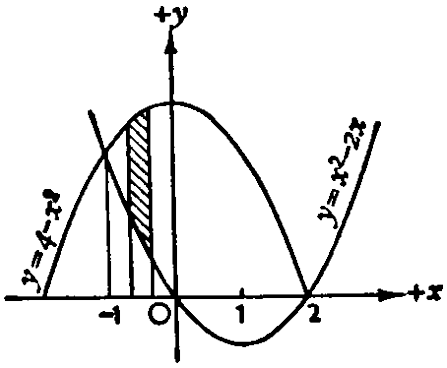


SUMMER PACKET

AP CALCULUS AB

Notre Dame High School

Lawrenceville, New Jersey

Name_____Block__&__

Honors Calculus Summer Review Packet

I. Introduction:

- This is the Honors Calculus Summer Review Packet. It has multiple parts, so please read this introduction thoroughly. By completing this work carefully and accurately, you will be academically prepared to begin Honors Calculus in September/January.

- If you have already completed Pre-Calculus, complete this work over the summer while the material is still familiar. If you are not scheduled for Calculus Honors until Second Semester, be sure to periodically look over the material throughout the first semester.

- If you are taking Pre-Calculus during the first semester next year, **you are still responsible for completing this packet!** You are prepared to complete much of this even after only Algebra 2, so get those parts done over the summer. Then I suggest you periodically and systematically work on completing this throughout the first semester. **Do not wait until you have finished Pre-Calculus** to begin this packet because you will be too overwhelmed in those couple days between semesters!

II. Suggestions:

- Show any necessary work.
- If there is any confusion, there will be time for brief questions during the first several days of class.
- Feel free to get together and work on this, but **everyone must hand in his/her own assignment with the appropriate work.**
- You will be using a TI-83 graphing calculator in this course, so be sure yours is in good working condition.

III. **Timeline** for the first week of class (brief general questions will be addressed in class):

- Day 1: Discuss policies/procedures, clarify standard graphs;
- Day 2: Brief general questions will be addressed in class.
- Day 3: Graphing Quiz (without a calculator!), answer questions;
- Day 4: Clarify theory and answer any final questions;
- Day 5: Summer Packet Test and Packet due.

IV. Graphs: be sure you are able to graph the following equations <u>without a calculator</u>:

$y=x$	$y=[x]$	$y=\frac{1}{x}$	$y=\sqrt{x}$	$y=\sin x$	$y=\csc x$
$y=\lvert x\rvert$	$y=x^3$	$y=c$	$y=e^x$	$y=\cos x$	$y=\sec x$
$y=x^2$	$y=\sqrt[3]{x}$	$x=c$	$y=\ln x$	$y=\tan x$	$y=\cot x$

V. **Readings:** The pages referenced are located in the "Calculus Online Textbook Excerpt" with the math work on the ND website. It is recommended that you **DO NOT PRINT the Online Textbook Excerpt**, as it is 68 pages, but rather view it on your computer or other mobile device. For your convenience, both textbook pages and PDF pages have been provided in this Packet.

A. "What is Calculus?" ((TB p. xix-xxi *I* PDF p. 1-3)

B. Chapter 1 (TB p.1-56/PDF p.4-53) in its entirety. You are responsible for all terminology and information contained therein.

C. Chapter 6 – selected passages
TB p. 327 *I* PDF p. 54: Logarithm Properties (middle of the page)
TB p. 328 *I* PDF p. 55: The number "e"- information and definition (middle of the page)
TB p. 342-346 *I* PDF p. 56-60 - stop at the theorems on p. 346
TB p. 350-351 *I* PDF p. 61-62- stop after "Properties of natural exponential function"
TB p. 357-358 *I* PDF p. 63-64 - stop at "Differentiation and Integration".

VI. Algebra/Pre-Calculus Outline:

Besides completing the actual written work, you need to be familiar and comfortable with the information in the following outline. **Before Unit 1 begins**, it is expected that you are familiar and comfortable with the terminology, notation, procedures, and graphs listed.

A. Functions

1. Formal definition and notation
2. Terminology
   a. domain and range
   b. odd and even
   c. increasing and decreasing
3. Inverse functions
   a. finding inverses
   b. testing if an inverse is also a function
   c. testing if two given functions are inverses
4. Composite functions
5. Intervals
   a. open (neighborhood)
   b. closed
   c. half-open half-closed
   d. notation
6. Compound/Piecewise Functions – evaluating

B. Linear Algebra

1. Graphing
2. Slopes
   a. formal definition
   b. parallel and perpendicular lines
3. Three forms and names
   a. point-slope
   b. slope-y-intercept
   c. standard/general **(new Ax+By+C=0)**
4. Finding linear equations including parallel and perpendicular to a given line

C. Absolute Values

1. Graphing
2. Solving

D. Trigonometry

1. Unit Circle
   a. definition
   b. equation
2. Radian measure
3. Graphs of all 6 functions $[-2\pi;2\pi]$
4. Identities:
   a. Pythagorean
   b. Reciprocal
   c. Cofunction
5. Solving equations

E. Quadratic equations

1. Graphing
2. Solving
   a. factoring
   b. completing the square
   c. quadratic formula

F. Miscellaneous

1. Distance formula
2. Midpoint formula

G. Exponents and logarithms

1. General notation
2. Logarithm Rules
3. Change of base formula
4. “e” “ln”
   a. definition of “e”
   b. relationship between “e” and “ln”
5. Other bases
6. General evaluating and solving

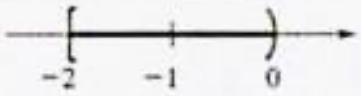
Do Print, Solve and Hand in this Problem set: show all necessary work.

I.I. Real Numbers and the Real Line

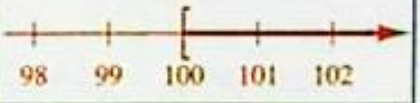
In Exercises 1-12, complete the table by filling in the appropriate interval notation, set notation, and graph on the real line:

	Set-builder Notation	Interval Notation	Graph
1.	$\{x \in \mathbb{R} \mid x < 3 \text{ or } x \geq 5\}$		
2.		$(-4,0) \cup (1,\infty)$	
3.			
4.	$\{x \in \mathbb{R} \mid -5 \leq x < 4\}$		
5.		$(-\infty,-2) \cup (2,\infty)$	
6.			
7.	$\{x \in \mathbb{R} \mid x < -3 \text{ or } -2 \leq x \leq 3\}$		
8.		$(-\infty,-4] \cup (-2,\infty)$	
9.			
10.	$\{x \in \mathbb{R} \mid x \leq -2 \text{ or } -1 < x < 4\}$		

11)

Interval notation	*Set notation*	*Graph*
		
$(-\infty, -4]$		
	$\{x: 3 \leq x \leq \frac{11}{2}\}$	
$(-1, 7)$		

12)

Interval notation	*Set notation*	*Graph*
		
	$\{x: 10 < x\}$	
$(\sqrt{2}, 8]$		
	$\{x: \frac{1}{3} < x \leq \frac{22}{7}\}$	

Solve inequality and graph the solution on the real line:

13. $2x>3$

14. $2x+7<3$

15. $3x+1\geq 2x+2$

16. $0\leq x+3<5$

17. $-1<-\frac{x}{3}<1$

18. $x>\frac{1}{x}$

19. $\frac{x}{2}-\frac{x}{3}>5$

20. $\left|\frac{x}{2}\right|>3$

21. $|x+2|<5$

22. $|3x+1|\geq 4$

23. $|9-2x|<1$

24. $2x^2+1<9x-3$

25. In the manufacture and sale of a certain product, the revenue for selling x units is R=115.95x and the cost of producing x units is C=95x+750. In order for a profit to be realized, R must be greater than C. For what values of x will this product return a profit?

26. A utility company has a fleet of vans. The annual operating cost of each van is estimated to be C=0.32m+2300, where C is measured in dollars and m is measured in miles. If the company wants the annual operating cost of each van to be less than $10,000, then m must be less than what value?

27. A business had annual retail sales of $110,000 in 1993 and $224,000 in 1996. Assuming that the annual increase in sales followed a linear pattern, what were the retail sales in 1995?

I.II. The Cartesian Plane

In exercises 28-33, a) plot the points, b) find the distance between the points, and c) find the midpoint of the line segment joining the points:

28. (2,1), (4,5)

29. (-3,2), (3,-2)

30. $(\frac{1}{2},1),(-\frac{3}{2},-5)$

31. $(\frac{2}{3},-\frac{1}{3}),(\frac{5}{6},1)$

32. $(1,\sqrt{3}),(-1,1)$

33. $(-2,0),\ (0,\sqrt{2})$

In exercises 34-35, find x so that the distance between the points is 5:

34. (0,0), (x,-5)

35. (2,-1), (x,2)

In exercises 36-37, find y so that the distance between the points is 8:

36. (0,0), (3,y)

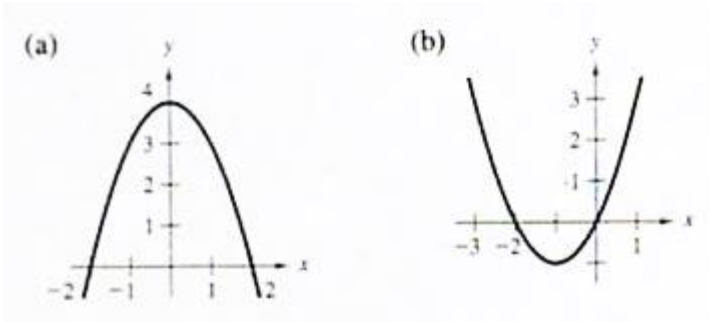
37. (5,1), (5,y)

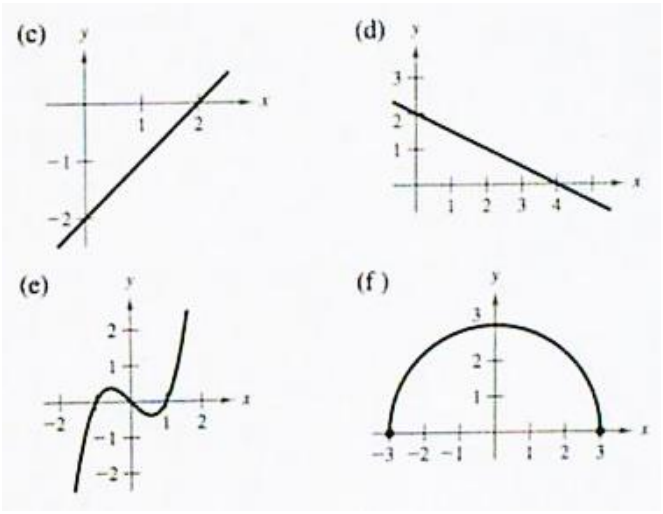
I.III. Graphs of Equations

In exercises 38-43, match the given equation with its graph:

38. y=x-2 39. $y = -\frac{1}{2}x + 2$ 40. $y=x^2+2x$

41. $y=\sqrt{9-x^2}$ 42. $y=4-x^2$ 43. $y=x^3-x$





In exercises 44-46, find intercepts:

44. y=(x-1)(x-3)

45. $y^2=x^3-4x$

46. xy=4

In exercises 47-50, sketch the graph of each equation, and identify the intercepts:

47. y=x-2

48. y=2x-3

49. $y=-\frac{1}{2}x+2$

50. $y=x^2+3$

In exercises 51-53, find the points of intersection of the graphs of the equations:

51. 2x-3y=13, 5x+3y=1

52. $x^2+y^2=25$, 2x+y=10

53. $x^2+y=4$, 2x-y=1

In exercises 54-55, find the sales necessary to break even (R=C) for the given cost C of x units and the given revenue R obtained by selling x units:

54. C=8650x+250,000

R=9950x

55. C=5.5√x+10,000

R=3.29x

I.IV. Lines in the Plane

In exercises 56-57, plot the given pair of points and find the slope of the line passing through them:

56. (-2,1), (4,-3)

57. $(\frac{7}{8},\frac{3}{4}),(\frac{5}{4},-\frac{1}{4})$

In exercises 58-59. use the given point on the line and the slope of the line to find three additional points that the line passes through:

58. (-3,4), m is undefined

59. (-2,-2), m=2

In exercises 60-61, find the slope and y-intercept (if possible) of the line specified by the given equation:

60. 6x-5y=15

61. y=-1

In exercises 62-67, find an equation for the line that passes through the given points, and sketch the graph of the line:

62. (2,1), (0,-3)

63. (-3,-4), (1,4)

64. (0,0), (-1,3)

65. (-3,6), (1,2)

66. (1,-2),(3,-2)

67. ($\left(\frac{7}{8},\frac{3}{4}\right),\left(\frac{5}{4},-\frac{1}{4}\right)$

In exercises 68-73, find an equation of the line that passes through the given point and has the indicated slope:

68. (0,3) m=$\frac{3}{4}$

69. (-1,2) m is undefined

70. (0,0) m=$\frac{2}{3}$

71. (-2,4), m=-$\frac{3}{5}$

72. (0,2) m=4

73. (0,4) m=0

In exercises 74-79, write an equation of the line through the given point (a) parallel to the given line and (b) perpendicular to the given line:

74. (2,1) 4x-2y=3

75. (-3,2) x+y=7

76. $\left(\frac{7}{8},\frac{3}{4}\right)$ 5x+3y=0

77. (-6,4) 3x+4y=7

78. (2,5) x=4

79. (-1,0) y=-3

I.V. Functions.

80. Given f(x)=2x-3, find the following:

(a) f(0) (b) f(-3) (c) f(b) (d) f(x-1)

81. Given f(x)=3x-1, find $\frac{f(x)-f(1)}{x-1}$

In exercises 82-91, find the domain and range of the given function, and sketch its graph:

82. f(x)=4-x

83. f(x)=$\frac{1}{3}x$

84. f(x)=4-x^2

85. f(x)=$\frac{4}{x}$

86. f(x)= $\sqrt{x-1}$

87. f(x)= $\frac{1}{2}x^3+2$

88. f(x)= $\sqrt{9-x^2}$

89. f(x)= $\sqrt{25-x^2}$

90. f(x)=|x-2|

91. f(x)= $\frac{|x|}{x}$

92. Use the graph of f(x)= $\sqrt{x}$ to sketch the graph of each of the following:

(a) y= $\sqrt{x}+2$

(b) y= $-\sqrt{x}$

(c) y= $\sqrt{x-2}$

(d) y= $\sqrt{x+3}$

(e) y= $\sqrt{x}-4$

(f) y= $2\sqrt{x}$

93. Given $f(x)=\frac{1}{x}$ and $g(x)=x^2-1$, find the following:

(a) f(g(2)) (b) g(f(2)) (c) $f(g(\frac{1}{\sqrt{2}}))$

(d) $g(f(\frac{1}{\sqrt{2}}))$ (e) g(f(x)) (f) f(g(x))

In exercises 94-95, find the composite functions f(g(x)) (which is the same as f∘g) and g(f(x)) (which is the same as g∘f) . What is the domain of each function? Are the two composite functions equal?

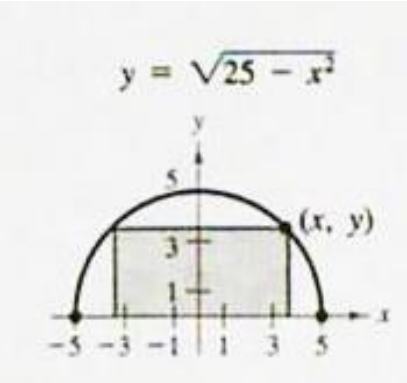
94. $f(x)=x^3$, $g(x)=\sqrt[3]{x}$ 95. $f(x)=x^2-1$, $g(x)=x$

In exercises 96-97, find the real zeros of the function:

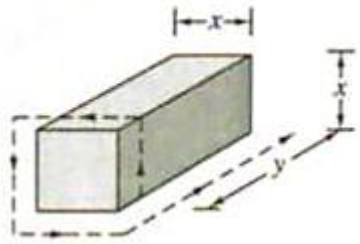
96. $f(x)=x^2-9$ 97. $f(x)=x^3-x$

98. An open box is to be made from a square piece of material 12 inches on a side, by cutting equal squares from each corner and turning up the sides. Express the Volume V as a function of x.

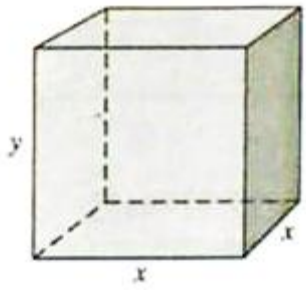
99. A rectangle is bounded by the x-axis and the semicircle $y=\sqrt{25-x^2}$. Write the area A of the rectangle as a function of x.



100. A rectangular package with square cross-sections has a combined length and girth (perimeter of a cross section) of 108 inches. Express the Volume V as a function of x.



101. A closed box with a square base of side x has a surface area of 100 square feet. Express the volume V of the box as a function of x.



I.VI. Trigonometric Functions

In exercises 102-103, express the given angle in radian measure as a multiple of π:

#102. a) 30º b) 150º c) 315º d) 120º

#103. a) -20º b) -240º c) -270º d) 144º

In exercises 104-105, express the given angle in degree measure:

#104. a) $\frac{3\pi}{2}$ b) $\frac{7\pi}{6}$

#105. a) $\frac{7\pi}{3}$ b) $-\frac{11\pi}{30}$

In exercises 106-109, solve the given equation for θ $(0 \le \theta < 2\pi)$. For some of the equations, you should use the trigonometric identities:

#106. $2\sin^2\theta = 1$

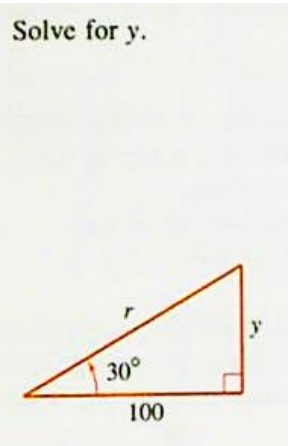
#107. $\tan^2\theta = 3$

#108. $\tan^2\theta - \tan\theta = 0$

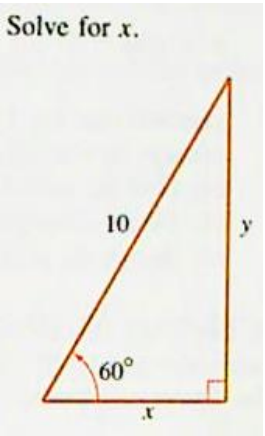
#109. $2\cos^2\theta - \cos\theta = 1$

In exercises 110-113, solve for x, y, or r as indicated:

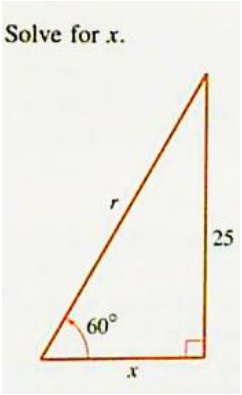
#110.



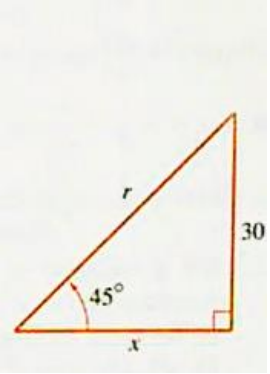
#111.



#112.



#113.



I.VII. Logarithmic, Exponential, and other Transcendental Functions

In exercises 114-121, use the properties of logarithms to write each expression as a sum, difference, or multiple of logarithms:

114. $\ln\frac{2}{3}$ 115. ln(xyz) 116. $\ln\frac{xy}{z}$ 117. $\ln(\frac{x^2-1}{x^3})^3$

118. $\ln\sqrt{a-1}$ 119. $\ln\sqrt{2^3}$ 120. $\ln\frac{1}{5}$ 121. $\ln 3e^2$

In exercises 122-127, write each expression as a logarithm of a single quantity:

122. ln(x-2)-ln(x+2) 123. 3lnx+2lny-4lnz 124. $\frac{1}{3}\left[2\ln(x+3)+\ln x-\ln(x^2-1)\right]$

126. 2[lnx-ln(x+1)-ln(x-1)] 127. $2\ln 3-\frac{1}{2}\ln(x^2+1)$ 128. $\frac{3}{2}[\ln(x^2+1)-\ln(x+1)-\ln(x-1)]$

In exercises 129-132, solve for x:

129. (a) $e^{\ln x}=4$ (b) $\ln e^{2x}=3$

130. (a) $e^{\ln 2x}=12$ (b) $\ln e^{-x}=0$

131. (a) $\ln x=2$ (b) $e^{x}=4$

132. (a) $\ln x^{2}=10$ (b) $e^{-4x}=5$

In exercises 133-138, solve for x (or b):

133. (a) $\log_{10}1000=x$ (b) $\log_{10}.1=x$

134. (a) $\log_4 \frac{1}{64} = x$ (b) $\log_5 25=x$

135. (a) $\log_3 x=-1$ (b) $\log_2 x=-4$

136. (a) $\log_b 27=3$ (b) $\log_b 125=3$

137. (a) $x^{2}-x=\log_5 25$ (b) $3x+5=\log_2 64$

138. (a) $\log_3 x+\log_3(x-2)=1$ (b) $\log_{10}(x+3)-\log_{10}x=1$

In exercises 139-142, find the inverse of f(x), then graph both f(x) and $f^{-1}(x)$:

139. $f(x)=3x$

140. $f(x)=x^3+1$

141. $f(x)=x^2$

142. $f(x)=2x-3$