



# TONBRIDGE SCHOOL

Scholarship Examination 2018

## MATHEMATICS I

Monday, 30th April 2018  
11.30 a.m.

**Time allowed: 1 hour 30 minutes**

*Answer as many questions as you can.  
Questions 1 to 5 are worth 8 marks each;  
Questions 6 to 9 are worth 15 marks each.  
Total marks: 100*

*All answers must be supported by adequate explanation.  
Calculators may be used in any question.*

1. A journey consists of two stages: Stage 1 is  $x$  metres long and Stage 2 is  $y$  metres long. By forming and solving two simultaneous equations, find  $x$  and  $y$  from the following information:
- If I run Stage 1 at 4 m/s and walk Stage 2 at 2 m/s, the journey takes 63 seconds;
  - If I walk Stage 1 at 2 m/s and run Stage 2 at 4 m/s, the journey takes 60 seconds.

[8 marks]

2. A field contains a total of 308 sheep and goats. The ratio of sheep to goats is 4 : 7.
- (a) How many sheep must be added to the field of 308 animals in order than the ratio of sheep to goats becomes 7 : 4?
- (b) How many goats must be removed from the field of 308 animals in order than the ratio of sheep to goats becomes 7 : 4?
- (c) If equal numbers of sheep and goats are added to the field of 308 animals so that the ratio of sheep to goats becomes 2 : 3, how many animals are now in the field?

[8]

3. For a number like 31428, we say that 28 are its last two digits.

- (a) Write down the last two digits of  $16$ ,  $16^2$ ,  $16^3$ ,  $16^4$ ,  $16^5$ ,  $16^6$ ,  $16^7$ .
- (b) What pattern do you notice in your answers to (a)?
- (c) Use your answer to (b) to predict the last two digits of the following numbers:  
(i)  $16^{85}$ , (ii)  $16^{98}$ , (iii)  $2^{368}$ , (iv)  $8^{348}$ .

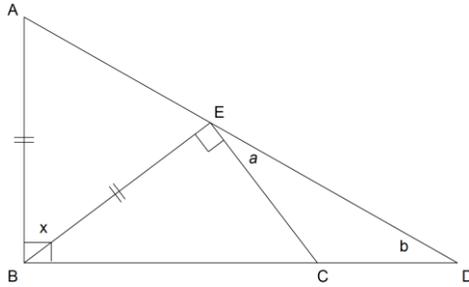
[8]

4. If a ball is dropped down a hole that passes right through an object in the Solar System of radius  $R$ , it will return to its starting point after time  $T$  given, in appropriate units, by  $T = 2\pi\sqrt{\frac{R}{g}}$  where  $g$  is a measure of the strength of gravity on the object.

- (a) Find  $T$  for the Earth where  $R = 6.4 \times 10^6$  and  $g = 9.8$ .
- (b) Find  $R$  for the Moon where  $T = 6600$  and  $g = 1.6$ .
- (c) Find  $g$  for a small asteroid where  $R = 2400$  and  $T = 720$ .

[8]

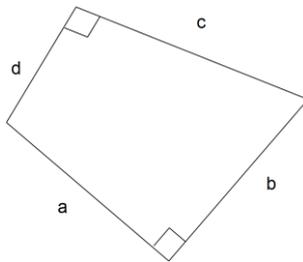
5. In the figure below,  $ABD$  and  $BEC$  are both right-angled triangles and the marked lengths  $AB, BE$  are equal. Also angle  $\hat{ABE} = x$ , angle  $\hat{CED} = a$  and angle  $\hat{CDE} = b$ .



- (a) If  $x = 30^\circ$ , find angles  $a$  and  $b$ .
- (b) Use algebra to show that  $a = b$  whatever the value of  $x$ .

[8]

6. The quadrilateral in the figure has two opposite right angles with side-lengths (in cm) denoted by  $a, b, c, d$ .



- (a) If  $a = 5, b = 6, c = 7$ , find  $d$ .
- (b) If  $a = 11.5, b = 3.5$  and  $c = d$ , find  $c$ .
- (c) Give an example of such a quadrilateral for which  $a, b, c, d$  are different whole numbers.

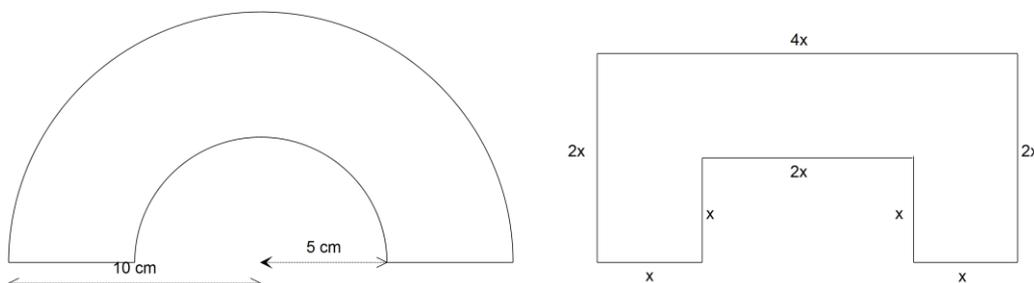
[15]

7. Two identical spheres each have radius 3 cm. When their centres are distance  $x$  apart, the overlapping region common to both has volume  $y \text{ cm}^3$  given by the formula  $y = \frac{1}{12} \pi(12+x)(6-x)^2$  where  $\pi$  has its usual meaning.

- (a) When  $x = 2$ , show that  $y = 58.6$  (correct to 1 decimal place).
- (b) Find the values of  $y$  for  $x = 0, 1, 3, 4, 5, 6$ .
- (c) Choosing sensible scales, use your values in (a) and (b) to plot a graph of  $y$  against  $x$ .
- (d) Making your method clear, find the value of  $x$  for which the overlapping volume is half of the volume of one of the spheres.
- (e) Explain carefully whether you can continue your graph for values of  $x$  greater than 6.

[15]

8. The left-hand figure below is made up two semi-circular arcs of radii 5 cm and 10 cm together with two straight line segments. The right hand figure is made up of a large rectangle measuring  $2x$  cm by  $4x$  cm with a smaller one measuring  $x$  cm by  $2x$  cm removed from it.



- (a) Find the value of  $x$  if both shapes have the same perimeter.  
 (b) Find the value of  $x$  if both shapes have the same area.

[15]

9. The terms  $F_1, F_2, F_3, \dots$  of the Fibonacci sequence are given by  $F_1 = 1, F_2 = 1, F_3 = 2, \dots$  where each term is the sum of the previous two.

- (a) Write down the next five terms of the Fibonacci sequence.  
 (b) Consider this statement about the Fibonacci sequence:

*“If you take any three successive terms in the Fibonacci sequence and find the difference between the squares of the largest and smallest, the answer is in the Fibonacci sequence.”*

Using your values of  $F_1, F_2, F_3, F_4, F_5$ , give three examples which verify this statement and explain carefully how you can predict which term of the Fibonacci sequence occurs as the answer.

- (c) Use your answer to (b) to find the values of  $x, y, a, b$  below:  
 (i)  $F_{50}^2 - F_{48}^2 = F_x$ , (ii)  $F_{y+1}^2 - F_{y-1}^2 = F_{152}$ , (iii)  $F_{70}^2 - F_{66}^2 = F_a + F_b$ .

[15]

END OF PAPER