مدرسة بيكن الخاصة
BEACON PRIVATE SCHOOL

# BEACON PRIVATE SCHOOL

# Curriculum Guide for Mathematics Middle School, Grades 6-10

*(Grade 10 being the Extension of Grade 9)*

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**The Mathematics Team wishes to clarify that they have been guided in the development of the Mathematic s Standards by the following main sources:**

***The California Core Curriculum State Standards,***

***The International Baccalaureate Syllabuses***
***The Ontario Guides to Effective Instruction in Mathematics***

***The NTCM Resources***
***The National Curriculum & Standards***

# Mathematics: Eight Big Ideas

***From the Ontario Standards for Teaching mathematics)***

***The following Big Ideas" describe the Attitudes and Habits of Mind students need to demonstrate and adopt in the learning and using mathematical knowledge and competencies .***

***We decided to edit them with the aim to making them easier to read and grasp.***

## 1) Make Sense of Problems and Persevere in Solving Them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

- They analyze givens, constraints, relationships, and goals.
- They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt.
- They consider analogous problems, and try special cases and simpler forms of the original problem to gain insight into its solution.
- They monitor and evaluate their progress and change course if necessary.
- Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?"
- Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## 2) Reason Abstractly and Quantitatively.

- Mathematically proficient students make sense of quantities and their relationships in problem situations.

They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to:

- ***Decontextualize*** - to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents; and the ability to
- ***Contextualize*** - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.
- Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

### 3) Construct Viable Arguments and Critique the Reasoning of Others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.

- They make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- They analyze situations by breaking them into cases, and can recognize and use counterexamples.
- They justify their conclusions, communicate them to others, and respond to the arguments of others.
- They reason inductively about data, making plausible arguments within the context from which the data arose.
- Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and - if there is a flaw in an argument - explain what it is.
- Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.
- Later, students learn to determine domains to which an argument applies.
- Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
- Students build proofs by induction and proofs by contradiction *(for grades IO - 12 only).*

### 4) Model with Mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

- In early grades, this might be as simple as writing an addition equation to describe a situation.
- In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
- By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
- They identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- They can analyze those relationships mathematically to draw conclusions.
- They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5) Use Appropriate Tools Strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem.

- These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.
- Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
- For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator.
- They detect possible errors by strategically using estimation and other mathematical strategies.
- When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
- Mathematically proficient students at various grade levels identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems.
- They use technological tools to explore and deepen their understanding of concepts.

## 6) Attend to Precision.

Mathematically proficient students try to communicate precisely to others.

- They try to use clear definitions in discussion with others and in their own reasoning.
- They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem.
- They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.
- In the elementary grades, students give carefully formulated explanations to each other.
- By the time they reach high school they have learned to examine claims and make explicit use of definitions.

### 7) Look for and Make Use of Structure.

Mathematically proficient students look closely to discern a pattern or structure.

- Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes based on the number of sides the shapes have.
- Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property.
- In the expression $x^2+9x+14$, older students can see the14 as 2x7 and the 9 as 2+7.
- They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
- They also can step back for an overview and shift perspective.
- They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.
- For example, they can see 5 - $3(x - y)2$ as 5 minus a positive number, times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

### 8) Look for and Express Regularity in Repeated Reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts.

- Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over, and conclude they have a repeating decimal.
- By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y-2) /(x- 1) = 3.
- Noticing the regularity in the way terms cancel when expanding expressions such as (x- l)(x+1), $(x$- l)(x$^2$+$x$+1), and $(x- l)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series.
- As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details.
- They continually evaluate the reasonableness of their intermediate results.

## Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for ***Mathematical Practice*** describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years.

Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for ***Mathematical Content*** are a balanced combination of ***Procedures and Understandings.***

Expectations that begin with the word ***"Understand"*** **are** ***Concepts*** that open good opportunities to connect the practices to the content.

Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut.

***In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.***

***In this respect, those Content Standards that set an expectation of understanding are potential "points of intersection" between:***

***The Standards for Mathematical Content, and the Standards for Mathematical Practice.***

These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

***(Adapted from the Ontario Guides to Effective Instruction in Mathematics)***

# Mathematics Learning: Essential Goals & Objectives

**Goals:**

> ***The essential goals of learning mathematics are for students to understand that mathematics is a powerful tool/or describing and analyzing the world, a highly effective tool/or solving real world problems, and a way of thinking in a logical and critical manner - far beyond seeing it as a series of/acts and equations to be memorized.***

**Objectives:**

**1 - By the end of Grade 6, students should:**

- Develop a ***Deep Understanding*** of the concepts of **Number Systems and Operations,** on whole numbers, fractions and decimals, and apply them in exploring and solving real-world problems
- Understand the basic ***Algebraic Thinking*** skills and apply them to analyze and understand the structures of the Number Systems, in preparation for the study of Algebra in the Middle school.
- Understand and practice the ***Measurement and the Data*** collection, analysis, and application.
- Understand the basic elements of ***Geometry*** and relations through the exploration of the physical world, identification and classification of shapes of objects and their properties and spatial relationships.

***We would like to clarify that the objectives for grades K-6 have served to divide the Mathematics Standards of this document into/our related Strands:***

- ***Number systems and Operations***
- ***Algebraic Thinking***
- ***Measurement & Data***
- ***Geometry***

**2 - By the End of Grade 10, students should:**

- Develop a ***Deep Understanding*** of the ***Algebraic Thinking and Theories,*** and apply them to solving problems at a higher and more abstract level of analysis and problem solving
- Develop a Deep Understanding of the ***Deductive Thinking*** processes through the study of geometry
- Be ready to enroll in the study of ***Calculus*** and ***Analytic Geometry*** in grades 11 and 12.

# Approaches to Teaching & Learning

It is important that learners acquire mathematical understanding by constructing their own meaning through ever-increasing levels of abstraction.
This construction spans over three phases:

## Constructing meaning, transferring meaning, applying meaning

### Constructing meaning about mathematics

Learners construct meaning based on their previous experiences and understanding, and by reflecting upon their interactions with objects and ideas. Therefore, involving learners in an active learning process, where they are provided with possibilities to interact with manipulatives and to engage in conversations with others, is paramount to this stage of learning mathematics. When making sense of new ideas learners either interpret these ideas to conform to their present understanding or generate a new understanding that accounts for what they perceive to be occurring.
This construct will continue to evolve as learners experience new situations and ideas, have an opportunity to reflect on their understandings and make connections about their learning.

### Transferring meaning into symbols

When learners have constructed their ideas about a mathematical concept, they will be able to transfer this understanding into symbols.
Symbolic notation can take the form of pictures, diagrams, modeling with concrete objects and mathematical notation. Learners should be given the opportunity to describe their understanding using their own method of symbolic notation, then learning to transfer them into conventional mathematical notation.

### Applying meaning with understanding

Applying with understanding can be viewed as the learners demonstrating and acting on their understanding. Through authentic activities, learners should independently select and use appropriate symbolic notation to process and record their thinking. These authentic activities should include a range of practical hands-on problem-solving activities and realistic situations that provide the opportunity to demonstrate mathematical thinking through presented or recorded formats. In this way, learners are able of applying their understanding of mathematical concepts as well as utilize mathematical skills and knowledge.
N In this way, students validate the meaning they construct from their experiences with mathematical situations. By explaining their ideas, theories and results, both orally and in writing, they invite constructive feedback and lay out alternative models of thinking for the class. Consequently, all benefit from this interactive process.

***Adapted from the Introduction to the PYP Mathematics Guide, 2014, and applies at all grade levels***

## Mathematics and Problem Solving

## A teaching/learning modality

***Problem solving is not considered to be an approach among many; rather it** is **the main strategy for teaching mathematics. Problem solving should be the mainstay of mathematical teaching and should be used daily***
*(Ontarion, A Guide to Effective Instruction in Mathematics, volume 2)*

***Too often we give children answers to remember rather than problems to solve***
*Roger Lewin*

***Problem solving** is **not only a goal of learning mathematics but also a major means of doing so***
*(NCTM, 2000, p.52)*

Teaching and learning mathematics ***through*** problem-solving situations to develop problem-solving skills is our priority.

Teaching ***through*** problem solving is a medium for teaching mathematical content.

In some way, we used to teach mathematics to prepare students to solve problems.

**In this approach, Students solve problems to discover and learn mathematics, structure and expand their learning and understandings, finally apply them to solving problems of a higher level.**

**In practical terms, teachers pose problems:**
1 - At the ***beginning of a unit,*** asking students to utilize their own strategies and knowledge in order to develop new understandings and create connections,

***2 - Throughout the unit,*** to develop the students' attitudes and ability to solve problems.

3 - ***And at the end of the unit,*** to solve complex problems using the newly acquired knowledge and strategies .

Teachers should present to students engaging problems as a way of motivating them to investigate mathematical concepts and to develop and apply their own understanding of those concepts.

Most concepts and computations can be taught through problem solving. The proposed situations could be real situations, as well as fictive ones.

***For the above reasons, we have not listed problem solving as one of the standards of achievement, as we are using problem solving as the main engine for learning mathematics, not a purpose for that!***

## Guidelines for Assessment

## Assessment for Learning instead of Assessment of Learning:

As clearly stated in the title of this section, teachers are invited to go beyond the assessment of ***what*** students have learned, and involve students in reflecting about ***why*** and ***how*** they have learned, then use the results of these assessments and reflections to guide their teaching and the learning of students.

As for all subjects at all grade levels, Assessment needs to be seen as part of the teaching- learning activity, and not as a means to grade students, rank them, or make decisions about their success or failure - worst, make judgments on teachers or on the school!

***Formative Assessments,*** in all forms, ranging from observation, communication, written test, Rubrics, and other forms, should serve as a main guide to the planning and teaching continuum.

Formative assessment results should focus on clear descriptions of the strengths and weaknesses, and include recommendations for improvement.

Discussions between the teacher and the student are of essence.

Grades in any form (percent and/or letters of any symbolic form and ranking lack the essence of

clarifying the problems and finding the solutions that lead to better learning.

Portfolios add the benefit of providing a dynamic image of the student progress and needs.

***Summative Assessments*** may include some forms of paper and pencil components, but should essentially focus on the students' performance in the Tasks expected at the end of the Units of Study.

***The most important contribution of assessments to the learning of students lies in the extent to which assessment results***:

- Describe the strengths and weaknesses of a student performance
- Invite students to reflect on their performance
- Guide students to devise different approaches to improve their performance

## جدول التوزيع الزمني لمحتوى مقرر الرياضيات للمرحلة الإعدادية (الصفوف ٦-٩)
## Table of Distribution of the Mathematics Content in the Middle School (Gr. 6–9)

**(Added Page)**

The Mathematics Concepts cover in each grade level four essential strands:

Strand 1: Number Systems
Strand 2: Algebraic Thinking
Strand 3: Measurement and Data
Stand 4: Space and Geometry

Strand 1 is a review of the Elementary Arithmetic and an opportunity to a deeper understanding on the different types of Numbers, starting from the Natural Whole numbers, through the Real Numbers, and a preparation for the introduction of Complex Numbers in the Secondary Classes.

Strand 2 is the hard core of the Mathematics Curriculum. As seen below, we recommend that this long Strand be divided into three Periods of 6 to 8 weeks each, and taught in alternating periods.

Strand 3 focuses on Measurement, mainly in statistical situations and moving into probabilistic thinking. It touches also on geometric measurements associated with basic geometric shapes.

Strand 4 moves geometric manipulation into the field on deductive thinking and the study of Euclidean Geometry as an Axiomatic Model. Through the Geometric Models, Deductive Thinking expands to the Binary Logic Thinking and its universal application various areas.

In practical terms, the weight and allocated study period to each Strand change from grade level to grade level, as shown in the below Table

| | **Grade 6** | **Grade 7** | **Grade 8** | **Grade 9** |
|---|---|---|---|---|
| **Stand 1**<br>**Number Systems** | 6 Weeks | 5 Weeks | 4 Weeks | 3 Weeks |
| **Strand 2**<br>**Algebraic Thinking**<br>***(Divided into three Periods: 2A, 2B, 2C, each of around 6 – 8 Weeks.*** | 20 Weeks | 21 Weeks | 22 Weeks | 24 Weeks |
| **Strand 3**<br>**Measurement / Data** | 5 Weeks | 5 Weeks | 5 Weeks | 4 Weeks |
| **Strand 4**<br>**Space & Geometry** | 5 Weeks | 5 Weeks | 5 Weeks | 5 Weeks |
| **TOTAL**<br>**Instruction Period** | **36 Weeks** | **36 Weeks** | **36 Weeks** | **36 Weeks** |

***The recommended Sequence of Instruction is:***
***Strand 1, Strand 2A, Strand 3, Strand 2B, Strand 4, Strand 2C***

## جدول توزيع درجات الطالب بالرياضيات للمرحلة الإعدادية (الصفوف ٦-٩)
## Table of Distribution of the Mathematics Grades in the Middle School (Gr. 6–9)

**(Added Page)**

As described in the previous pages of this Mathematics Guide, and particularly about the Assessment Principles in the study of mathematics, our focus at Beacon Private School is not mainly on Summative Assessments expressed through numbers or letters, rather more on Formative Assessment that aims at improving learning, through Units of Inquiry that make student think about mathematics and use mathematics as a learning component of Inquiry rather than a memorization of facts and formulas and a numeric representation of facts memorized.

To that effect, Teachers share with the Learners (students) observations about their work, and their skills in learning through interacting, collaborating, communicating, researching, thinking, reflecting, and problem-solving.

This sharing plays a much bigger role in helping students Learn, than simple number grades that indicate how much a students can repeat a correct answer, and how much he/she cannot!

This does not exclude the use of grades, especially when students get closer and closer to Higher Secondary Classes, and in a progressively increasing manner, in order to align with Forms of Assessment still commonly used.

The following Table offers a distribution of grades that describes the student achievement in mathematics, in three dimensions of studying and Learning:

- Dimension 1: The Student's Learning in Class, i.e. during the acquisition of knowledge trough listening to and communicating with others (teachers and classmates)
- Dimension 2: The Student's Learning with others during Units of Inquiry and their contribution to the Inquiry Team
- Dimension 3: The Assessment of the individual learning and evidence of gaining understandings, developing competencies, and demonstrating maturing personal and social positive attitudes.

| | Grade 6 | Grade 7 | Grade 8 | Grade 9 |
|---|---|---|---|---|
| **Dimension 1: Learning in Class, assessed through a variety of Tests and Exams** | 20% | 20% | 25% | 25% |
| **Dimension 2: Contribution to and Learning within Inquiry Teams** | 40% | 40% | 35% | 35% |
| **Dimension 3: Student's Balanced Development in Concepts, Competencies and Character** | 40% | 40% | 40% | 40% |

## General Practices

As in all subject guides, the focus of the guide is on the subject topics and specific instructional activities, leaving the practices common to all or most subjects to be included in other documents, mainly in the academic part of the Teachers' and Students' Handbooks. These include the following:

### Lesson Planning and Plans

In the Subject' Guides we list elements of the lesson planning and delivery that are specific to the subject and/or uncommon, unfamiliar to large numbers of teachers, or of major importance to new teaching methodologies. To that effect, we have highlighted in the Mathematics Guide the Role of Problem-Solving in the teaching and learning of mathematics, the Eight Big Ideas, and the principle of assessment for learning - not of learning.

These are cornerstones that affect and shape the whole teaching/learning structure and must be mentioned and highlighted.

Furthermore, mathematics teachers are expected to plan their instruction, in collaboration with each other and the guidance of their department head, taking into consideration the principles listed above, in addition to their own teaching styles, the students of their classes and the textbooks and teaching resources available.

These generally change from year to year and require the lesson plans to change accordingly, while the standards and educational principles remain unchanged for much longer periods. As for the Time allocated for each Unit plan and its sections, this is an essential part of the Lesson Plan.

### Assessment Tools

Specific Guidelines for Assessment are also listed in the Guide, but some assessment tools, including portfolios, rubrics and few generic samples, apply to all subject areas and grade level, and shall be included in the academic section of the teachers' handbooks.

### Grading Systems

The Grading Systems apply to the educational program of the schools and/or the program adopted by the school.

***Please refer to the Teachers' Handbook where various grading systems are described and discussed.***

### Reporting Characteristics

We have included in the Guide the importance of reporting through explanatory statements describing the status, strengths, and needs for improvement - not only numbers or letters that do not provide such meanings.

***The reporting approach is part of the school's policy that applies to all subjects - not related to any specific subject.***

### Allocation of Instructional Time

***Tables indicating clearly the times allocated to every subject in every grade level are part of the Policy Manual and the Handbooks.***

### Space Requirements and Arrangement

While the teaching and learning of mathematics require manipulative kits and drawing sets, they do not require any special classroom or special furniture and equipment.

# BEACON PRIVATE SCHOOL

## AL-HIDD, BAHRAIN

# Mathematics Subject Guide

# Grades 6-8

# Introduction

January 2019

## Mathematics & Science Connections, Grades 6 – 8

***(The below connections are found in the Science Guide Too)***

- Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (I). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

- Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; I, ml; hr, min, sec. Within a single system of measurement express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. Generate a conversion table.

- Reason abstractly and quantitatively
- Model with mathematics
- Use appropriate tools strategically

- Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units-whole numbers, halves, or quarters

- Apply Operations and Algebraic Thinking

- Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded across the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

- Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

- Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding

- Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

- Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

- Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of Zero in each situation

- Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).

- Summarize numerical data sets in relation to their context.

- Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

- Use ratio and rate reasoning to solve real-world and mathematical problems
- Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

# Grade 6
# Preamble

In grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division, and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus, students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular, negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students under- stand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically.

Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability.

Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected

Students in grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in grade 7 by drawing polygons in the coordinate plane.

| Mathematics Standards<br>Grade 6<br>Strand 1: Number Systems (NS) | |
|---|---|
| **Apply and extend previous understandings of multiplication and division to divide fractions by fractions.** | |
| 6-NS-01 | Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. |
| 6-NS-02 | Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3* |
| 6-NS-03 | Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* |
| 6-NS-04 | Interpret and compute quotients of fractions, and solve word problems involving division of unit fractions by non-zero whole numbers, division of whole numbers by unit fractions and division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3.*<br>*(In general, (a/b) ÷ (c/d) = ad/bc).*<br>*How much chocolate will each person get if 3 people share 1/2 kg of chocolate equally?*<br>*How wide is a rectangular strip of land with length 3/4 meter and area 1/2 square meter?* |
| **Compute fluently with multi-digit numbers and find common factors and multiples** | |
| 6-NS-05 | Fluently divide multi-digit numbers using the standard algorithm. Check the reasonableness of calculated results, for example, by estimating. |
| 6-NS-06 | Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. Use and explain mental methods to multiply and divide whole numbers and decimals. Use calculator methods to multiply and divide by decimal to convert fractions to decimals, and to calculate with numbers involving several digits. Reduce a fraction to its simplest form by cancelling a common factor |
| 6-NS-07 | Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* Identify prime numbers and find the prime factorization of a number. |
| **Apply and extend previous understandings of numbers to the system of rational numbers.** | |
| 6-NS-08 | Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. |
| 6-NS-09 | Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.<br>a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., – (–3) = 3, and that 0 is its own opposite.<br>b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.<br>c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. |
| 6-NS-10 | Understand ordering and absolute value of rational numbers.<br>a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*<br>b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*<br>c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write \|–30\| = 30 to describe the size of the debt in dollars.*<br>d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.* |

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| 6-NS-11 | Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate |

| **Mathematics Standards<br>Grade 6<br>Strand 2: Algebraic Thinking & Operations (AT)** | |
|---|---|
| **Ratios and Proportional Relations.<br>Understand ratio concepts and use ratio reasoning to solve problems.** | |
| 6-AT-01 | Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* |
| 6-AT-02 | Understand the concept of a unit rate *a/b* associated with a ratio *a:b with* $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.* (Expectations for unit rates in this grade are limited to non-complex fractions) |
| 6-AT-03 | Use ratio and rate reasoning to solve real-world and mathematical problems (Students may use a calculator. (Exclude the use of % key so that students understand how the calculation is carried out), e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.<br>a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.<br>b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*<br>c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole (limit to parts of 25%, 50%, 75% and multiples of 10%), given a part and the percent.<br>d. Use ratio reasoning to convert measurement units (Include converting local currencies to US dollars or British pound, and vice-versa) , manipulate and transform units appropriately when multiplying or dividing quantities. |
| **Expressions and Equations:<br>Apply and extend previous understandings of arithmetic to algebraic expressions** | |
| 6-AT-04 | Write and evaluate numerical expressions involving whole-number exponents. |
| 6-AT-05 | Write, read, and evaluate expressions in which letters stand for numbers.<br>a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*<br>b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product*<br>c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real- world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.* |
| 6-AT-06 | Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y* |
| 6-AT-07 | Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.* |

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| **Reason about and solve one-variable equations and inequalities** | |
| 6-AT-08 | Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.<br>6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. |
| 6-AT-09 | Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which $p$, $q$ and $x$ are all nonnegative rational numbers. |
| 6-AT-10 | Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real- world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. |
| **Represent and analyze quantitative relationships between dependent and independent variables** | |
| 6-AT-11 | Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.* |

| **Mathematics Standards<br>Grade 6<br>Strand 3: Measurement & Data (MD)** | |
|---|---|
| **Solve real-world and mathematical problems involving angle measure, area, surface area, and volume.** | |
| 6-MD-01 | Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. |
| 6-MD-02 | Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.<br>Use the equivalence of 1 liter and 1 000 cm3 to find the volume of a liquid. |
| 6-MD-03 | Find the surface area of a cube, given the length of an edge; the edge of a cube, given its volume or surface area; one dimension of a cuboid, given its volume and the other two dimensions (Limit to cubes and cuboids with integer dimensions. Exclude use of √ sign). |
| 6-MD-04 | Find and calculate unknown angles in geometric figures, involving angles in a straight line, around a point or vertically opposite angles; the angle sum of a triangle; angle properties of an isosceles, equilateral, right-angled and scalene triangle; angle properties of a square, rectangle or parallelogram. |
| 6-MD-05 | Identify equal lengths in isosceles, equilateral triangle, a square, rectangle and parallelogram. Find and calculate unknown lengths involving sides properties of an isosceles, equilateral, of a square, rectangle or parallelogram. |
| **Statistics and Probability:<br>Develop understanding of statistical variability** | |
| 6-MD-06 | Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* |
| 6-MD-07 | Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape. Calculate a mean and find the median of a set of data; draw conclusions about a set of data |
| 6-MD-08 | Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. |

| Summarize and describe distributions. | |
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| 6-MD-09 | Display numerical data in plots on a number line, including dot plots, histograms, and box plots (use ICT to generate graphs, charts and tables, including bar charts and pie charts). |
| 6-MD-10 | Summarize numerical data sets in relation to their context, such as by:<br>a. Report the number of observations.<br>b. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement.<br>c. Give quantitative measures of center (median and/or mean) and variability (interquartile range). Describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.<br>d. Relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. |

| Mathematics Standards<br>Grade 6<br>Strand 4: Space & Geometry (SG) | |
|---|---|
| **Draw, construct and describe geometrical figures and describe the relationships between them.** | |
| 6-SG-01 | Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. Draw the reflection of a 2-D shape in a given mirror line and about one of its vertices. |
| 6-SG-02 | Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems |
| 6-SG-03 | Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. |

# Grade 7

## Preamble

In grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

1- Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships. They distinguish proportional relationships from other relationships.

2- Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents, as different representations of rational numbers, using the symbols $<, >, \geq, \leq, =, \neq$. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division; they estimate and round numbers, checking results to see if they are reasonable. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3- Students continue their work with area from grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

4- Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences. They understand and use the probability scale from 0 to 1, and find the probabilities of simple vents in simple contexts. They know that the total probability of all mutually exclusive outcomes is 1, and use this to solve proble

***Grade 7 teachers should review end consolidate Grade 6 standards where necessary***

| Mathematics Standards<br>**Grade 7**<br>**Strand 1: Number Systems (NS)** | |
|---|---|
| **Calculate with fractions and use them to solve problems.** | |
| 7-NS-01 | Order fractions by creating a common denominator or by converting them to decimals. |
| 7-NS-02 | Use written or mental methods to add, subtract, multiply and divide fractions, including combined operations; relate operations with fractions to situations and models, use mental method to find the fraction of a number or quantity. |
| **Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** | |
| 7-NS-03 | Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.<br>a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*<br>b. Understand $p + q$ as the number located a distance $\|q\|$ from $p$, in the positive or negative direction depending on whether $q$ is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.<br>c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.<br>d. Apply properties of operations as strategies to add and subtract rational numbers. |
| 7-NS-04 | Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.<br>a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*<br>b. Understand $p + q$ as the number located a distance $\|q\|$ from $p$, in the positive or negative direction depending on whether $q$ is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.<br>c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.<br>d. Apply properties of operations as strategies to add and subtract rational numbers. |
| 7-NS-05 | Solve real-world and mathematical problems involving the four operations with rational numbers. |

| Mathematics Standards<br>Grade 7<br>Strand 2: Algebraic Thinking & Operations (AT) | |
|---|---|
| **Analyze proportional relationships and use them to solve real-world and mathematical problems** | |
| 7-AT-01 | Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 km in each 1/4 hour, compute the unit rate as the complex fraction ½/¼ km per hour, equivalently 2 km per hour.* Compare two ratios. *Example: Fruit drink A is made from 1part orange juice and 9 parts pineapple juice. Fruit drink B is made from 1part orange juice and 4 parts pineapple juice. Which has more orange juice, 1litre of fruit drink A or 1litre of fruit drink B?* |
| 7-AT-02 | Recognize and represent proportional relationships between quantities.<br>a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.<br>b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.<br>c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*<br>d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate. |
| 7-AT-03 | Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error, average speed, average rate.* |
| 7-AT-04 | Generate sequences and plot graphs of functions. |
| 7-AT-05 | Extend and find missing terms in numeric or geometric patterns or sequences using words, diagrams or symbols (use keys on a graphics calculator to generate term-to-term and position-to-term sequences), generalize the relationship between one term of a sequence and the next, or between the number of the term and the term, using words or symbol, and vice-versa, identify intercepts on axes (on paper or using ICT). |
| **Expressions & Equations: Use properties of operations to generate equivalent expressions.** | |
| 7-AT-06 | Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. Simplify algebraic expressions with one or two variables by collecting like terms and multiplying a single term over a bracket. Example: 7 + 2t + 3t; 3d +9 + 4(d-2).<br>Evaluate formulae and linear expressions by substituting integers for letters. |
| 7-AT-07 | Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.<br>*For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* |
| **Solve real-life and mathematical problems using numerical and algebraic expressions and equations** | |
| 7-AT-08 | Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* |
| 7-AT-09 | Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.<br>a. Solve word problems leading to equations of the form $px + q = r$ ; $p(x + q) = r$; and $px + q = p'x + q'$, where $p, q, p', q'$ and specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*<br>b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where $p, q,$ and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.<br>*For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solution.* |

| Mathematics Standards<br>Grade 7<br>Strand 3: Measurement & Data (MD) | |
|---|---|
| **Use random sampling to draw inferences about a population.** | |
| 7-MD-01 | Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. |
| 7-MD-02 | Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.* |
| **Draw informal comparative inferences about two populations**. | |
| 7-MD-03 | Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.* |
| 7-MD-04 | Use measures of mean, mode, median and range for numerical data (of a small set of data) from random samples to draw informal comparative inferences about two populations.<br>*For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.* |
| 7-MD-05 | Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. |
| 7-MD-06 | Know that the total probability of all mutually exclusive outcomes is 1, and use this to solve a problem; solve simple problems based on equally likely outcomes for a single event. |

| Mathematics Standards<br>Grade 7<br>Strand 4: Space & Geometry (SG) | |
|---|---|
| **Draw, construct, and describe geometrical figures and describe the relationships between them.** | |
| 7-SG-01 | Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. |
| 7-SG-02 | Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions: parallel and perpendicular lines, draw circles and arc and use circles and arcs to construct Islamic patterns, construct angle bisectors and perpendicular bisectors. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. |
| 7-SG-03 | Identify, sketch, label and describe angle, side, diagonal and symmetry properties of plane figures: triangles (isosceles, equilateral, right-angled), quadrilaterals (square, rectangle, rhombus, parallelogram, trapezium, kite), polygons (pentagon, hexagon, octagon, decagon) |
| 7-SG-04 | Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. |
| **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.** | |
| 7-SG-05 | Know the formulas for the area and circumference of a circle, and use them (including the areas used in grade 6) to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. |
| 7-SG-06 | Use facts about supplementary, complementary, vertical, adjacent angles, corresponding, alternate, angle properties of isosceles, equilateral, right-angled and scalene triangles, including the angle sum and exterior angle properties, angle properties of squares, rectangles, parallelograms and rhombuses, including angle properties related to their diagonals and angle bisectors and perpendicular bisectors in a multi-step problem to write and solve simple equations for an unknown angle in a figure. |
| 7-SG-07 | ve real world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. |

## Preamble
## Grade 8

**In grade 8**, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

1- Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems.

Students recognize equations for proportions *(y/x = m* or *y = mx)* as special linear equations *(y = mx + b),* understanding that the constant of proportionality (*m*) is the slope, and the graphs are lines through the origin. They understand that the slope (*m*) of a line is a constant rate of change, so that if the input or *x*-coordinate changes by an amount A, the output or *y*-coordinate changes by the amount *m*A. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span versus height for students in a classroom). At this grade, fitting the model and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students

to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and *y*-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

Students simplify, factorize, evaluate algebraic expressions and formulae, and find the sum or difference of simple algebraic fractions with integer denominators.

2- Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations, they identify the intervals where functions increase, decrease or are constant.

3- Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students solve problems using the sum of the angles in a triangle, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean theorem and its converse, and can explain why the Pythagorean theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume and surface area by solving problems involving prisms, cones, cylinders, and spheres. They solve problems involving speed or density.

| Mathematics Standards<br>Grade 8<br>Strand 1: Number Systems (NS) | |
|---|---|
| **Know that there are numbers that are not rational, and approximate them by rational numbers.** | |
| 8-NS-01 | Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion that repeats eventually into a rational number. |
| 8-NS-02 | Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them. *Show that Square Root of 2, √2, is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* |

| Mathematics Standards<br>Grade 8<br>Strand 2: Algebraic Thinking & Operations (AT) | |
|---|---|
| **Work with radicals and integer exponents** | |
| 8-AT-01 | Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $32 \times 3^{-5} = 3^{-3} = 1/33 = 1/27$* |
| 8-AT-02 | Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is irrational |
| 8-AT-03 | Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as $3 \times 10^8$ and the population of the world as $7 \times 10^9$, and determine that the world population is more than 20 times larger than the population of the United States.* |
| 8-AT-04 | Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. Interpret scientific notation that has been generated by technology. |
| 8-AT-05 | Use mental, written and calculator methods for the four operations to solve problems involving whole numbers, decimals, fractions, rationals, and rounding answers appropriately (based on approximations and by considering the context of the problem). |
| 8-AT-06 | Perform operations on simple expressions containing variables; add and subtract simple algebraic expressions. Factorize simple algebraic expressions. *Examples: factorize: $ax + ay$; $2m^2 + 6mn$* |
| **Understand the connections between proportional relationships, lines, and linear equations** | |
| 8-AT-07 | Graph proportional relationships (including percentages), interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* |
| 8-AT-08 | Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. |
| **Analyze and solve linear equations and pairs of simultaneous linear equations** | |
| 8-AT-09 | Solve linear equations in one variable.<br>a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). Determine whether given number values satisfy a given linear equation.<br>b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive, associative and commutative properties and collecting like terms |

| | |
|---|---|
| 8-AT-10 | Analyze and solve pairs of simultaneous linear equations.<br>a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.<br>b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.*<br>c. Solve real-world and mathematical problems leading to linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair* |
| **Define, evaluate and compare functions** | |
| 8-AT-11 | Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in grade 8) |
| 8-AT-12 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change*. |
| 8-AT-13 | Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function A = s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line* |
| **Use functions to model relationships between quantities** | |
| 8-AT-14 | Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values |
| 8-AT-15 | Describe qualitatively the functional relationship between two quantities by analyzing a graph (*e.g., where the function is increasing or decreasing, linear or nonlinear)*.<br>Sketch a graph that exhibits the qualitative features of a function that has been described verbally (explore situations involving density, average speed, distance or time…) |
| 8-AT-16 | Generate sequences, extend and find missing terms in numeric, algebraic or geometric patterns or sequences, generate the relationship between one term of a sequence and the next (using a spreadsheet or graphics calculator), or describe the *n*th term, using symbols. |

| **Mathematics Standards<br>Grade 8<br>Strand 3: Measurement & Data (MD)** | |
|---|---|
| **Investigate patterns of association in bivariate data** | |
| 8-MD-01 | Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. |
| 8-MD-02 | Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line |
| 8-MD-03 | Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* |
| 8-MD-04 | Compare in general terms two data sets using the mean, median, or range or the shape of distribution |
| 8-MD-05 | Use a data from experiments to estimate probability for favorable outcomes; understand that different outcomes may result from repeating experiment; use problem conditions to calculate theoretical probabilities for possible outcomes; list systematically all the possible outcomes of an experiment. |

| Mathematics Standards<br>Grade 8<br>Strand 4: Space & Geometry (SG) | |
|---|---|
| **Understand congruence and similarity using physical models, transparencies, or geometry software** | |
| 8-SG-01 | Verify experimentally the properties of rotations, reflections, and translations:<br>a. Lines are taken to lines, and line segments to line segments of the same length.<br>b. Angles are taken to angles of the same measure.<br>c. Parallel lines are taken to parallel lines. |
| 8-SG-02 | Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them |
| 8-SG-03 | Describe the effect of dilations, translations, rotations, reflections and enlargement by a whole-number scale factor using a given center of enlargement on two-dimensional figures using coordinates; draw these transformations of a simple 2-D shape and the combination of two of them |
| 8-SG-04 | Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about interior and exterior angles of polygons about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so* |
| 8-SG-05 | Identify similar figures; know that corresponding sides of similar figures are in the same ratio |
| **Understand and apply the Pythagorean Theorem** | |
| 8-SG-06 | Explain a proof of the Pythagorean theorem and its converse |
| 8-SG-07 | Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions |
| 8-SG-08 | Apply the Pythagorean theorem to find the distance between two points in a coordinate system |
| **Solve real-world and mathematical problems involving measures and volume of cylinders, cones, and spheres** | |
| 8-SG-09 | Know the formulas for the volumes of cuboids, cones, cylinders, and spheres and use them to solve real-world and mathematical problems. Find the surface areas of cubes and cuboids and related solids. Recognize that measurements are not precise *(e.g. give the upper and lower bounds of a measurement recorded as 15 cm to the nearest centimeter)* |
| 8-SG-10 | Visualize, describe and draw simple 3-D solids in different orientations |

# Grades 9 & 10

## Preamble

The mathematics standards and topics in grades 9 & 10 form an integrated syllabus covering every school year topics from all four major strands:

**Number Systems and Operations, Algebraic Thinking and Operations,**
**Space & Geometry, Data & Measurement**

The concepts of sets and relations are introduced in a more formal approach, and are applied in Number Sets (Natural, Whole, Rational, Irrational, Real, and Complex numbers).

Operations on Polynomials define them as sets.

Points that are characterized by a specific property form a set - named “locus”.

Addition of vectors defines them as sets.

Finally, relations between sets lead to understanding Functions as specific relations between numerical sets.

The elements of the Set Theory bring the various mathematical stands into a higher level of unity and deeper understanding.

In grades 9 and 10 students are expected to fluently manipulate the deductive thinking, mainly in geometry.

In parallel with the geometric thinking, analytic geometry is used as another approach to the study of space.

For example, parallelism of lines is associated with the relations between angles formed by the intersection of two parallel lines by a third line. From the analytics point on view, parallelism of two lines is associated with having the same slope in a Cartesian plane.

In grade 10 students move from statistical description on events to the concept of probability.

By the end of grade 10, students are expected to have reached an advanced level in the study of elements of pre-calculus and calculus in grades 11 and 12.

By the end of grade 10, students are expected to have reached an advanced level in the study of function and their observed variation rates, in preparation for the introduction of derivatives and the study on tangents to curves on different points of the curve, and be thus introduced to the essential elements of pre-calculus and calculus in grades 11 and 12.

| Grades 9 – 10<br>Mathematics & Science Connections |
|---|
| - Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases<br>- Model with mathematics<br>- Reason abstractly and quantitatively and use units to solve problems.<br>- Reason quantitatively and use units to solve problems.<br>- Represent data with plots on the real number line (dot plots, histograms, and box plots).<br>- Understand statistics as a process for making inferences about population parameters based on a random sample from that population.<br>- Write a function that describes a relationship between two quantities<br>- In grades11 and 12, use the elements of calculus, analytic geometry, vectors and transformations, mainly in the study and problem solving in of physics |

| Mathematics Standards<br>Grade 9<br>Strand 1: Number Systems (NS) | |
|---|---|
| **Write equivalent forms of algebraic expressions to solve problems** | |
| 9-NS-01 | **Identify and use number sets.**<br>Identify the number sets. Know when a real number is irrational, i.e. when it is not a member of ; Use and understand the symbols associated with set theory. |
| 9-NS-02 | **Operate on sets and number sets**<br>- Understand the meaning of the ***union*** of two sets A and B and that this is denoted by A ∪ B, and the meaning of the ***intersection*** of two sets A and B, denoted by A ∩ B, and represent these sets in a Venn diagram;<br>- Represent the ***complement*** of set A as A′, such as A ∪ A′ = E. |
| 9-NS-03 | **Use properties of rational and irrational numbers**<br>Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. |
| 9-NS-04 | **Extend the properties of exponents to rational exponents**<br>Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.<br>For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3} = 5^1 = 5$<br>Rewrite expressions involving radicals and rational exponents using the properties of exponents. |
| 9-NS-05 | **Introduce arithmetic operations on polynomials**<br>Understand that polynomials form a system analogous to the integers, namely they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials (Polynomials that simplify to quadratics are the expectation at this level). |

| Mathematics Standards<br>Grade 9<br>Strand 2: Algebraic Thinking (AT) | |
|---|---|
| **Write equivalent forms of algebraic expressions to solve problems** | |
| 9-AT-01 | **Operate with polynomials and rational expressions.**<br>Perform arithmetic operations on polynomials: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.<br>Add, subtract, multiply and divide algebraic fractions. Expand expressions of the forms:<br>$a(x \pm b)$ ; $(x + a)^2$ ; $(x - a)^2$ ; $(x + a)(x - a)$ ; $(x \pm a)(x \pm b)$ |
| 9-AT-02 | **Factor polynomials**.<br>Interpret the structure of expressions: Use the structure of an expression to identify way to rewrite it.<br>In particular Expand expressions of the forms: $a(x \pm b)$ ; $(x + a)^2$ ; $(x - a)^2$ ; $(x + a)(x - a)$ ; $(x \pm a)(x \pm b)$<br>Factorize algebraic expressions, by removing common factors from expressions such as:<br>$ax \pm ay$ ; $ax + a^2x^2$ ; $a^2x^2 - b^2y^2$ ; $x^2 + 2ax + a^2$ |
| 9-AT-03 | **Write expressions in equivalent forms to solve problems.**<br>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines (At this level, the limit is quadratic expressions of the form $ax^2 + bx + c$ |
| **Use formulas and algebraic expressions, including iterative and recursive forms, to model and solve problems.** | |
| 9-AT-04 | **Create equations that describe numbers or relationships.**<br>Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R. (At this level, limit to formulas that are linear in the variable of interest, or to formulas involving squared or cubed variables.)<br>Write and solve linear equations, including simple cases of fractional linear equations, and apply these skills to solving problems; verify the solution. |
| 9-AT-05 | **Interpret the structure of expressions.**<br>Interpret expressions that represent a quantity in terms of its context.<br>Interpret parts of an expression, such as terms, factors, and coefficients (At this level, limit to linear expressions, exponential expressions with integer exponents and quadratic expressions.) |
| **Equations and Functions: Use linear functions or inequalities to model and solve problems** | |
| 9-AT-06 | **Create equations that describe numbers or relationships.**<br>- Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions<br>- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. |
| 9-AT-07 | **Understand solving equations as a process of reasoning and explain the reasoning**<br>Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. |
| 9-AT-08 | **Solve equations and inequalities in one variable:**<br>Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. |
| 9-AT-09 | **Represent and solve equations and inequalities graphically**<br>- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).<br>- Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.<br>- Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. |
| 9-AT-10 | **Build a function that models a relationship between two quantities.**<br>Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms (At this level, formal recursive notation is not used. Instead, use of informal recursive notation is intended) |

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| 9-AT-11 | **Understand the concept of a function and use function notation.**<br>- Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.<br>*For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*<br>- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*. |
| 9-AT-12 | **Interpret functions that arise in applications in terms of the context.**<br>- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries (*At this level, focus on linear, exponential quadratic functions)<br>- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (At this level, focus on linear and exponential functions.) |

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| 9-AT-13 | **Analyze functions using different representations.**<br>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.□ Graph linear and quadratic functions and show intercepts, maxima, and minima. |
| 9-AT-14 | **Construct and compare linear, quadratic and exponential models and solve problems**<br>Distinguish between situations that can be modeled with linear functions and with quadratic functions. Prove that:<br>***- Linear functions grow by equal differences over equal intervals***<br>***- Exponential functions grow by equal factors over equal intervals***<br>Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a given table).<br>- Solve problems involving fractions, percentages, ratios and proportions. |
| 9-AT-15 | **Solve systems of equations**<br>- Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.<br>- Write and solve simultaneous linear equations with two unknowns by elimination and by substitution, and apply these skills to solving problems; verify the solution.<br>Find the approximate solutions of equations such as $x^3 + x = 20$ using ICT and trial and improvement methods. |

| Mathematics Standards<br>Grade 9<br>Strand 3: Measurement & Data (MD) | |
|---|---|
| **Measurements in geometric figures** | |
| 9-MD-01 | **Measurements in Geometric Figures**<br>- Use the length, area, and volume of geometric figures to solve problems.<br>- In a circle, find arc length, area of sectors<br>- In three-dimensional figures including sphere, cylinder, cube, and right prism find the lateral area, surface area, and volume. |
| 9-MD-02 | **Reason quantitatively and use units to solve problems.**<br>- Use units as a way to understand problems and to guide the solution of multi-step problems.<br>- Choose and interpret units consistently in formulas.<br>- Choose and interpret the scale and the origin in graphs and data displays.<br>- Define appropriate quantities for the purpose of descriptive modeling.<br>- Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. |
| 9-MD-03 | **Explain volume formulas and use them to solve problems.**<br>- Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a right prism, a cylinder, pyramid, and cone. |
| **Data, statistics and probability** | |
| 9-MD-04 | **Summarize, represent, and interpret data on two categorical and quantitative variables.** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.<br>- Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.<br>- Informally assess the fit of a function by plotting and analyzing residuals.<br>- Fit a linear function for a scatter plot that suggests a linear association. |
| 9-MD-05 | **Interpret linear models.** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. |
| 9-MD-06 | **Summarize, represent, and interpret data on a single count or measurement variable.** Represent data with plots on the real number line (dot plots, histograms, and box plots).<br>- Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.<br>- Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). |
| 9-MD-07 | **Summarize, represent, and interpret data on two categorical and quantitative variables.**<br>- Summarize categorical data for two categories in two-way frequency tables.<br>- Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies).<br>- Use relative frequency as an estimate of probability and use this to compare outcomes of experiments.<br>- Recognize possible associations and trends in the data.<br>- Compute (using technology) and interpret the correlation coefficient of a linear fit.<br>- Know that if events A and B are mutually exclusive, the probability of event (A or B) is the sum of the probabilities of A and of B.<br>- Compare experimental and theoretical probability in different contexts. |
| 9-MD-08 | **Construct and compare linear, quadratic, and exponential models and solve problems:**<br>- Distinguish between situations that can be modeled with linear functions and with exponential functions. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.<br>- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). |

| Mathematics Standards<br>Grade 9<br>Strand 4: Space & Geometry (SG) | |
|---|---|
| **Elements of Euclidean Geometry** | |
| 9-SG-01 | **Point, line, and circle.**<br>- Know precise definitions of line segment, angle, circle, perpendicular line, and parallel line, based on the undefined notions of point, line, distance along a line, and distance around a circular arc (At this level, distance around a circular arc is not addressed).<br>- Practice using rulers and compass to draw parallel and perpendicular lines, perpendicular bisector, and angle bisector. |
| **Geometric Proof by Deductive Reasoning** | |
| 9-SG-02 | Identify congruent triangles and their corresponding angles and sides. Know the conditions of congruence and determine whether two triangles are congruent |
| 9-SG-03 | Use knowledge of angles and properties of 2-D shapes to conjecture or deduce properties in a given plane figure. Identify similar triangles and their corresponding angles and sides. |
| 9-SG-04 | Use the properties of congruence or similarity of triangles to solve problems, e.g. find unknown sides or angles of similar or congruent triangles, figures, and sides or angles of similar or congruent figures. |
| 9-SG-05 | **Solve problem involving right-angled triangle**.<br>- State and apply Pythagoras' theorem.<br>- Know the sine, cosine and tangent ratios for a right-angled triangle and apply them to calculate an angle or a side of a triangle. |
| **Coordinate Geometry in a Cartesian set plane** | |
| 9-SG-06 | **Use coordinates to prove simple geometric theorems algebraically.**<br>- Use coordinates to prove simple geometric theorems algebraically.<br>- Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (using the distance formula)<br>- Find the point on a directed line segment between two given points that partitions the segment in a given ratio (At this level, focus on finding the midpoint of a segment).<br>- Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (e.g., using the distance formula) |

# BEACON PRIVATE SCHOOL

## Mathematics Subject Guide
## Grade 10

**FOR FUTURE REFERENCE**

**Details of Grade 10 are included in the following pages to clarify how**

**they extend the**

**Instructional Materials of Grade 9**

**January 2019**

| Mathematics Standards<br>Grade 10<br>Strand 1: Number Systems (NS) | |
|---|---|
| **Extend the understand of Sets to non-numerical elements.** | |
| 10-NS-01 | **Operate on number sets and non-numerical sets.**<br>- Understand the meaning of the ***union*** of two sets A and B and that this is denoted by A ∪ B, and the meaning of the ***intersection*** of two sets A and B, denoted by A ∩ B, and represent these sets in a Venn diagram;<br>- Represent the ***complement*** of set A as A′, such as A ∪ A′ = E. |
| 10-NS-02 | **Extend on the use and properties of geometric and other sets.**<br>- Understand the meaning of a locus of points is a set and apply union and intersection of loci as sets. |
| 10-NS-03 | **Define Relations between two sets and Laws of Composition within sets and apply these definitions to numerical and non-numerical sets.**<br>Understand that polynomials form a set analogous to the integers, namely they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials (Polynomials that simplify to quadratics are the expectation at this level). |
| 10-NS-04 | **Perform arithmetic operations with complex numbers**<br>- Know there is a complex number i such that $i^2 = -1$, and every expression in the form a + bi, with a and b real, is a complex number.<br>- Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. |
| 10-NS-05 | 9. **Use complex numbers in polynomial identities and equations**<br>- Solve quadratic equations with real coefficients that have complex solutions.<br>- Extend polynomial identities to the complex numbers. For example, rewrite $x^2+4$ as $(x + 2i)(x – 2i)$.<br>- Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. |

| Mathematics Standards<br>Grade 10<br>Strand 2: Algebraic Thinking & Operations (AT) | |
|---|---|
| **Work with radicals and integer exponents** | |
| 10-AT-01 | **Interpret the structure of expressions**<br>Interpret expressions that represent a quantity in terms of its context. (Quadratic and exponential are the expectations at this level.)<br>a. Interpret parts of an expression, such as terms, factors, and coefficients<br>b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on $P$.<br>c. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$ thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. |
| 10-AT-02 | **Write expressions in equivalent forms to solve problems**<br>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.<br>- Factor a quadratic expression to reveal the zeros of the function it defines. |
| **Use algebraic expressions, including iterative and recursive forms, to model and solve problems.** | |
| 10-AT-03 | **Build a function that models a relationship between two quantities**<br>Write a function that describes a relationship between two quantities (Quadratic and exponential functions are the expectation at this level.).<br>- Determine an explicit expression, a recursive process, or steps for calculation from a context<br>- Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. |
| 10-AT-04 | **Construct and compare linear, quadratic, and exponential models and solve problems**<br>Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing in a linear, quadratic or (more generally) as a polynomial function. |

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|---|---|
| **Sequences and patterns** | |
| 10-AT-05 | **Generate sequences from term-to-term definitions,** investigate the growth of simple patterns, generalizing algebraic relationships to model the behavior of the pattern. |
| 10-AT-06 | **Generate sequences from term-to-term definitions,** investigate the growth of simple patterns, generalizing algebraic relationships to model the behavior of the pattern. |
| 10-AT-07 | **Identify and sum geometric sequences**, and know the conditions under which an infinite geometric series can be summed. Convert any recurrent decimal to an exact fraction. |
| 10-AT-08 | **Identify number pattern contained in Pascal's triangle.** |
| **Use linear functions to model and solve problems; justify results** | |
| 10-AT-09 | **Apply the properties of proportional reasoning**<br>Apply them in a range of problems. |
| 10-AT-10 | **Relate proportion to linear function**: translate the statement $y$ is proportional to $x$ into the equation $y = kx$<br>Know that a straight line in the explicit form $y = mx + c$ represents a function, but that a straight line in the implicit form $ax + by + c = 0$ may, or may not, represents a function. |
| 10-AT-11 | **Construct the Cartesian equation of a straight line**<br>From its graph, or from the knowledge of the coordinates of two points on the line, or from the coordinates of one point on the line and the gradient of the line. |
| 10-AT-12 | **Know the condition for two straight lines to be parallel or perpendicular.**<br>**Interpret the solution set of simultaneous equations.** |
| **Use functions to model and solve problems. Justify results.** | |
| 10-AT-13 | **Analyze functions using different representations**<br>- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.<br>- Graph functions and show intercepts, maxima, and minima. (Linear, exponential, quadratic, square root, cube root, absolute value, step, and piecewise-defined functions are the expectations at this level.) |
| 10-AT-14 | **Analyze functions using different representations**<br>Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (Linear, exponential, quadratic, absolute value, step, and piecewise-defined functions are the expectations at this level.) |
| 10-AT-15 | **Use equations and inequalities that describe numbers or relationships**<br>Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. |
| 10-AT-16 | **Solve equations and inequalities in one variable:**<br>- Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.<br>- Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$. |
| 10-AT-17 | **Create equations that describe numbers or relationships:**<br>- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.<br>- Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.<br>(Using linear and exponential, including formulas involving quadratic terms are the expectations) |
| 10-AT-18 | **Solve systems of equations:**<br>- Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.<br>- For example, find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$. |
| 10-AT-19 | **Translate between the geometric description and the equation for a conic section**<br>- Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.<br>- Derive the equation of a parabola given as the locus of the set of point that are at the same distance from a fixed point (focus) and a fixed line (directrix). |

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| 10-AT-20 | **Interpret functions that arise in applications in terms of the context**<br>- For a quadratic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (Quadratic functions are the expectation at this level)<br>- Relate the domain of a quadratic function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function<br>- Calculate and interpret the average rate of change of a quadratic function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |
| 10-AT-21 | **Analyze functions using different representations**<br>- Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$ , $y = (0.97)^t$ ; $y = (1.01)^{12t}$ ; $y = (1.2)^{t/10}$ ; and classify them as representing exponential growth or decay. (Linear, exponential, quadratic, absolute value, step, and piecewise-defined functions are the expectations at this level)<br>- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (Linear, exponential, quadratic, absolute value, step, and piecewise-defined functions are the expectations at this level). |
| 10-AT-22 | **Build new functions from existing ones (**Quadratic and absolute value functions are the expectation at this level)<br>- Identify the effect on the graph of replacing f(x) by f(x) +k, k.f(x), f(k.x), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.<br>- Find inverse functions.. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. |

| **Mathematics Standards<br>Grade 10<br>Strand 3: Measurement & Data (MD)** | |
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| **Data: Statistics** | |
| 10-MD-01 | **Analyzing data**<br>Calculate and use measures of central tendency and spread: arithmetic mean, median, variance and standard deviation. |
| 10-MD-02 | **Present the findings, make inferences and draw conclusions**<br>Construct histograms, (grouping continuous data when necessary), draw stem-and-leaf diagram and box-and-whisker plots. |
| 10-MD-03 | **Make correlation between two random variables**<br>Draw scatter diagram and draw the line of best fit. |
| **Data: Probability** | |
| 10-MD-04 | **Understand independence and conditional probability and use them to interpret data** |
| 10-MD-05 | **Describe events as subsets of a sample space** (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). |
| 10-MD-06 | **Apply the Addition Rule**:<br>P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. |
| 10-MD-07 | **Understand independency of two events**<br>Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. |

| Mathematics Standards<br>Grade 10<br>Strand 4: Space & Geometry (SG) | |
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| **Use parallelism or perpendicularity of lines and segments to prove theorems and solve problems** | |
| 10-SG-01 | Prove theorems about parallelograms. Theorems include: Opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. |
| 10-SG-02 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio. |
| 10-SG-03 | **Understand similarity in terms of similarity transformations**<br>- Verify experimentally the properties of dilations given by a center and a scale factor: A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.<br>- Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. |
| ***Revise Congruence and Similarity from grade 9 and use them in solving more complex problems*** | |
| 10-SG-04 | ***Prove theorems about triangles using similarity.***<br>*Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*<br>*Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.* |
| 10-SG-05 | ***Prove theorems about lines and angles.***<br>*Theorems include: Vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* |
| 10-SG-06 | ***Prove theorems about triangles***.<br>*Theorems include: Measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* |
| 10-SG-07 | **Define trigonometric ratios and solve problems involving right triangles**<br>- Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.<br>- Explain and use the relationship between the sine and cosine of complementary angles.<br>- Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. |
| 10-SG-08 | **Understand and apply theorems about circles**<br>- Prove that all circles are similar.<br>- Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.<br>- Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.<br>- Construct a tangent line from a point outside a given circle to the circle. |
| 10-SG-09 | **Find arc lengths and areas of sectors of circles**<br>Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. |
| 10-SG-10 | **Determine the locus of an object moving according to a rule, including those arising in simple physical situations.**<br>(Essential cases: Segment bisector, angle bisector, circle, parabola) |
| **Use coordinates to prove simple geometric theorems algebraically** | |
| 10-SG-11 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio. |
| 10-SG-12 | Use coordinates to prove simple geometric theorems algebraically.<br>For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0,2). |
| 10-SG-13 | Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in geometric and algebraic terms. |

| **Vectors** | |
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| 10-SG-14 | **Consider coordinate systems as grids for moving around space in two dimensions**; understand position vector, unit vector and components of a vector. |
| 10-SG-16 | **Interpret a translation as a vector displacement**; know that a vector displacement from A to B depends only on the starting point A and the finish point B and not on intermediate steps from A to C to D to … to B, and that the vector sum of all these separate displacements from A to B is equivalent to the resultant displacement from A to B directly. |
| 10-SG-17 | **Add and subtract two vectors and draw corresponding vector diagrams** |
| 10-SG-18 | **Multiply a vector by a scalar** and know that this amounts to stretching the vector; calculate the magnitude and direction of a vector; use vectors to calculate displacement and velocity in a range of contexts. |

# Grades 11 & 12

# Comments

**In grades 11 & 12 students focus on the external examinations required and accredited by universities.**

**They will use the skills and learning approaches acquired in the previous school levels, and use them in learning the requirements of the external exams they have enrolled in.**

**To that effect, the school shall adopt the selected standards from the following sources:**
**In grades 11 & 12 students may follow one of two academic tracks:**

- **The International Baccalaureate Diploma Track**
- **The American Advanced Placement Track**

**Students enrolled in the IB Diploma Program shall naturally adhere to the stipulations of the IB Math Higher Level curriculum, and sit for the relevant IB certificate.**

**Students enrolled in American High School Program shall naturally adhere to the stipulations of the Mathematics Advanced Placement curriculum, and sit for the relevant AP examinations.**

***For program details of the courses in mathematics, teachers and students are referred to the appropriate Mathematics Subject Guides***