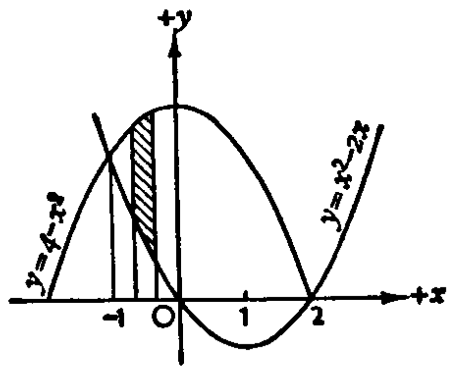


SUMMER PACKET

AP CALCULUS BC

Notre Dame High School

Lawrenceville, New Jersey

Name_________________________Block__&__

AP Calculus BC Summer Review Packet

I. Introduction:

- This is the AP Calculus BC Summer Review Packet. It has multiple parts, so please read this introduction thoroughly. By completing this work carefully and accurately, you will be academically prepared to begin AP Calculus BC.
- AP Calculus BC starts where Calculus AB left off, completing your understanding of the theory and applications of integral calculus, and further applying the concepts of calculus to parametric functions, polar functions, and series. To be successful in Calculus BC, the presumption is that you already have an exhaustive understanding of differential calculus and a fair introduction to integral calculus from AP Calculus AB. The objective of this course is to make you work daily toward gaining a deeper, more intuitive understanding of what calculus is and how it works. The course is rigorous and much is expected of you. There is a lot challenging material to learn in a short time and most of the material is new. You must be aware of this and plan to be committed to working to the best of your ability.
- Complete this work over the summer while the material is still familiar. Then, since you will not be taking Calculus BC until Second Semester, be sure to periodically look over the material throughout the first semester.

II. Suggestions:

Here are some things you need to bear in mind as you tackle this assignment:

- ❖ Do not wait too long before you start this work. There is a lot to be done, but if you pace yourself, it won't take too long.
- ❖ **Please show any necessary work.** Not all exercises may need work, but if you need to do work, hand it in. **THIS POINT CANNOT BE EMPHISIZED ENOUGH!!!**
- ❖ Feel free to get together to work on this, but everyone must hand in his/her own assignment with **the appropriate work.**
- ❖ You will be using a graphing calculator in this course, so be sure yours is in good working condition. You need to be an adept user of technology, minimally competent in the four required calculator uses on the AP exam:

1. Plot the graph of a function within an arbitrary viewing window;
2. Find the zeros of functions (solve equations numerically)
3. Numerically calculate the derivative of a function;
4. Numerically calculate the value of a definite integral.

You should also be comfortable using the calculator to store functions using the "Y=menu." For example, a typical free-response question about area or volume may require you to evaluate definite integrals using the calculator. By storing functions in the calculator as Y1(X) or Y2(X), you only need to enter the functions carefully one time. You do not need to reenter them each time that you have to use numerical integration on the calculator. This is particularly helpful when the integrand involves the sum or difference of two functions. This saves you time and decreases the chances of making entry mistakes (particularly with parentheses, exponents, or radicals) under the pressure of the exam.

III. Timeline for the First Couple Days of Class:

Day 1: Brief, general questions will be addressed in class;

Day 2: Summer Packet Quiz and Packet due.

IV. Problems: can be found at the end of this outline.

V. Graphs: be sure you are able to graph the following equations <u>without a calculator</u>:

y=x	y=[x]	$y=\frac{1}{x}$	$y=\sqrt{x}$	y=sinx	y=cscx
y=|x|	$y=x^3$	y=c	$y=e^x$	y=cosx	y=secx
$y=x^2$	$y=\sqrt[3]{x}$	x=c	y=lnx	y=tanx	y=cotx

VI. AP Calculus AB Outline:

Besides completing the actual written work, you need to be familiar and comfortable with the information in the following outline. It is expected that you are familiar and comfortable with the terminology, notation, procedures, and graphs listed.

A. Limits and Their Properties
1.Finding limits graphically and numerically
2. Evaluating Limits analytically
3. Continuity and One-Sided Limits
4. Infinite Limits

B. Differentiation
1. Relationship between Derivatives and the Tangent line
2. Basic differentiation rules and rates of change
3. Product and Quotient Rules
4. Higher order derivatives
5. Chain Rule
6. Implicit Derivatives

C. Applications of Differentiation
1.Related Rates
2. Extrema on an Interval
3. Intervals of Increase and Decrease and the First Derivative Test
4. Concavity and the Second Derivative Test
5. Rolle's Theorem
6. Intermediate Value Theorem (IVT)
7. Mean Value Theorem (MVT)
8. Curve Sketching

D. Integration
1.Antiderivatives and Indefinite Integration
2. Area
3. Riemann Sums and Definite Integrals
4. The Fundamental Theorem of Calculus (FTC)
5. Integration by substitution
6. Numerical Integration

E. Logarithmic, Exponential, and other Transcendental Functions
a. Differentiation
b. Integration

1.The Natural Logarithmic Function

2. Exponential Functions

3. Bases Other than e

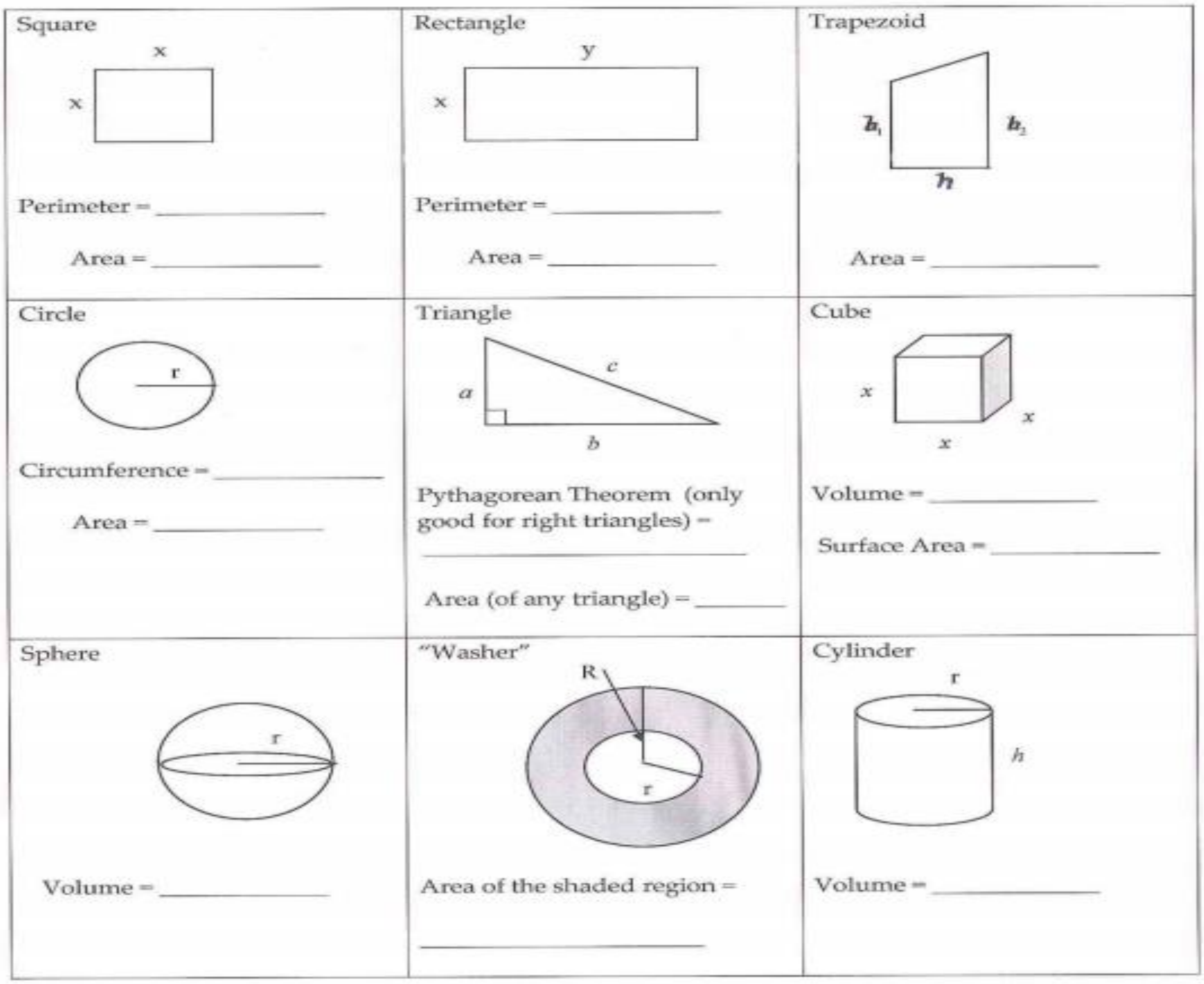
4.Inverse Trigonometric Functions

F. Differential Equations
1.Slope Fields
2. Separation of Variables
3. First-Order Linear Differential Equations

G. Applications of Integration
1. Area of a Region between two curves
2. Volume: The Disk Method

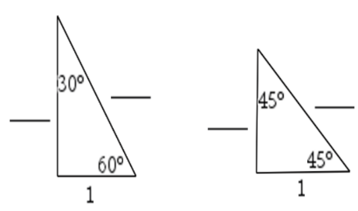
Geometry

You will use each of the following formulas in AP Calculus. Complete each of the following.



Square x x Perimeter = ____________ Area = ____________	Rectangle y x Perimeter = ____________ Area = ____________	Trapezoid b_1 b_2 h Area = ____________
Circle r Circumference = ____________ Area = ____________	Triangle a c b Pythagorean Theorem (only good for right triangles) = ____________________ Area (of any triangle) = ________	Cube x x x Volume = ____________ Surface Area = ____________
Sphere r Volume = ____________	"Washer" R r Area of the shaded region = ____________________	Cylinder r h Volume = ____________

Basic Right Angle Trigonometry



If point *P* is on the terminal side of ϑ, find all 6 trigonometric functions of ϑ. (Answers need not be rationalized.)

1.) $P(-2,4)$

Special Angles

Evaluate each of the following.

2.) $\left(\cos\frac{2\pi}{3}-\tan\frac{3\pi}{4}\right)^2$

3.) $\left(\sin\frac{11\pi}{6}-\tan\frac{5\pi}{6}\right)\left(\sin\frac{11\pi}{6}+\tan\frac{5\pi}{6}\right)$

Trigonometric Identities

Verify the following identities:

4.) $\frac{1-\sec x}{1-\cos x}=-\sec x$

Solving Trigonometric Equations

Solve the equation on the interval $[0,2\pi)$. Do not use a calculator.

5.) $\sin^2 x=\sin x$

Graphical Solutions to Equations and Inequalities

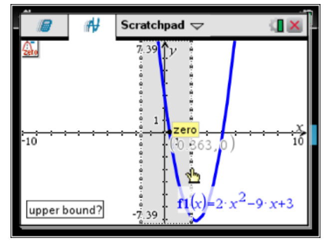
You have a TI graphing calculator. So, when are we going to use it? So far, no mention has been made of it. Yet, a graphing calculator is a tool that is required on the AP Calculus exam. For about 25% of the exam, a calculator is permitted. So, it is vital you are comfortable using it.

There are several settings of the calculator you should make. First, so you don't get into rounding difficulties, it is suggested you show at least three decimal places (and preferably more). This is standard on the AP Calculus exam, so it's best we start getting used to it.

Also, be sure that your calculator's Angle Setting is in Radian mode throughout the year. To make these changes go to "mode" and select 'Radian" mode.

You must know how to graph functions on your TI-calculator.

<u>How to find zeros (solutions or *x*-intercepts) of a function.</u>



Step 1: Enter the left side of the equation that's already set equal to zero (for example, $2x^2 - 9x + 3 = 0$) into the *y1*(*x*) entry line.

Step 2: Select 2nd - Trace - Zero.

Step 3: Move the cursor to a position you feel is left of the zero you wish to find first and press Enter. Then move the cursor to the right of the desired zero you wish to find and press Enter. Click enter one more time. You will notice that the ordered pair for the zero will show up automatically once it falls within the range of your lower and upper bound.

You can find relative Maximum and Minimum values in a similar way.

<u>How to find the intersection of two functions.</u>

Step 1: Enter each side of the equation (for example, $x^3 = 2x - 3$ into the *y1*(*x*) and *y2*(*x*) entry lines.

Step 2: Select 2nd - Trace - Intersect

Step 3: Make cursor be close enough to a point of intersection, click enter, enter, enter.

This problem could also be solved by setting the above equation equal to zero, and by finding zeros instead.

We will learn much more about the functionality of this calculator in the first several days of class. While many of these problems on this page can be done with other graphing utilities (desmos, etc.), I highly recommend that you wait to do these problems when you have a TI graphing calculator.

Using the TI graphing calculator (Continued)

Use your graphing calculator to find the zeros of each of the following functions. Make sure each equation is set equal to zero first.

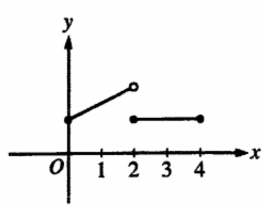
6.) $3x^3 - x - 5 = 0$

7.) $2x^2 - 1 = 2^x$

8.) $2\ln(x+1) = 5\cos x$ on $[0, 2\pi)$

Limits and Continuity

(C – Calculator Active, NC – Non-Calculator part)

9 NC	$\lim_{x\to\infty} \frac{(2x-1)(3-x)}{(x-1)(x+3)}$ is (A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent
10 NC	$\lim_{x\to 0} \frac{5x^4+8x^2}{3x^4-16x^2}$ is (A) $-\frac{1}{2}$ (B) 0 (C) 1 (D) $\frac{5}{3}$ (E) nonexistent
11 NC	$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$ Let f be the function defined above. Which of the following statements about f are true? I. f has a limit at $x = 2$. II. f is continuous at $x = 2$. III. f is differentiable at $x = 2$. (A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III
12 NC	 Graph of f The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true? I. $\lim_{x\to 2^-} f(x)$ exists. II. $\lim_{x\to 2^+} f(x)$ exists. III. $\lim_{x\to 2} f(x)$ exists. (A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

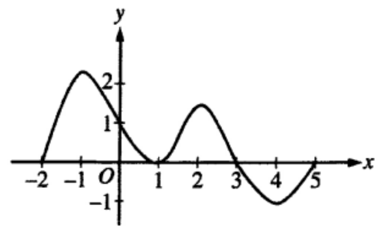
13NC	If $f(x)=2x$, $g(x)=x^2$, and $h(x)=2^x$, which of the following limits is equal to 0? (A) $\lim_{x\to\infty}\frac{g(x)}{f(x)}$ (B) $\lim_{x\to\infty}\frac{g(x)}{h(x)}$ (C) $\lim_{x\to\infty}\frac{h(x)}{f(x)}$ (D) $\lim_{x\to\infty}\frac{h(x)}{g(x)}$ (E) $\lim_{x\to\infty}\frac{h(x)}{f(x)g(x)}$
14NC	$\lim_{x\to5}\frac{2x^2-50}{x^2-15x+50}=$ (A) -4 (B) -1 (C) 0 (D) 1 (E) 2
15NC	$\lim_{x\to2}\frac{e^{2x}-e^4}{x-2}=$ (A) e (B) $2e$ (C) $2e^2$ (D) e^4 (E) $2e^4$

Derivatives

16NC	If $f(x)=(x-1)(x^2+2)^3$, then $f'(x)=$ (A) $6x(x^2+2)^2$ (B) $6x(x-1)(x^2+2)^2$ (C) $(x^2+2)^2(x^2+3x-1)$ (D) $(x^2+2)^2(7x^2-6x+2)$ (E) $-3(x-1)(x^2+2)^2$
17NC	8. If $f(x)=\cos(3x)$, then $f'\left(\frac{\pi}{9}\right)=$ (A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$
18NC	If $f(x)=e^{(2/x)}$, then $f'(x)=$ (A) $2e^{(2/x)}\ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

19NC	If $\sin(xy) = x$, then $\frac{dy}{dx} =$ (A) $\frac{1}{\cos(xy)}$ (B) $\frac{1}{x\cos(xy)}$ (C) $\frac{1-\cos(xy)}{\cos(xy)}$ (D) $\frac{1-y\cos(xy)}{x\cos(xy)}$ (E) $\frac{y(1-\cos(xy))}{x}$
20NC	If $f(x) = x^2 + 2x$, then $\frac{d}{dx}(f(\ln x)) =$ (A) $\frac{2\ln x + 2}{x}$ (B) $2x\ln x + 2x$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$ (E) $\frac{2x+2}{x}$
21NC	What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$? (A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2
22NC	Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$? (A) $-\frac{1}{2}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) The value of $g'(3)$ cannot be determined from the information given.
23C	The derivative of the function f is given by $f'(x) = x^2\cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$? (A) One (B) Two (C) Three (D) Four (E) Five

24NC



Graph of f'

The graph of f', the derivative of f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

(A) $[-2, 1]$ only

(B) $[-2, 3]$

(C) $[3, 5]$ only

(D) $[0, 1.5]$ and $[3, 5]$

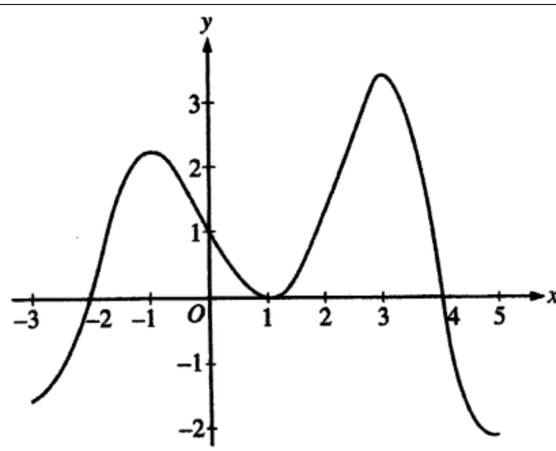
(E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$

25C

What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ and $y = x + 5$?

(A) 10.667 (B) 11.833 (C) 14.583 (D) 21.333 (E) 32

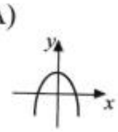
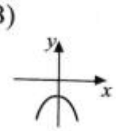
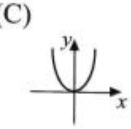
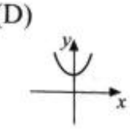
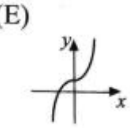
26C



Graph of f'

The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 3$. At which of the following values of x does f have a relative maximum?

(A) −2 only (B) 1 only (C) 4 only (D) −1 and 3 only (E) −2, 1, and 4

27C	The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$.) (A) -108π (B) -72π (C) -48π (D) -24π (E) -16π
28NC	If $f(x) = e^{-x} + \sin x - \cos x$, then $f''(0) =$ (A) -2 (B) -1 (C) 0 (D) 1 (E) 2
29NC	The slope of the line tangent to the curve $3x^2 - 2xy + y^2 = 11$ at the point $(1, -2)$ is (A) $-\frac{1}{6}$ (B) 0 (C) 1 (D) $\frac{5}{3}$ (E) 10
30NC	$\frac{d}{dx}\left[\ln(\sec x)\right] =$ (A) $\cos x$ (B) $\tan x$ (C) $\cos x \cot x$ (D) $\frac{\sec x \tan x}{x}$ (E) $\frac{\sec x}{x} + \ln\left(\sec x \tan x\right)$
31NC	If $f''(0) < 0$, $f'(0) = 0$, and $f(0) > 0$, which of the following could be the graph of $y = f(x)$? (A)  (B)  (C)  (D)  (E) 
32NC	Given that $f(-3) = 4$ and $f'(-3) = 2$, which of the following is the tangent line approximation of $f(-3.1)$? (A) 3.8 (B) 3.9 (C) 4.0 (D) 4.1 (E) 4.2
33NC	If $x^2 y + y^2 + 4 = 0$, then when $x = 2$, the value of $\frac{dy}{dx}$ is (A) -2 (B) -1 (C) 0 (D) 2 (E) nonexistent

34NC	The slope to the tangent line to the graph of $y = \tan 2x$ at $x = \frac{\pi}{8}$ is (A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$ (E) 4
35NC	If $x^2 - y^2 = 5$, what is the value of $\frac{d^2y}{dx^2}$ at the point (3, 2)? (A) $-\frac{13}{8}$ (B) $-\frac{11}{8}$ (C) $-\frac{7}{8}$ (D) $-\frac{5}{8}$ (E) $-\frac{1}{4}$
36NC	The velocity of a particle at time t is given by the function $v(t) = t^3 - \sin t + 2$. What is the acceleration of the particle at time $t = 2\pi$? (A) $6\pi - 1$ (B) $6\pi + 1$ (C) $12\pi^2 - 1$ (D) $12\pi^2$ (E) $12\pi^2 + 1$

Integration

37NC	$\int \frac{1}{x^2}\,dx =$ (A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$
38NC	$\int (\sin(2x) + \cos(2x))\,dx =$ (A) $\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$ (B) $-\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) + C$ (C) $2\cos(2x) + 2\sin(2x) + C$ (D) $2\cos(2x) - 2\sin(2x) + C$ (E) $-2\cos(2x) + 2\sin(2x) + C$
39NC	(table below)

x	0	1	2	3
$f''(x)$	5	0	−7	4

The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

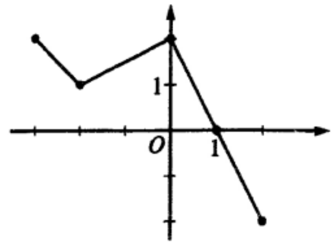
(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$.

40NC



Graph of f

The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$

41C

x	-4	-3	-2	-1
$f(x)$	0.75	-1.5	-2.25	-1.5
$f'(x)$	-3	-1.5	0	1.5

The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x)\,dx$?

(A) -4.5 (B) -2.25 (C) 0 (D) 2.25 (E) 4.5

42NC

$\int_{1}^{8} x^{-\frac{2}{3}}\,dx =$

(A) $-\frac{31}{48}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{3}$ (D) 1 (E) 3

43NC

$\int_{\pi/6}^{\pi/4} 2\sin(2x)\cos(2x)\,dx =$

(A) $-\frac{3}{8}$ (B) $\frac{1}{8}$ (C) $\frac{5\pi^2}{288}$ (D) $\frac{1}{4}$ (E) $\frac{3}{8}$

44NC

The area of the region bounded by the graphs of $y = x^2$ and $y = \sqrt{x}$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) 1

45NC	If $\int_0^3 e^{\sin x}\,dx = k$, then $\int_1^2 x\,e^{\sin(4-x^2)}\,dx =$ (A) $-\frac{k}{2}$ (B) $-\frac{k}{3}$ (C) $\frac{k}{6}$ (D) $\frac{k}{3}$ (E) $\frac{k}{2}$
46NC	What is the x-coordinate of the point of inflection of the graph of $y = x^3 + 3x^2 - 45x + 81$? (A) -9 (B) -5 (C) -1 (D) 1 (E) 3
47NC	The area of the region enclosed by the graph of $y = 5 - x^2$ and the line $y = 1$ is (A) $\frac{4}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{16}{3}$ (E) $\frac{32}{3}$
48NC	The average value of e^{3x} on the interval [0, 4] is (A) $\frac{e^{12}-1}{12}$ (B) $\frac{e^{12}}{12}$ (C) $\frac{e^{12}-1}{4}$ (D) $\frac{e^{12}}{4}$ (E) $\frac{e^{12}-1}{3}$