing and solving practical problems involving equal sharing with two sharers.

Computation and Estimation

K.6   The student will model and solve single-step story and picture problems with sums to 10 and differences within 10, using concrete objects.

Measurement and Geometry

K.7   The student will recognize the attributes of a penny, nickel, dime, and quarter and identify the number of pennies equivalent to a nickel, a dime, and a quarter.

K.8   The student will investigate the passage of time by reading and interpreting a calendar.

K.9   The student will compare two objects or events, using direct comparisons, according to one or more of the following attributes: length (longer, shorter), height (taller, shorter), weight (heavier, lighter), temperature (hotter, colder), volume (more, less), and time (longer, shorter).

K.10   The student will
   a) identify and describe plane figures (circle, triangle, square, and rectangle);
   b) compare the size (smaller, larger) and shape of plane figures (circle, triangle, square, and rectangle); and
   c) describe the location of one object relative to another (above, below, next to) and identify representations of plane figures (circle, triangle, square, and rectangle) regardless of their positions and orientations in space.

Probability and Statistics

K.11   The student will
   a) collect, organize, and represent data; and
   b) read and interpret data in object graphs, picture graphs, and tables.

Patterns, Functions, and Algebra

K.12   The student will sort and classify objects according to one attribute.

K.13   The student will identify, describe, extend, create, and transfer repeating patterns.

Grade One

The first-grade standards place emphasis on counting, comparing, and ordering sets of up to 110 objects; recognizing and describing simple repeating and growing patterns; and tracing, describing, and sorting plane figures. Students' understanding of number is expanded through recognizing and describing part-whole relationships for numbers up to 10, solving story and picture problems using addition and subtraction within 20; using nonstandard units to measure; and organizing and interpreting data. Fractional concepts will be expanded through sharing scenarios involving halves and fourths.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

1.1   The student will
   a) count forward orally by ones to 110, starting at any number between 0 and 110;
   b) write the numerals 0 to 110 in sequence and out-of-sequence;
   c) count backward orally by ones when given any number between 1 and 30; and
   d) count forward orally by ones, twos, fives, and tens to determine the total number of objects to 110.

1.2   The student, given up to 110 objects, will
   a) group a collection into tens and ones and write the corresponding numeral;
   b) compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words *greater than, less than* or *equal to*; and
   c) order three or fewer sets from least to greatest and greatest to least.

1.3   The student, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth.

1.4   The student will
   a) represent and solve practical problems involving equal sharing with two or four sharers; and
   b) represent and name fractions for halves and fourths, using models.

1.5   The student, given a familiar problem situation involving magnitude, will
   a) select a reasonable order of magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, 500); and
   b) explain the reasonableness of the choice.

Computation and Estimation

1.6   The student will create and solve single-step story and picture problems using addition and subtraction within 20.

1.7   The student will
   a) recognize and describe with fluency part-whole relationships for numbers up to 10; and
   b) demonstrate fluency with addition and subtraction within 10.

Measurement and Geometry

1.8   The student will determine the value of a collection of like coins (pennies, nickels, or dimes) whose total value is 100 cents or less.

1.9   The student will investigate the passage of time and
   a) tell time to the hour and half-hour, using analog and digital clocks; and
   b) read and interpret a calendar.

1.10   The student will use nonstandard units to measure and compare length, weight, and volume.

1.11   The student will
   a) identify, trace, describe, and sort plane figures (triangles, squares, rectangles, and circles) according to number of sides, vertices, and angles; and
   b) identify and describe representations of circles, squares, rectangles, and triangles in different environments, regardless of orientation, and explain reasoning.

Probability and Statistics

1.12   The student will
   a) collect, organize, and represent various forms of data using tables, picture graphs, and object graphs; and
   b) read and interpret data displayed in tables, picture graphs, and object graphs, using the vocabulary *more, less, fewer, greater than, less than,* and *equal to*.

Patterns, Functions, and Algebra

1.13   The student will sort and classify concrete objects according to one or two attributes.

1.14   The student will identify, describe, extend, create, and transfer growing and repeating patterns.

1.15   The student will demonstrate an understanding of equality through the use of the equal symbol.

Grade Two

The second-grade standards extend the study of number and spatial sense to include three-digit whole numbers and solid geometric figures. Students will continue to learn, use, and gain proficiency in addition and subtraction within 20. Students will begin to use U.S. Customary units to measure length and weight; predict and use simple probability; and create and interpret pictographs and bar graphs. Students will work with a variety of patterns and will develop an understanding of equality.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

2.1   The student will
   a) read, write, and identify the place and value of each digit in a three-digit numeral, with and without models;
   b) identify the number that is 10 more, 10 less, 100 more, and 100 less than a given number up to 999;
   c) compare and order whole numbers between 0 and 999; and
   d) round two-digit numbers to the nearest ten.

2.2   The student will
   a) count forward by twos, fives, and tens to 120, starting at various multiples of 2, 5, or 10;
   b) count backward by tens from 120; and
   c) use objects to determine whether a number is even or odd.

2.3   The student will
   a) count and identify the ordinal positions first through twentieth, using an ordered set of objects; and
   b) write the ordinal numbers 1st through 20th.

2.4   The student will
   a) name and write fractions represented by a set, region, or length model for halves, fourths, eighths, thirds, and sixths;
   b) represent fractional parts with models and with symbols; and
   c) compare the unit fractions for halves, fourths, eighths, thirds, and sixths, with models.

Computation and Estimation

2.5   The student will
   a) recognize and use the relationships between addition and subtraction to solve single-step practical problems, with whole numbers to 20; and

b) demonstrate fluency with addition and subtraction within 20.

2.6 The student will
a) estimate sums and differences;
b) determine sums and differences, using various methods; and
c) create and solve single-step and two-step practical problems involving addition and subtraction.

Measurement and Geometry

2.7 The student will
a) count and compare a collection of pennies, nickels, dimes, and quarters whose total value is $2.00 or less; and
b) use the cent symbol, dollar symbol, and decimal point to write a value of money.

2.8 The student will estimate and measure
a) length to the nearest inch; and
b) weight to the nearest pound.

2.9 The student will tell time and write time to the nearest five minutes, using analog and digital clocks.

2.10 The student will
a) determine past and future days of the week; and
b) identify specific days and dates on a given calendar.

2.11 The student will read temperature to the nearest 10 degrees.

2.12 The student will
a) draw a line of symmetry in a figure; and
b) identify and create figures with at least one line of symmetry.

2.13 The student will identify, describe, compare, and contrast plane and solid figures (circles/spheres, squares/cubes, and rectangles/rectangular prisms).

Probability and Statistics

2.14 The student will use data from probability experiments to predict outcomes when the experiment is repeated.

2.15 The student will
a) collect, organize, and represent data in pictographs and bar graphs; and
b) read and interpret data represented in pictographs and bar graphs.

Patterns, Functions, and Algebra

2.16 The student will identify, describe, create, extend, and transfer patterns found in objects, pictures, and numbers.

2.17 The student will demonstrate an understanding of equality through the use of the equal symbol and the use of the not equal symbol.

Grade Three

The third-grade standards place emphasis on developing an understanding of, and solving problems that involve multiplication and division through 10 × 10. Students will apply knowledge of place value and the properties of addition and multiplication as strategies for solving problems. Concrete models and pictorial representations will be used to introduce addition and subtraction with fractions and the concept of probability as the measurement of chance. Students will use standard units (U.S. Customary and metric) to measure temperature, length, and liquid volume. Properties of shapes, points, line segments, rays, angles, vertices, and lines will be explored and students will identify polygons with 10 or fewer sides, combine and subdivide polygons, and name the resulting polygon(s).

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

3.1   The student will
   a)   read, write, and identify the place and value of each digit in a six-digit whole number, with and without models;
   b)   round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and
   c)   compare and order whole numbers, each 9,999 or less.

3.2   The student will
   a)   name and write fractions and mixed numbers represented by a model;
   b)   represent fractions and mixed numbers with models and symbols; and
   c)   compare fractions having like and unlike denominators, using words and symbols (>, <, =, or ≠), with models.

Computation and Estimation

3.3   The student will
   a)   estimate and determine the sum or difference of two whole numbers; and
   b)   create and solve single-step and multistep practical problems involving sums or differences of two whole numbers, each 9,999 or less.

3.4   The student will
   a)   represent multiplication and division through 10 × 10, using a variety of approaches and models;
   b)   create and solve single-step practical problems that involve multiplication and division through 10 x 10; and
   c)   demonstrate fluency with multiplication facts of 0, 1, 2, 5, and 10; and

    d) solve single-step practical problems involving multiplication of whole numbers, where one factor is 99 or less and the second factor is 5 or less.

3.5   The student will solve practical problems that involve addition and subtraction with proper fractions having like denominators of 12 or less.

Measurement and Geometry

3.6   The student will
    a) determine the value of a collection of bills and coins whose total value is $5.00 or less;
    b) compare the value of two sets of coins or two sets of coins and bills; and
    c) make change from $5.00 or less.

3.7   The student will estimate and use U.S. Customary and metric units to measure
    a) length to the nearest $\frac{1}{2}$ inch, inch, foot, yard, centimeter, and meter; and
    b) liquid volume in cups, pints, quarts, gallons, and liters.

3.8   The student will estimate and
    a) measure the distance around a polygon in order to determine its perimeter using U.S. Customary and metric units; and
    b) count the number of square units needed to cover a given surface in order to determine its area.

3.9   The student will
    a) tell time to the nearest minute, using analog and digital clocks;
    b) solve practical problems related to elapsed time in one-hour increments within a 12-hour period; and
    c) identify equivalent periods of time and solve practical problems related to equivalent periods of time.

3.10   The student will read temperature to the nearest degree.

3.11   The student will identify and draw representations of points, lines, line segments, rays, and angles.

3.12   The student will
    a) define polygon;
    b) identify and name polygons with 10 or fewer sides; and
    c) combine and subdivide polygons with three or four sides and name the resulting polygon(s).

3.13   The student will identify and describe congruent and noncongruent figures.

Probability and Statistics

3.14   The student will investigate and describe the concept of probability as a measurement of chance and list possible outcomes for a single event.

3.15 The student will
    a) collect, organize, and represent data in pictographs or bar graphs; and
    b) read and interpret data represented in pictographs and bar graphs.

Patterns, Functions, and Algebra

3.16 The student will identify, describe, create, and extend patterns found in objects, pictures, numbers and tables.

3.17 The student will create equations to represent equivalent mathematical relationships.

Grade Four

The fourth-grade standards place emphasis on multiplication and division with whole numbers and solving problems involving addition and subtraction of fractions and decimals. Students will develop fluency with multiplication through 12 x 12 and the corresponding division facts as they become proficient in multiplying larger numbers. Students will apply knowledge of place value and the properties of addition and multiplication as strategies for solving problems. Students also will refine their estimation skills for computations and measurements. Students will identify and describe representations of points, lines, line segments, rays, and angles, including endpoints and vertices. Students will describe and compare characteristics of plane and solid figures. Concrete models and pictorial representations will be used to solve problems involving perimeter and area, patterns, probability, and equivalence of fractions and decimals.

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The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

4.1   The student will
  a) read, write, and identify the place and value of each digit in a nine-digit whole number;
  b) compare and order whole numbers expressed through millions; and
  c) round whole numbers expressed through millions to the nearest thousand, ten thousand, and hundred thousand.

4.2   The student will
  a) compare and order fractions and mixed numbers, with and without models;
  b) represent equivalent fractions; and
  c) identify the division statement that represents a fraction, with models and in context.

4.3   The student will
  a) read, write, represent, and identify decimals expressed through thousandths;
  b) round decimals to the nearest whole number;
  c) compare and order decimals; and
  d) given a model, write the decimal and fraction equivalents.

Computation and Estimation

4.4   The student will
a)   demonstrate fluency with multiplication facts through 12 x 12, and the corresponding division facts;
b)   estimate and determine sums, differences, and products of whole numbers;
c)   estimate and determine quotients of whole numbers, with and without remainders; and
d)   create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication, and single-step practical problems involving division with whole numbers.

4.5   The student will
a)   determine common multiples and factors, including least common multiple and greatest common factor;
b)   add and subtract fractions and mixed numbers having like and unlike denominators; and
c)   solve single-step practical problems involving addition and subtraction with fractions and mixed numbers.

4.6   The student will
a)   add and subtract with decimals; and
b)   solve single-step and multistep practical problems involving addition and subtraction with decimals.

Measurement and Geometry

4.7   The student will solve practical problems that involve determining perimeter and area in U.S. Customary and metric units.

4.8   The student will
a)   estimate and measure length and describe the result in U.S. Customary and metric units;
b)   estimate and measure weight/mass and describe the result in U.S. Customary and metric units;
c)   given the equivalent measure of one unit, identify equivalent measures of length, weight/mass, and liquid volume between units within the U.S. Customary system; and
d)   solve practical problems that involve length, weight/mass, and liquid volume in U.S. Customary units.

4.9   The student will solve practical problems related to elapsed time in hours and minutes within a 12-hour period.

4.10   The student will
a)   identify and describe points, lines, line segments, rays, and angles, including endpoints and vertices; and
b)   identify and describe intersecting, parallel, and perpendicular lines.

4.11   The student will identify, describe, compare, and contrast plane and solid figures according to their characteristics (number of angles, vertices, edges, and the number and shape of faces) using concrete models and pictorial representations.

4.12   The student will classify quadrilaterals as parallelograms, rectangles, squares, rhombi, and/or trapezoids.

Probability and Statistics

4.13   The student will
   a) determine the likelihood of an outcome of a simple event;
   b) represent probability as a number between 0 and 1, inclusive; and
   c) create a model or practical problem to represent a given probability.

4.14   The student will
   a) collect, organize, and represent data in bar graphs and line graphs;
   b) interpret data represented in bar graphs and line graphs; and
   c) compare two different representations of the same data (e.g., a set of data displayed on a chart and a bar graph, a chart and a line graph, or a pictograph and a bar graph).

Patterns, Functions, and Algebra

4.15   The student will identify, describe, create, and extend patterns found in objects, pictures, numbers, and tables.

4.16   The student will recognize and demonstrate the meaning of equality in an equation.

Grade Five

The fifth-grade standards place emphasis on number sense with whole numbers, fractions, and decimals. This focus includes concepts of prime and composite numbers, identifying even and odd numbers, and solving problems using order of operations for positive whole numbers. Students will develop proficiency in the use of fractions and decimals to solve practical problems. Students will collect, display, and analyze data in a variety of ways and solve probability problems, using a sample space, a tree diagram, or the Fundamental Counting Principle. Students will also solve problems involving volume, area, and perimeter. Students will be introduced to expressions with a variable. Students will solve problems using strategies including place value and the properties of addition and multiplication. All of these skills assist in the development of the algebraic concepts needed for success in the middle grades.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

5.1 The student, given a decimal through thousandths, will round to the nearest whole number, tenth, or hundredth.

5.2 The student will
   a) represent and identify equivalencies among fractions and decimals, with and without models; and
   b) compare and order fractions, mixed numbers, and/or decimals in a given set, from least to greatest and greatest to least.

5.3 The student will
   a) identify and describe the characteristics of prime and composite numbers; and
   b) identify and describe the characteristics of even and odd numbers.

Computation and Estimation

5.4 The student will create and solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of whole numbers.

5.5 The student will
   a) estimate and determine the product and quotient of two numbers involving decimals; and
   b) create and solve single-step and multistep practical problems involving addition, subtraction, and multiplication of decimals, and create and solve single-step practical problems involving division of decimals.

5.6   The student will
    a) solve single-step and multistep practical problems involving addition and subtraction with fractions and mixed numbers; and
    b) solve single-step practical problems involving multiplication of a whole number, limited to 12 or less, and a proper fraction, with models.

5.7   The student will simplify whole number numerical expressions using the order of operations.

Measurement and Geometry

5.8   The student will
    a) solve practical problems that involve perimeter, area, and volume in standard units of measure; and
    b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.

5.9   The student will
    a) given the equivalent measure of one unit, identify equivalent measurements within the metric system; and
    b) solve practical problems involving length, mass, and liquid volume using metric units.

5.10   The student will identify and describe the diameter, radius, chord, and circumference of a circle.

5. 11   The student will solve practical problems related to elapsed time in hours and minutes within a 24-hour period.

5. 12   The student will classify and measure right, acute, obtuse, and straight angles.

5. 13   The student will
    a) classify triangles as right, acute, or obtuse and equilateral, scalene, or isosceles; and
    b) investigate the sum of the interior angles in a triangle and determine an unknown angle measure.

5.14   The student will
    a) recognize and apply transformations, such as translation, reflection, and rotation; and
    b) investigate and describe the results of combining and subdividing polygons.

Probability and Statistics

5.15   The student will determine the probability of an outcome by constructing a sample space or using the Fundamental (Basic) Counting Principle.

5. 16   The student, given a practical problem, will
    a) represent data in line plots and stem-and-leaf plots;
    b) interpret data represented in line plots and stem-and-leaf plots; and
    c) compare data represented in a line plot with the same data represented in a stem-and-leaf plot.

5.17   The student, given a practical context, will
- a) describe mean, median, and mode as measures of center;
- b) describe mean as fair share;
- c) describe the range of a set of data as a measure of spread; and
- d) determine the mean, median, mode, and range of a set of data.

Patterns, Functions, and Algebra

5.18   The student will identify, describe, create, express, and extend number patterns found in objects, pictures, numbers and tables.

5.19   The student will
- a) investigate and describe the concept of variable;
- b) write an equation to represent a given mathematical relationship, using a variable;
- c) use an expression with a variable to represent a given verbal expression involving one operation; and
- d) create a problem situation based on a given equation, using a single variable and one operation.

Grade Six

The sixth-grade standards provide a transition from the emphasis placed on whole number arithmetic in the elementary grades to foundations of algebra. The standards include a focus on rational numbers and operations involving rational numbers. Students will use ratios to compare data sets; recognize decimals, fractions, and percents as ratios; solve single-step and multistep problems, using positive rational numbers; and gain a foundation in the understanding of and operations with integers. Students will solve problems involving area and perimeter, and begin to graph in a coordinate plane. In addition, students will build on the concept of graphical representation of data developed in the elementary grades and develop concepts regarding measures of center. Students will solve linear equations and inequalities in one variable, and use algebraic terminology. Students will represent proportional relationships using two variables as a precursor to the development of the concept of linear functions.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

6.1   The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a:b*.

6.2   The student will
   a) represent and determine equivalencies among fractions, mixed numbers, decimals, and percents; and
   b) compare and order positive rational numbers.

6.3   The student will
   a) identify and represent integers;
   b) compare and order integers; and
   c) identify and describe absolute value of integers.

6.4   The student will recognize and represent patterns with whole number exponents and perfect squares.

Computation and Estimation

6.5   The student will
   a) multiply and divide fractions and mixed numbers;
   b) solve single-step and multistep practical problems involving addition, subtraction, multiplication, and division of fractions and mixed numbers; and
   c) solve multistep practical problems involving addition, subtraction, multiplication, and division of decimals.

6.6   The student will
  a) add, subtract, multiply, and divide integers;
  b) solve practical problems involving operations with integers; and
  c) simplify numerical expressions involving integers.

Measurement and Geometry
6.7   The student will
  a) derive π (pi);
  b) solve problems, including practical problems, involving circumference and area of a circle; and
  c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles.

6.8   The student will
  a) identify the components of the coordinate plane; and
  b) identify the coordinates of a point and graph ordered pairs in a coordinate plane.

6.9   The student will determine congruence of segments, angles, and polygons.

Probability and Statistics
6.10   The student, given a practical situation, will
  a) represent data in a circle graph;
  b) make observations and inferences about data represented in a circle graph; and
  c) compare circle graphs with the same data represented in bar graphs, pictographs, and line plots.

6.11   The student will
  a) represent the mean of a data set graphically as the balance point; and
  b) determine the effect on measures of center when a single value of a data set is added, removed, or changed.

Patterns, Functions, and Algebra
6.12   The student will
  a) represent a proportional relationship between two quantities, including those arising from practical situations;
  b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
  c) determine whether a proportional relationship exists between two quantities; and
  d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

6.13   The student will solve one-step linear equations in one variable, including practical problems that require the solution of a one-step linear equation in one variable.

 6.14   The student will
  a) represent a practical situation with a linear inequality in one variable; and
  b) solve one-step linear inequalities in one variable, involving addition or subtraction, and graph the solution on a number line.

Grade Seven

The seventh-grade standards continue to emphasize the foundations of algebra. The standards address the concept of and operations with rational numbers by continuing their study from grade six. Students will build on the concept of ratios to solve problems involving proportional reasoning. Students will solve problems involving volume and surface area and focus on the relationships among the properties of quadrilaterals. Probability is investigated through comparing experimental results to theoretical expectations. Students continue to develop their understanding of solving linear equations and inequalities in one variable by applying the properties of real numbers. Students discern between proportional and non-proportional relationships and begin to develop a concept of slope as rate of change.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense
7.1   The student will
        a)   investigate and describe the concept of negative exponents for powers of ten;
        b)   compare and order numbers greater than zero written in scientific notation;
        c)   compare and order rational numbers;
        d)   determine square roots of perfect squares; and
        e)   identify and describe absolute value of rational numbers.

Computation and Estimation
7.2   The student will solve practical problems involving operations with rational numbers.

7.3   The student will solve single-step and multistep practical problems, using proportional reasoning.

Measurement and Geometry
7.4   The student will
        a)   describe and determine the volume and surface area of rectangular prisms and cylinders; and
        b)   solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders.

7.5   The student will solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar quadrilaterals and triangles.

7.6   The student will
     a)   compare and contrast quadrilaterals based on their properties; and
     b)   determine unknown side lengths or angle measures of quadrilaterals.

7.7   The student will apply translations and reflections of right triangles or rectangles in the coordinate plane.

Probability and Statistics

7.8   The student will
     a)   determine the theoretical and experimental probabilities of an event; and
     b)   investigate and describe the difference between the experimental probability and theoretical probability of an event.

7.9   The student, given data in a practical situation, will
     a)   represent data in a histogram;
     b)   make observations and inferences about data represented in a histogram; and
     c)   compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.

Patterns, Functions, and Algebra

7.10   The student will
     a)   determine the slope, m, as rate of change in a proportional relationship between two quantities and write an equation in the form $y = mx$ to represent the relationship;
     b)   graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in $y = mx$ form where m represents the slope as rate of change;
     c)   determine the y-intercept, b, in an additive relationship between two quantities and write an equation in the form $y = x + b$ to represent the relationship;
     d)   graph a line representing an additive relationship between two quantities given the y-intercept and an ordered pair, or given the equation in the form $y = x + b$, where b represents the y-intercept; and
     e)   make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

7.11   The student will evaluate algebraic expressions for given replacement values of the variables.

7.12   The student will solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable.

7.13   The student will solve one- and two-step linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and graph the solution on a number line.

Grade Eight

The eighth-grade standards continue to build on the concepts needed for success in high school level algebra, geometry, and statistics. Students will explore real numbers and the subsets of the real number system. Proportional reasoning is expounded upon as students solve a variety of problems. Students find the volume and surface area of more complex three-dimensional figures and apply transformations to geometric shapes in the coordinate plane. Students will verify and apply the Pythagorean Theorem creating a foundation for further study of triangular relationships in geometry. Students will represent data, both univariate and bivariate data, and make predictions by observing data patterns. Students build upon the algebraic concepts developed in the standards for grades six and seven mathematics, which include simplifying algebraic expressions, solving multistep equations and inequalities, and graphing linear functions. The grade eight standards are vital to providing a solid foundation in Algebra I for students in middle school mathematics.

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies to facilitate problem solving. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts or for proficiency in basic computations.

The acquisition of specialized mathematical vocabulary and language is crucial to a student's understanding and appreciation of the subject and fosters confidence in mathematics communication and problem solving.

Problem solving is integrated throughout the content strands. The development of problem-solving skills is a major goal of the mathematics program at every grade level. The development of skills and problem-solving strategies must be integrated early and continuously into each student's mathematics education.

Number and Number Sense

8.1   The student will compare and order real numbers.

8.2   The student will describe the relationships between the subsets of the real number system.

8.3   The student will
   a) estimate and determine the two consecutive integers between which a square root lies; and
   b) determine both the positive and negative square roots of a given perfect square.

Computation and Estimation

8. 4   The student will solve practical problems involving consumer applications.

Measurement and Geometry

8.5   The student will use the relationships among pairs of angles that are vertical angles, adjacent angles, supplementary angles, and complementary angles to determine the measure of unknown angles.

8.6   The student will
   a) solve problems, including practical problems, involving volume and surface area of cones and square-based pyramids; and
   b) describe how changing one measured attribute of a rectangular prism affects the volume and surface area.

8.7   The student will
   a) given a polygon, apply transformations, to include translations, reflections, and dilations, in the coordinate plane; and
   b) identify practical applications of transformations.

8.8   The student will construct a three-dimensional model, given the top or bottom, side, and front views.

8.9   The student will
   a) verify the Pythagorean Theorem; and
   b) apply the Pythagorean Theorem.

8.10   The student will solve area and perimeter problems, including practical problems, involving composite plane figures.

Probability and Statistics

8.11   The student will
   a) compare and contrast the probability of independent and dependent events; and
   b) determine probabilities for independent and dependent events.

8.12   The student will
   a) represent numerical data in boxplots;
   b) make observations and inferences about data represented in boxplots; and
   c) compare and analyze two data sets using boxplots.

8.13   The student will
   a) represent data in scatterplots;
   b) make observations about data represented in scatterplots; and
   c) use a drawing to estimate the line of best fit for data represented in a scatterplot.

Patterns, Functions, and Algebra

8.14   The student will
   a) evaluate an algebraic expression for given replacement values of the variables; and
   b) simplify algebraic expressions in one variable.

8.15   The student will
   a) determine whether a given relation is a function; and
   b) determine the domain and range of a function.

8.16   The student will
  a)   recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;
  b)   identify the slope and *y*-intercept of a linear function, given a table of values, a graph, or an equation in *y* = *mx* + *b* form;
  c)   determine the independent and dependent variable, given a practical situation modeled by a linear function;
  d)   graph a linear function given the equation in *y* = *mx* + *b* form; and
  e)   make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

8.17   The student will solve multistep linear equations in one variable with the variable on one or both sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable.

8.18   The student will solve multistep linear inequalities in one variable with the variable on one or both sides of the inequality symbol, including practical problems, and graph the solution on a number line.

Algebra I

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. All students are expected to achieve the Algebra I standards. The study of Algebra I assists students in generalizing patterns or modeling relevant, practical situations with algebraic models. In order to assist students in developing meaning and connecting algebraic concepts to geometry and statistics, consideration should be given to the sequential development of concepts and skills by using concrete materials to assist students in making the transition from the numeric to the symbolic. Connections between Algebra I and other subject areas through practical applications may assist in helping students attach meaning to the abstract concepts of algebra.

These standards require students to use algebra as a tool for representing and solving a variety of practical problems. Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze behaviors of functions. These standards include a transformational approach to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions. Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Expressions and Operations

A.1   The student will
   a) represent verbal quantitative situations algebraically; and
   b) evaluate algebraic expressions for given replacement values of the variables.

A.2   The student will perform operations on polynomials, including
   a) applying the laws of exponents to perform operations on expressions;
   b) adding, subtracting, multiplying, and dividing polynomials; and
   c) factoring completely first- and second-degree binomials and trinomials in one
      variable.

A.3   The student will simplify
   a) square roots of whole numbers and monomial algebraic expressions;
   b) cube roots of integers; and
   c) numerical expressions containing square or cube roots.

Equations and Inequalities

A.4   The student will solve
   a) multistep linear equations in one variable algebraically;
   b) quadratic equations in one variable algebraically;
   c) literal equations for a specified variable;
   d) systems of two linear equations in two variables algebraically and graphically; and
   e) practical problems involving equations and systems of equations.

A.5   The student will
a) solve multistep linear inequalities in one variable algebraically and represent the solution graphically;
b) represent the solution of linear inequalities in two variables graphically;
c) solve practical problems involving inequalities; and
d) represent the solution to a system of inequalities graphically.

A.6   The student will
a) determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line;
b) write the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line; and
c) graph linear equations in two variables.

Functions

A.7   The student will investigate and analyze linear and quadratic function families and their characteristics both algebraically and graphically, including
a) determining whether a relation is a function;
b) domain and range;
c) zeros;
d) intercepts;
e) values of a function for elements in its domain; and
f) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs.

Statistics

A.8   The student, given a data set or practical situation, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.

A.9   The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems, using mathematical models of linear and quadratic functions.

Geometry

This course is designed for students who have successfully completed the standards for Algebra I. All students are expected to achieve the Geometry standards. The course includes an emphasis on developing reasoning skills through the exploration of geometric relationships including properties of geometric figures, trigonometric relationships, and mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods.

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques, including algebraic skills, should be used to implement these standards. Graphing utilities (calculators, computers, and other technology tools) and dynamic geometry applications will be used to assist in teaching and learning.

Reasoning, Lines, and Transformations

G.1   The student will use deductive reasoning to construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
   a) identifying the converse, inverse, and contrapositive of a conditional statement;
   b) translating a short verbal argument into symbolic form; and
   c) determining the validity of a logical argument.

G.2   The student will use the relationships between angles formed by two lines intersected by a transversal to
   a) prove two or more lines are parallel; and
   b) solve problems, including practical problems, involving angles formed when parallel lines are intersected by a transversal.

G.3   The student will solve problems involving symmetry and transformation. This will include
   a) investigating and using formulas for determining distance, midpoint, and slope;
   b) applying slope to verify and determine whether lines are parallel or perpendicular;
   c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
   d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

G.4   The student will construct and justify the constructions of
   a) a line segment congruent to a given line segment;
   b) the perpendicular bisector of a line segment;
   c) a perpendicular to a given line from a point not on the line;
   d) a perpendicular to a given line at a given point on the line;
   e) the bisector of a given angle,
   f) an angle congruent to a given angle;
   g) a line parallel to a given line through a point not on the line; and
   h) an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Triangles

G.5   The student, given information concerning the lengths of sides and/or measures of angles in triangles, will solve problems, including practical problems. This will include
    a)   ordering the sides by length, given angle measures;
    b)   ordering the angles by degree measure, given side lengths;
    c)   determining whether a triangle exists; and
    d)   determining the range in which the length of the third side must lie.

G.6   The student, given information in the form of a figure or statement, will prove two triangles are congruent.

G.7   The student, given information in the form of a figure or statement, will prove two triangles are similar.

G.8   The student will solve problems, including practical problems, involving right triangles. This will include applying
    a)   the Pythagorean Theorem and its converse;
    b)   properties of special right triangles; and
    c)   trigonometric ratios.

Polygons and Circles

G.9   The student will verify and use properties of quadrilaterals to solve problems, including practical problems.

G.10   The student will solve problems, including practical problems, involving angles of convex polygons. This will include determining the
    a)   sum of the interior and/or exterior angles;
    b)   measure of an interior and/or exterior angle; and
    c)   number of sides of a regular polygon.

G.11   The student will solve problems, including practical problems, by applying properties of circles. This will include determining
    a)   angle measures formed by intersecting chords, secants, and/or tangents;
    b)   lengths of segments formed by intersecting chords, secants, and/or tangents;
    c)   arc length; and
    d)   area of a sector.

G.12   The student will solve problems involving equations of circles.

Three-Dimensional Figures

G.13   The student will use surface area and volume of three-dimensional objects to solve practical problems.

G.14   The student will apply the concepts of similarity to two- or three-dimensional geometric figures. This will include
   a) comparing ratios between lengths, perimeters, areas, and volumes of similar figures;
   b) determining how changes in one or more dimensions of a figure affect area and/or volume of the figure;
   c) determining how changes in area and/or volume of a figure affect one or more dimensions of the figure; and
   d) solving problems, including practical problems, about similar geometric figures.

Algebra, Functions, and Data Analysis

This course is designed for students who have successfully completed the standards for Algebra I and may benefit from additional support in their transition to Algebra II. Within the context of mathematical modeling and data analysis, students will study functions and their behaviors, systems of inequalities, probability, experimental design and implementation, and analysis of data. Data will be generated through practical applications arising from science, business, and finance. Students will solve problems that require the formulation of linear, quadratic, exponential, or logarithmic equations or a system of equations.

Through the investigation of mathematical models and interpretation/analysis of data from relevant, applied contexts and situations, students will strengthen conceptual understandings in mathematics and further develop connections between algebra and statistics. Students should use the language and symbols of mathematics in representations and communication, both orally and in writing, throughout the course.

These standards include a transformational approach to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Algebra and Functions

AFDA.1   The student will investigate and analyze linear, quadratic, exponential, and logarithmic function families and their characteristics. Key concepts include
   a) domain and range;
   b) intervals on which a function is increasing or decreasing;
   c) absolute maxima and minima;
   d) zeros;
   e) intercepts;
   f) values of a function for elements in its domain;
   g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;
   h) end behavior; and
   i) vertical and horizontal asymptotes.

AFDA.2   The student will use knowledge of transformations to write an equation, given the graph of a linear, quadratic, exponential, and logarithmic function.

AFDA.3   The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems using models of linear, quadratic, and exponential functions.

AFDA.4   The student will use multiple representations of functions for analysis, interpretation, and prediction.

AFDA.5   The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

Data Analysis

AFDA.6   The student will calculate probabilities. Key concepts include
      a) conditional probability;
      b) dependent and independent events;
      c) mutually exclusive events;
      d) counting techniques (permutations and combinations); and
      e) Law of Large Numbers.

AFDA.7   The student will
      a) identify and describe properties of a normal distribution;
      b) interpret and compare z-scores for normally distributed data; and
      c) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

AFDA.8   The student will design and conduct an experiment/survey. Key concepts include
      a) sample size;
      b) sampling technique;
      c) controlling sources of bias and experimental error;
      d) data collection; and
      e) data analysis and reporting.

Algebra II

Students enrolled in Algebra II are assumed to have mastered those concepts outlined in the Algebra I standards. A thorough treatment of advanced algebraic concepts will be provided through the study of functions, equations, inequalities, systems of equations, polynomials, rational and radical equations, complex numbers, and sequences and series. Emphasis will be placed on practical applications and modeling throughout the course of study. Oral and written communication concerning the language of algebra, logic of procedures, and interpretation of results should also permeate the course.

These standards include a transformational approach to graphing functions. Transformational graphing uses translation, reflection, dilation, and rotation to generate a "family of functions" from a given "parent" function and builds a strong connection between algebraic and graphic representations of functions. Students will vary the coefficients and constants of an equation, observe the changes in the graph of the equation, and make generalizations that can be applied to many graphs.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Expressions and Operations
AII.1   The student will
   a)   add, subtract, multiply, divide, and simplify rational algebraic expressions;
   b)   add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents; and
   c)   factor polynomials completely in one or two variables.

AII.2   The student will perform operations on complex numbers and express the results in simplest form using patterns of the powers of i.

Equations and Inequalities
AII.3   The student will solve
   a)   absolute value linear equations and inequalities;
   b)   quadratic equations over the set of complex numbers;
   c)   equations containing rational algebraic expressions; and
   d)   equations containing radical expressions.

AII.4   The student will solve systems of linear-quadratic and quadratic-quadratic equations, algebraically and graphically.

Functions
AII.5   The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first n terms, determining the nth term, and evaluating summation formulas. Notation will include Σ and a_n.

AII.6   For absolute value, square root, cube root, rational, polynomial, exponential, and
            logarithmic functions, the student will
            a)   recognize the general shape of function families; and
            b)   use knowledge of transformations to convert between equations and the corresponding
                 graphs of functions.

AII.7   The student will investigate and analyze linear, quadratic, absolute value, square root, cube
            root, rational, polynomial, exponential, and logarithmic function families algebraically and
            graphically. Key concepts include
            a)   domain, range, and continuity;
            b)   intervals in which a function is increasing or decreasing;
            c)   extrema;
            d)   zeros;
            e)   intercepts;
            f)   values of a function for elements in its domain;
            g)   connections between and among multiple representations of functions using verbal
                 descriptions, tables, equations, and graphs;
            h)   end behavior;
            i)   vertical and horizontal asymptotes;
            j)   inverse of a function; and
            k)   composition of functions algebraically and graphically.

AII.8   The student will investigate and describe the relationships among solutions of an equation,
            zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

Statistics

AII.9   The student will collect and analyze data, determine the equation of the curve of best fit in
            order to make predictions, and solve practical problems, using mathematical models of
            quadratic and exponential functions.

AII.10   The student will represent and solve problems, including practical problems, involving
            inverse variation, joint variation, and a combination of direct and inverse variations.

AII.11   The student will
            a)   identify and describe properties of a normal distribution;
            b)   interpret and compare *z*-scores for normally distributed data; and
            c)   apply properties of normal distributions to determine probabilities associated with areas
                 under the standard normal curve.

AII.12   The student will compute and distinguish between permutations and combinations.

Trigonometry

The standards below outline the content for a one-semester course in trigonometry. Trigonometry includes the study of trigonometric definitions, applications, graphing, and solving trigonometric equations and inequalities. Emphasis should also be placed on using connections between right triangle ratios, trigonometric functions, and circular functions. In addition, applications and modeling should be included throughout the course of study. Oral and written communication concerning the language of mathematics, logic of procedure, and interpretation of results should also permeate the course.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Triangular and Circular Trigonometric Functions

T.1   The student, given a point on the terminal side of an angle in standard position, or the value of the trigonometric function of the angle, will determine the sine, cosine, tangent, cotangent, secant, and cosecant of the angle.

T.2   The student will develop and apply the properties of the unit circle in degrees and radians.

Graphs of Trigonometric Functions

T.3   The student, given one of the six trigonometric functions in standard form, will
   a) state the domain and the range of the function;
   b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
   c) sketch the graph of the function by using transformations for at least a two-period interval; and
   d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

T.4   The student will graph the six inverse trigonometric functions.

Equations and Identities

T.5   The student will verify basic trigonometric identities and make substitutions, using the basic identities.

T.6   The student will solve trigonometric equations and inequalities.

T.7   The student will determine the value of any trigonometric function and inverse trigonometric function.

Applications of Trigonometric Functions

T.8   The student will create and solve practical problems involving triangles.

T.9   The student will solve problems, including practical problems, involving
   a) arc length and area of sectors in circles using radians and degrees; and
   b) linear and angular velocity.

Computer Mathematics

This course is intended to provide students with experiences in using computer programming techniques and skills to solve problems that can be set up as mathematical models. Students enrolled in Computer Mathematics are assumed to have studied the concepts and skills in Algebra I and beginning geometry.

Even though computer ideas should be introduced in the context of mathematical concepts, problem solving should be developed in the most general sense, making the techniques applicable by students in many other environments. Strategies include defining the problem; developing, refining, and implementing a plan; and testing and revising the solution. Programming, ranging from simple programs involving only a few lines to complex programs involving subprograms, should permeate the entire course. Programming concepts, problem-solving strategies, and mathematical applications should be integrated throughout the course.

These standards identify fundamental principles and concepts in the field of computer science that will be used within the context of mathematical problem solving in a variety of applications. As students develop and refine skills in logic, organization, and precise expression, they will apply those skills to enhance learning in all disciplines.

Problem Solving
COM.1   The student will design and apply computer programs to solve practical problems in mathematics arising from business and applications in mathematics.

Program Design
COM.2   The student will design, write, document, test, and debug a computer program.

COM.3   The student will write program specifications that define the constraints of a given problem.

COM.4   The student will design an algorithm to solve a given problem.

COM.5   The student will divide a given problem into modules by task and implement the solution.

COM.6   The student will translate mathematical expressions into programming expressions by declaring variables, writing assignment statements, and using the order of operations.

COM.7   The student will select and call library functions to process data, as appropriate.

COM.8   The student will implement conditional statements that include "if/then" statements, "if/then/else" statements, case statements, and Boolean logic.

COM.9   The student will implement pre-defined algorithms, including sort routines, search routines, and simple animation routines.

Program Implementation
COM.10   The student will design and implement the input phase of a program, which will include designing screen layout, getting information into the program by way of user interaction and/or file input, and validating input.

COM.11   The student will design and implement the output phase of a computer program, which will include designing output layout, accessing available output devices, using output statements, and labeling results.

COM.12   The student will design and implement computer graphics to enhance output.

COM.13   The student will implement various mechanisms for performing iteration with an algorithm.

COM.14   The student will select and implement appropriate data structures, including arrays (one- and/or two-dimensional), and objects.

Data Manipulation

COM.15   The student will define and use appropriate variable data types that include integer, real (fixed and scientific notation), character, string, Boolean, and object.

COM.16   The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, parameter passing, scope of variables, and local versus global variables.

Program Testing

COM.17   The student will test a program, using an appropriate set of data. The test data should include boundary cases and test all branches of a program.

COM.18   The student will debug a program using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, other debugging tools available in the programming environment), and identify the difference among syntax errors, runtime errors, and logic errors.

Probability and Statistics

The following standards outline the content of a one-year course in Probability and Statistics. If a one-semester course is desired, the standards with a dagger (†) would apply. The purpose of the course is to present basic concepts and techniques for collecting and analyzing data, drawing conclusions, and making predictions.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Descriptive Statistics

PS.1† The student will analyze graphical displays of univariate data, including dotplots, stemplots, boxplots, cumulative frequency graphs, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers.

PS.2† The student will analyze numerical characteristics of univariate data sets to describe patterns and departures from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers.

PS.3† The student will compare distributions of two or more univariate data sets, numerically and graphically, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features.

PS.4† The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points.

PS.5   The student will determine and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plots to assess linearity.

PS.6   The student will make logarithmic and power transformations to achieve linearity.

PS.7† The student, using two-way tables and other graphical displays, will analyze categorical data to describe patterns and departures from patterns and to determine marginal frequency and relative frequencies, including conditional frequencies.

Data Collection

PS.8† The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.

PS.9† The student will plan and conduct a survey. The plan will address sampling techniques and methods to reduce bias.

PS.10† The student will plan and conduct a well-designed experiment. The plan will address control, randomization, replication, blinding, and measurement of experimental error.

Probability

PS.11[†]   The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.

PS.12[†]   The student will determine probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the Law of Large Numbers concept, the addition rule, and the multiplication rule.

PS.13   The student will develop, interpret, and apply the binomial and geometric probability distributions for discrete random variables, including computing the mean and standard deviation for the binomial and geometric variables.

PS.14   The student will simulate probability distributions, including binomial and geometric.

PS.15   The student will identify random variables as independent or dependent and determine the mean and standard deviations for random variables and sums and differences of independent random variables.

PS.16[†]   The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities.

Inferential Statistics

PS.17   The student, given data from a large sample, will determine and interpret appropriate point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, difference between two means (independent and paired), and slope of a least-squares regression line.

PS.18   The student will apply and interpret the logic of an appropriate hypothesis-testing procedure. Tests will include large sample tests for proportion, mean, difference between two proportions, difference between two means (independent and paired); chi-squared tests for goodness of fit, homogeneity of proportions, and independence; and slope of a least-squares regression line.

PS.19   The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.

PS.20   The student will identify properties of a *t*-distribution and apply *t*-distributions to single-sample and two-sample (independent and matched pairs) *t*-procedures.

Discrete Mathematics

The following standards outline the content of a one-year course in Discrete Mathematics. If a one-semester course is desired, the standards with a dagger (†) would apply.

Discrete mathematics may be described as the study of mathematical properties of sets and systems that have a countable (discrete) number of elements. With the advent of modern technology, discrete (discontinuous) models have become as important as continuous models. In this course, the main focus is problem solving in a discrete setting. Techniques that are not considered in the current traditional courses of algebra, geometry, and calculus will be utilized. As students solve problems, they will analyze and determine whether a solution exists (existence problems), investigate how many solutions exist (counting problems), and focus on finding the best solution (optimization problems). Connections will be made to other disciplines. The importance of discrete mathematics has been influenced by computers.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Graphs

DM.1† The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated.

DM.2† The student will solve problems through investigation and application of circuits, cycles, Euler paths, Euler circuits, Hamilton paths, and Hamilton circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.

DM.3† The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization.

DM.4 The student will apply algorithms relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.

Election Theory and Fair Division

DM.5† The student will analyze and describe the issue of fair division in discrete and continuous cases.

DM.6† The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, runoff, sequential runoff, Borda count, and Condorcet winners.

DM.7 The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.

Computer Mathematics

DM.8 The student will describe and apply sorting algorithms and coding algorithms used in sorting, processing, and communicating information.

DM.9† The student will select, justify, and apply an appropriate technique to solve a logic problem.

Recursion and Optimization

DM.10   The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.

DM.11   The student will solve linear programming problems.

DM.12   The student will use the recursive process and difference equations with the aid of appropriate technology to generate
a) compound interest;
b) sequences and series;
c) fractals;
d) population growth models; and
e) the Fibonacci sequence.

DM.13   The student will apply the formulas of combinatorics in the areas of
a) the Fundamental (Basic) Counting Principle;
b) knapsack and bin-packing problems;
c) permutations and combinations; and
d) the pigeonhole principle.

Mathematical Analysis

Students enrolled in Mathematical Analysis are assumed to have mastered Geometry and Algebra II concepts. Mathematical Analysis develops students' understanding of algebraic and transcendental functions, parametric and polar equations, sequences and series, and vectors. The content of this course serves as appropriate preparation for a calculus course.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Functions

MA.1   The student will investigate and identify the properties of polynomial, rational, piecewise, and step functions and sketch the graphs of the functions.

MA.2   The student will investigate and identify the characteristics of exponential and logarithmic functions to graph the function, solve equations, and solve practical problems.

MA.3   The student will apply compositions of functions and inverses of functions to practical situations and investigate and verify the domain and range of resulting functions.

MA.4   The student will determine the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity.

MA.5   The student will investigate and describe the continuity of functions.

Analytic Geometry

MA.6   The student will investigate, graph, and identify the properties of conic sections from equations in vertex and standard form.

MA.7   The student will perform operations with vectors in the coordinate plane and solve practical problems using vectors.

MA.8   The student will identify, create, and solve practical problems involving triangles.

Equations

MA.9   The student will investigate and identify the characteristics of the graphs of polar equations.

MA.10   The student will use parametric equations to model and solve practical problems.

MA.11   The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.

Discrete Mathematics

MA.12   The student will expand binomials having positive integral exponents.

MA.13   The student will determine the sum of finite and infinite convergent series.

MA.14   The student will use mathematical induction to prove formulas and mathematical statements.