GEOMETRY

# Cumulative Review: Chapters 1–11

**In Exercises 1–12 classify each statement as true or false.**

A **1.** If point $A$ lies on $\overrightarrow{BC}$, but not on $\overline{BC}$, then $B$ is between $A$ and $C$.

**2.** A true conditional always has a true converse.

**3.** The statement "If $ac = bc$, then $a = b$" is true for all real numbers $a$, $b$, and $c$.

**4.** If two parallel lines are cut by transversal $t$ and $t$ is perpendicular to one of the lines, then $t$ must also be perpendicular to the other line.

**5.** If $\triangle ABC \cong \triangle DEF$ and $\angle A \cong \angle B$, then $\overline{DE} \cong \overline{EF}$.

**6.** If the opposite sides of a quadrilateral are congruent and the diagonals are perpendicular, then the quadrilateral must be a square.

**7.** If $\triangle GBS \sim \triangle JFK$, then $\frac{JF}{JK} = \frac{GB}{GS}$.

**8.** The length of the altitude to the hypotenuse of a right triangle is always the geometric mean between the lengths of the legs.

**9.** In any right triangle, the sine of one acute angle is equal to the cosine of the other acute angle.

**10.** If an angle inscribed in a circle intercepts a major arc, then the measure of the angle must be between 180 and 360.

**11.** The angle bisectors of an obtuse triangle intersect at a point that is equidistant from the three vertices.

**12.** If $JK = 10$, then the locus of points in space that are 4 units from $J$ and 5 units from $K$ is a circle.

**13.** Two lines that do not intersect are either __?__ or __?__.

**14.** In $\triangle RST$, $m\angle R = 2x + 10$, $m\angle S = 3x - 10$, and $m\angle T = 4x$.
**a.** Find the numerical measure of each angle.
**b.** Is $\triangle RST$ scalene, isosceles, or right? Why?

**15.** Use inductive thinking to guess the next number: 10, 9, 5, −4, −20, __?__.

**16.** If a diagonal of an equilateral quadrilateral is drawn, what method could be used to show that the two triangles formed are congruent?

**17.** A trapezoid has bases with lengths $x + 3$ and $3x - 1$ and a median of length 11. Find the value of $x$.

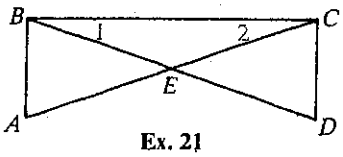
**18.** If 4, 7, and $x$ are the lengths of the sides of a triangle and $x$ is an integer, list the possible values for $x$.

**19.** Describe the locus of points in space that are 4 cm or less from a given point $P$.

**20.** Two similar rectangles have diagonals of $6\sqrt{3}$ and 9. Find the ratio of their perimeters and the ratio of their areas.

**21.** Given: $\overline{AB} \perp \overline{BC}$; $\overline{DC} \perp \overline{BC}$; $\overline{AC} \cong \overline{BD}$
Prove: $\triangle BCE$ is isosceles.

**22.** Given: Quad. $EFGH$; $\overline{EF} \cong \overline{HG}$; $\overline{EF} \parallel \overline{HG}$
Prove: $\angle EHF \cong \angle GFH$



**Ex. 21**

**23.** Use an indirect proof to show that no triangle has sides of length $x$, $y$, and $x + y$.

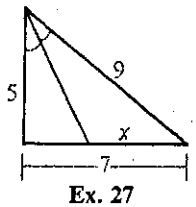
**24.** The legs of a right triangle are 4 cm and 8 cm long. What is the length of the median to the hypotenuse?

**25.** If a 45°-45°-90° triangle has legs of length $5\sqrt{2}$, find the length of the altitude to the hypotenuse.

**26.** The altitude to the hypotenuse of a 30°-60°-90° triangle divides the hypotenuse into segments with lengths in the ratio ___?___:___?___.

**27.** Find the value of $x$ in the diagram.

**28.** In $\triangle DEF$, $m\angle F = 42$, $m\angle E = 90$, and $DE = 12$. Find $EF$ to the nearest integer. (Use the table on page 311.)

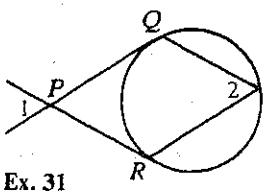


**Ex. 27**

**29.** In right $\triangle XYZ$ with hypotenuse $\overline{XZ}$ if $\cos X = \frac{7}{10}$ and $XZ = 24$, then to the nearest integer $XY =$ ___?___.

**30.** If a tree is 20 m high and the distance from point $P$ on the ground to the base of the tree is also 20 m, then the angle of elevation of the top of the tree from point $P$ is ___?___.

**31.** If $\overline{PQ}$ and $\overline{PR}$ are tangents to the circle and $m\angle 1 = 58$, find $m\angle 2$.



**Ex. 31**

**32.** $\triangle ABC$ is an isosceles right triangle with hypotenuse $\overline{AC}$ of length $2\sqrt{2}$. If medians $\overline{AD}$ and $\overline{BE}$ intersect at $M$, find $AD$ and $AM$.

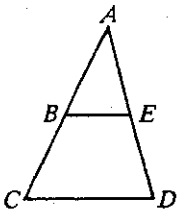
**33.** Draw two segments and let their lengths be $x$ and $y$. Construct a segment of length $t$ such that $t = \frac{2x^2}{y}$.

**34.** An equilateral triangle has perimeter 12 cm. Find its area.

**35.** Find the area of an isosceles trapezoid with legs 7 and bases 11 and 21.

**36.** **a.** Find the length of a 200° arc in a circle with diameter 24.
**b.** Find the area of the sector determined by this arc.

**37.** $B$ and $E$ are the respective midpoints of $\overline{AC}$ and $\overline{AD}$. Given that $AB = 9$, $BE = 6$, and $AE = 8$, find:
**a.** the perimeter of $\triangle ACD$
**b.** the ratio of the areas of $\triangle ABE$ and $\triangle ACD$



NAME _____ DATE _____ SCORE _____

# Mixed Review
## *Chapters 1–12*

**Directions:** Write answers in the spaces provided.

1. Given $\triangle ABC \sim \triangle DEF$, $AB = 8$, $BC = 6$, and $DE = 18$.

   **a.** Find $EF$. _____

   **b.** What is the ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$?

   _____

2. In $\triangle TOP$, $\overline{TO} \cong \overline{OP}$. If $m\angle T = 42$, find $m\angle O$. _____

3. Find the geometric mean between 12 and 4. _____

4. Write the contrapositive of the following statement: "If two angles of a triangle are not congruent, then the triangle is not isosceles."

   _____

5. A triangle has sides of lengths 4, 6, and $6\sqrt{2}$. Is the triangle acute, right, or obtuse? _____

6. In quad. $QUAD$, $\overline{QA} \perp \overline{UD}$, and $\overline{QU} \cong \overline{UA} \cong \overline{AD}$.

   **a.** What special kind of quadrilateral is $QUAD$? _____

   **b.** If $m\angle DQA = 62$, find $m\angle DAQ$. _____

7. In $\triangle CAR$, $m\angle C = 15x$, $m\angle A = 22x$, and $m\angle R = 6x + 8$.

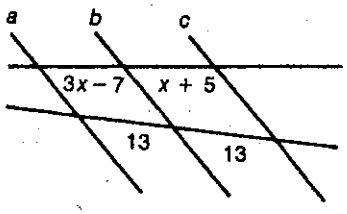
   Name the shortest side of $\triangle CAR$. _____

8. $ABCD$ is a square with a diagonal of length 6.

   **a.** Find the length of a side. _____

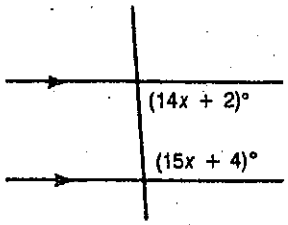
   **b.** Find the area of the square. _____

9. If $a \parallel b \parallel c$, find the value of $x$.




_____

10. Find the value of $x$.




_____

11. $\overline{MN}$ is a median of $\triangle LMP$. If $m\angle MNL = 75$, which is longer, $\overline{LM}$ or $\overline{MP}$? _____

12. A right cone has volume $100\pi$ and height 12. Find its total area. _____

13. Name the property of equality illustrated by the statement "If $RS + ST = RT$, then $RT = RS + ST$." _____

*(continued)*

NAME _____ DATE _____ SCORE _____

## Mixed Review *(continued)*

**14.** The length of an altitude of equilateral $\triangle LAT$ is $5\sqrt{3}$. Find the perimeter of $\triangle LAT$. _____

In Exercises 15 and 16 use a scientific calculator or the table on page 311 of the textbook.

**15.** $\triangle ABC$ is a right triangle with right $\angle C$. If $BC = 24$ and $AC = 7$, find the measure of $\angle A$ to the nearest integer. _____

**16.** A 15 ft ladder leans against a building with the bottom of the ladder 6 ft from the building. Find the angle (to the nearest degree) that the ladder forms with the ground. _____

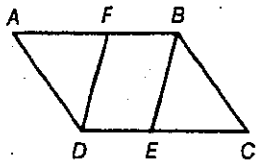
**17.** A base angle of an isosceles trapezoid is 45° and the lengths of the bases are 12 and 18. Find the area and the perimeter of the trapezoid.

area = _____, perimeter = _____

**18.** Complete the proof by supplying the missing reasons.

Given: $\square ABCD$; $\square FBED$

Prove: $\triangle AFD \cong \triangle CEB$




Proof:

| Statements | Reasons |
|---|---|
| 1. $\square ABCD$, $\square FBED$ | 1. _____ |
| 2. $\angle A \cong \angle C$ | 2. _____ |
| 3. $\overline{AD} \cong \overline{BC}$ | 3. _____ |
| 4. $\angle AFD$ and $\angle BFD$ are supplementary; $\angle CEB$ and $\angle DEB$ are supplementary. | 4. _____ |
| 5. $\angle BFD \cong \angle DEB$ | 5. _____ |
| 6. $\angle AFD \cong \angle CEB$ | 6. _____ |
| 7. $\triangle AFD \cong \triangle CEB$ | 7. _____ |

**19.** A 45°-45°-90° triangle has legs of length $6\sqrt{2}$. Find the length of the altitude to the hypotenuse. _____

**20.** The legs of a right triangle are 6 cm and 8 cm long. What is the length of the median to the hypotenuse? _____

**21.** Describe the locus of points 5 cm or less from a given point $P$.

_____

*(continued)*

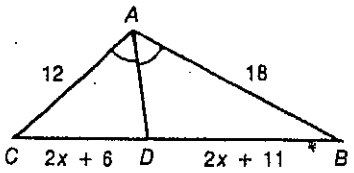
NAME _____ DATE _____ SCORE _____

## Mixed Review *(continued)*

**22.** $\overline{PR}$ is tangent to $\odot O$ at $R$. A secant from $P$ to $\odot O$ intersects the circle first at point $A$ and then at point $B$. If $BP = 10$ and $RP = 8$, find $BA$. _____

**23.** In $\triangle ABC$, $\overrightarrow{AD}$ bisects $\angle CAB$. Find the value of $x$.

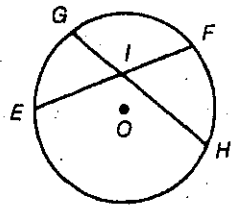
_____



Ex. 23

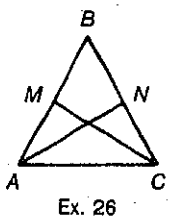
**24.** A cylinder has radius 8 and height 12. Find its volume. _____

**25.** Given $\odot O$ with chords $\overline{EF}$ and $\overline{GH}$. If $m\widehat{GF} = 82$ and $m\widehat{EH} = 160$, find $m\angle EIH$. _____



Ex. 25

**26.** In $\triangle ABC$, $\overline{CM}$ and $\overline{AN}$ are altitudes. If $\overline{CM} \cong \overline{AN}$, which congruence postulate or theorem can be used to prove $\triangle ACM \cong \triangle CAN$? _____



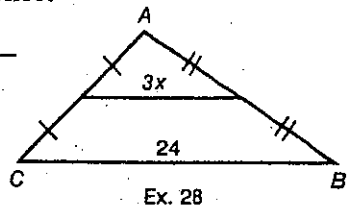
Ex. 26

**27.** Predict the next two numbers in the sequence:

$1, -\frac{1}{4}, \frac{1}{16}, -\frac{1}{64}, \ldots$ _____, _____

**28.** Find the value of $x$ in $\triangle ABC$.

_____



Ex. 28

**29.** A sphere has diameter 8.4 cm. Find its surface area to the nearest square centimeter. Use $\pi \approx 3.14$. _____

**30.** The areas of two similar prisms are 100 $cm^2$ and 25 $cm^2$. What is the ratio of their volumes? _____

**31.** In $\triangle ABC$, medians $\overline{AD}$, $\overline{BM}$, and $\overline{CN}$ intersect at $P$. If $CN = 48$, find $CP$ and $PN$. $CP$ _____, $PN$ _____

**32.** A point is chosen at random from the interior of right $\triangle RST$ with right angle $S$. If $M$ is the midpoint of $\overline{RT}$, what is the probability that the point is in the interior of $\triangle RMS$? _____

You have just finished an overall review of Geometry. Some other areas which you may want to consider reviewing are:

- the Pythagorean theorem and its converse,
- special right triangles,
- areas and volumes of solids and their formulas,
- and proofs starting with only "the given" and "the prove."

We hope this review helps you prepare for the exemption exam. Please make sure you also review your own tests, notes and your textbook. Best wishes.

ANSWERS:

**Chapters 1–12** **1. a.** 13.5 **b.** 16 : 81 **2.** 96 **3.** $4\sqrt{3}$ **4.** If a $\triangle$ is isos., then 2 $\measuredangle$s of the $\triangle$ are $\cong$. **5.** obtuse **6. a.** rhombus **b.** 62 **7.** $\overline{AC}$ **8. a.** $3\sqrt{2}$ **b.** 18 **9.** $x = 6$ **10.** $x = 6$ **11.** $\overline{MP}$ **12.** $90\pi$ **13.** Symmetric Prop. **14.** 30 **15.** 74 **16.** 66 **17.** 45; $30 + 6\sqrt{2}$ **18.** 1. Given

2. Opp. $\measuredangle$s of $\square$ are $\cong$. 3. Opp. sides of $\square$ are $\cong$. 4. $\angle$ Add. Post.; def. of supp. $\measuredangle$s 5. Opp. $\measuredangle$s of $\square$ are $\cong$. 6. Supps. of $\cong$ $\measuredangle$s are $\cong$. 7. AAS Thm. **19.** 6 **20.** 5 cm **21.** a sphere with center $P$, radius 5, and its interior **22.** 3.6 **23.** 2 **24.** $768\pi$ **25.** 121 **26.** HL **27.** $\frac{1}{256}, -\frac{1}{1024}$ **28.** 4 **29.** 222 $cm^2$ **30.** 8 : 1 **31.** $CP = 32$, $PN = 16$ **32.** $\frac{1}{2}$

 Geometry Answers

# Cumulative Review: Chapters 1–11

**In Exercises 1–12 classify each statement as true or false.**

A 1. If point $A$ lies on $\overrightarrow{BC}$, but not on $\overline{BC}$, then $B$ is between $A$ and $C$. False

2. A true conditional always has a true converse. **False**

3. The statement "If $ac = bc$, then $a = b$" is true for all real numbers $a$, $b$, and $c$. **False**

4. If two parallel lines are cut by transversal $t$ and $t$ is perpendicular to one of the lines, then $t$ must also be perpendicular to the other line. **True**

5. If $\triangle ABC \cong \triangle DEF$ and $\angle A \cong \angle B$, then $\overline{DE} \cong \overline{EF}$. **False**

6. If the opposite sides of a quadrilateral are congruent and the diagonals are perpendicular, then the quadrilateral must be a square. **False**

7. If $\triangle GBS \sim \triangle JFK$, then $\frac{JF}{JK} = \frac{GB}{GS}$. **True**

8. The length of the altitude to the hypotenuse of a right triangle is always the geometric mean between the lengths of the legs. **False**

9. In any right triangle, the sine of one acute angle is equal to the cosine of the other acute angle. **True**

10. If an angle inscribed in a circle intercepts a major arc, then the measure of the angle must be between 180 and 360. **False**

11. The angle bisectors of an obtuse triangle intersect at a point that is equidistant from the three vertices. **False**

12. If $JK = 10$, then the locus of points in space that are 4 units from $J$ and 5 units from $K$ is a circle. **False**

13. Two lines that do not intersect are either ___? or ___?. **parallel, skew**

14. In $\triangle RST$, $m\angle R = 2x + 10$, $m\angle S = 3x - 10$, and $m\angle T = 4x$.
   a. Find the numerical measure of each angle. **50; 50; 80**
   b. Is $\triangle RST$ scalene, isosceles, or right? Why? **isosceles; $2 \cong \measuredangle$**

15. Use inductive thinking to guess the next number: 10, 9, 5, −4, −20, ___?. **−45**

16. If a diagonal of an equilateral quadrilateral is drawn, what method could be used to show that the two triangles formed are congruent? **SSS**

17. A trapezoid has bases with lengths $x + 3$ and $3x - 1$ and a median of length 11. Find the value of $x$. **5**

18. If 4, 7, and $x$ are the lengths of the sides of a triangle and $x$ is an integer, list the possible values for $x$. **4, 5, 6, 7, 8, 9, 10**

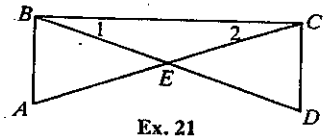
19. Describe the locus of points in space that are 4 cm or less from a given point $P$. **a sphere with ctr. $P$ and radius 4 cm, along with its interior**

20. Two similar rectangles have diagonals of $6\sqrt{3}$ and 9. Find the ratio of their perimeters and the ratio of their areas. **$2\sqrt{3}$:3; 4:3**



B 21. Given: $\overline{AB} \perp \overline{BC}$; $\overline{DC} \perp \overline{BC}$; $\overline{AC} \cong \overline{BD}$
Prove: $\triangle BCE$ is isosceles.

22. Given: Quad. $EFGH$; $\overline{EF} \cong \overline{HG}$; $\overline{EF} \parallel \overline{HG}$
Prove: $\angle EHF \cong \angle GFH$



Ex. 21

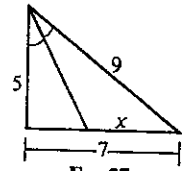
23. Use an indirect proof to show that no triangle has sides of length $x$, $y$, and $x + y$.

24. The legs of a right triangle are 4 cm and 8 cm long. What is the length of the median to the hypotenuse? **$2\sqrt{5}$ cm**

25. If a 45°-45°-90° triangle has legs of length $5\sqrt{2}$, find the length of the altitude to the hypotenuse. **5**

26. The altitude to the hypotenuse of a 30°-60°-90° triangle divides the hypotenuse into segments with lengths in the ratio ___?:___?. **1, 3**

27. Find the value of $x$ in the diagram. **4.5**



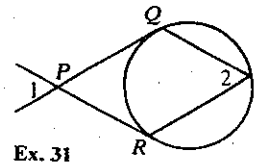
Ex. 27

28. In $\triangle DEF$, $m\angle F = 42$, $m\angle E = 90$, and $DE = 12$. Find $EF$ to the nearest integer. (Use the table on page 311.) **13**

29. In right $\triangle XYZ$ with hypotenuse $\overline{XZ}$ if $\cos X = \frac{7}{10}$ and $XZ = 24$, then to the nearest integer $XY =$ ___?. **17**

30. If a tree is 20 m high and the distance from point $P$ on the ground to the base of the tree is also 20 m, then the angle of elevation of the top of the tree from point $P$ is ___?. **45°**

31. If $\overline{PQ}$ and $\overline{PR}$ are tangents to the circle and $m\angle 1 = 58$, find $m\angle 2$. **61**



Ex. 31

32. $\triangle ABC$ is an isosceles right triangle with hypotenuse $\overline{AC}$ of length $2\sqrt{2}$. If medians $\overline{AD}$ and $\overline{BE}$ intersect at $M$, find $AD$ and $AM$. **$\sqrt{5}$; $\frac{2}{3}\sqrt{5}$**

33. Draw two segments and let their lengths be $x$ and $y$. Construct a segment of length $t$ such that $t = \frac{2x^2}{y}$.

34. An equilateral triangle has perimeter 12 cm. Find its area. **$4\sqrt{3}$ cm$^2$**

35. Find the area of an isosceles trapezoid with legs 7 and bases 11 and 21. **$32\sqrt{6}$**

36. a. Find the length of a 200° arc in a circle with diameter 24. **$\frac{40}{3}\pi$**
   b. Find the area of the sector determined by this arc. **$80\pi$**

37. $B$ and $E$ are the respective midpoints of $\overline{AC}$ and $\overline{AD}$. Given that $AB = 9$, $BE = 6$, and $AE = 8$, find:
   a. the perimeter of $\triangle ACD$ **46**
   b. the ratio of the areas of $\triangle ABE$ and $\triangle ACD$ **$\frac{1}{4}$**