Advanced Placement Calculus I - What Your Child Will Learn

I. Functions, Graphs, and Limits

A. Analysis of graphs

With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

B. Limits of functions (including one-sided limits)

An intuitive understanding of the limiting process is sufficient for this course.

1. Calculating limits using algebra
2. Estimating limits from graphs or tables of data

C. Asymptotic and unbounded behavior

1. Understanding asymptotes in terms of graphical behavior
2. Describing asymptotic behavior in terms of infinite limits and limits at infinity.
3. Comparing relative magnitudes of functions and their rates of change. (For example, contrasting exponential growth, polynomial growth, and logarithmic growth.)

D. Continuity as a property of functions

The central idea of continuity is that close values of the domain lead to close values of the range.

1. Understanding continuity in terms of limits

2. Geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

II. Derivatives

A. Concept of the derivative

The concept of the derivative is presented geometrically, numerically, and analytically, and is interpreted as an instantaneous rate of change.

1. Derivative defined as the limit of the difference quotient.
2. Relationship between differentiability and continuity.

B. Derivative at a point

1. Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are not tangents.
2. Tangent line to a curve at a point and local linear approximation.
3. Instantaneous rate of change as the limit of average rate of change.
4. Approximate rate of change from graphs and tables of values.

C. Derivative as a function

1. Corresponding characteristics of graphs of f and f'
2. Relationship between the increasing and decreasing behavior of f and the sign of f'
3. The Mean Value Theorem and its geometric consequences
4. Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa

D. Second Derivatives

1. Corresponding characteristics of the graphs of *f*, *f'*, and *f''*
2. Relationship between the concavity of f and the sign of *f''*.
3. Points of inflection as places where concavity changes

E. Applications of derivatives

1. Analysis of curves, including the notions of monotonicity and concavity
2. Optimization, both absolute (global) and relative (local) extrema
3. Modeling rates of change, including related rates problems
4. Use of implicit differentiation to find the derivative of an inverse function
5. Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration

F. Computation of derivatives

1. Basic rules for the derivative of sums, products, and quotients of functions
2. Chain rule and implicit differentiation

III. Integrals

A. Riemann sums

1. Concept of a Riemann sum over equal subdivisions
2. Computation of Riemann sums using left, right, and midpoint evaluation points

B. Interpretations and properties of definite integrals

1. Definite integral as a limit of Riemann sums
2. Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

   $$\int_a^b f'(x)dx - f(b) - f(a)$$

3. Basic properties of definite integrals. (for example, additivity and linearity.)

C. Applications of integrals

Appropriate integrals are used in a variety of applications to model physical, social or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to give accumulated change or using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of region, the volume of a solid with known cross sections, the average value of a function, and the distance traveled by a particle along a line.

D. Fundamental Theorem of Calculus

1. Use of the Fundamental Theorem to evaluate definite integrals
2. Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

E. Techniques of antidifferentiation

1. Antiderivatives following directly from basic derivatives
2. Antiderivatives by substitution of variables (including change of limits for definite integrals)

F. Applications of antidifferentiation

1. Finding specific antiderivatives using initial conditions, including applications to motion along a line
2. Solving separable differential equations and using them in modeling. In particular, studying the equation $y' = ky$ and exponential growth

G. Numerical approximations to definite integrals

Use of Riemann sums and the Trapezoidal Rule to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.