We’re in this together:
Supporting high-quality math teaching in uncertain times
About the Authors

Dr. Jo Boaler  
Nomellini & Olivier Professor of Education  
Stanford University  
USA

Tanya LaMar  
PhD Candidate, Mathematics  
Graduate School of Education  
Stanford University  
USA

Dr. Jennifer Langer-Osuna  
Assistant Professor of Mathematics Education  
Stanford University  
USA

Dr. Christine Suurtamm  
Professor of Mathematics Education  
University of Ottawa  
Canada

Dr. Jennifer Adams  
Editor-in-Chief  
Knowledgehook Signature Leadership Series  
Canada

ABOUT KNOWLEDGEHOOK

Knowledgehook, a leading educational technology company, empowers over 450 thousand teachers and parents to collaboratively support the mathematics learning journey of over 5 million students worldwide.

Winner of Google’s Game Changer Award and named Top Disruptor by BNN, its platform analyzes student understanding through engaging assessments, providing real-time personalized solutions to close learning gaps between classroom teaching and at-home learning. Designed by leading numeracy and research experts, Knowledgehook’s Instructional Guidance System is known for reinventing how online technology supports education and educators, while inspiring the problem solvers of tomorrow.

Our mission is to support the development of the teaching skills necessary for every teacher to make every student numerate.

Knowledgehook is an Instructional Guidance System (IGS) designed to professionally develop the highly-skilled math teachers who are necessary to improve student achievement. Our proprietary technology can uncover insights about every student’s learning needs from assessment data and then recommend expert-designed teaching activities to guide each teacher.
Why this Report?

The year 2020 has seen unprecedented disruptions to learning with more than 1.37 billion students out of school world-wide by the end of March (UNESCO, 2020). Global education organizations like the OECD have released reports to guide senior education leaders when responding to this disruption (OECD, 2020). This year has also been a year of protests about racial injustice. While the level of protests may be new, sadly, racial injustice is not. Longstanding gaps exist in equity of educational opportunities and outcomes for Black, Indigenous, and racialized groups of students (Berends & Penaloza, 2010; Lee, 2002; Truth and Reconciliation Commission of Canada, 2015).

So, what about disruptions and inequities in learning specifically for mathematics? What can Ministry and Department officials, district and school leaders, teachers and parents do to improve equity of outcomes in mathematics for all learners as they continue with remote instruction, return to their schools, or adjust to a blended delivery model of classroom and online learning?

WHY THIS FORMAT?

We’re in this together: Supporting High-quality Math Teaching in Uncertain Times is a short, practical report designed to be accessible to all stakeholders. It provides a narrative on research-based best practices across four categories:

1 Technology
2 Assessment
3 Curriculum
4 Pedagogy

The co-authors have deliberately placed the categories in this order to disrupt common thinking around how these categories should be approached. We lead with technology to acknowledge the current and dramatic shift in delivery of learning. Next is assessment, because the best time for assessment is not just at the end of a learning cycle, but also early and throughout the cycle to inform instruction and provide necessary feedback to students. We then discuss curriculum (the what) and we end with pedagogy (the how). Each category has a list of recommendations for consideration. Finally, we highlight key thoughts around implementation. Our hope is that this combination will help all educators support high-quality teaching and learning in math no matter what the circumstances.

WHY AN ASSETS-BASED APPROACH?

The co-authors firmly believe that an assets-based, rather than a deficits-based approach, is the only way forward. Students and teachers should be celebrated and feedback should begin with recognition of what students (and teachers) know and can do. Neuroscience has shown that all people are capable of learning to any level, and that risks and struggle should be embraced by all learners (Boaler, 2015; Luculano et al., 2015). Many people believe in the myth of the “math brain”, that only some people are capable of reaching high levels in mathematics. This has been disproven, it is known that all students can achieve any level of mathematics with high-quality teaching and growth mindset messages. Educators should find multiple opportunities to give students messages of growth mindset and belonging, to counter the ideas that have, in the past, held back particular populations, such as girls and students of color (Yeager & Walton, 2011; Walton et al., 2015). It is equally important that teachers have growth mindsets and that policy makers, district and school leaders, and teachers all believe in the ability of teachers to deliver the highest quality teaching, whether online or in person, and move forward with better, more equitable mathematics teaching and learning. The recommendations that follow are intended to help all of those in education deliver the highest quality, equity-focused mathematics instruction that will support all learners.
One of the biggest changes teachers are facing with the shift to online teaching is the presence of technology. Students continue to develop their technology literacy and use technological tools to enhance the power of mathematical thinking, but now technology has also become an important tool for teachers in the delivery of their lessons. High-quality mathematics teaching centers upon student thinking and collaboration (Boaler, 2002a, b; Boaler & Staples, 2008; Cobb, Gresalfi, and Hodge, 2009), which seems more challenging when students are separated and learning online. Technology can aid in supporting both online learning experiences and in person learning as well as synchronous and asynchronous learning through tools that are content specific or content neutral (NCTM, 2015).

Most importantly, technology can help teachers in creating online learning experiences that center student thinking and build a collaborative community. For example, students can collaborate on mathematical tasks within a digital whiteboard space. A digital whiteboard provides a wider range of tools beyond a typical word document, and allows students more freedom in the expression of ideas with options for drawing, adding pictures and figures to edit, sticky notes for commenting, and more. Students can also record video of their thoughts and reflections on their work which they share with peers for discussion. These videos can also serve as formative assessments so that the teacher can gauge student understanding and plan the next lesson according to student need. To interact with each other’s work and provide peer feedback, students can engage in what’s called a ‘digital gallery walk’, which is similar to an in-person gallery walk. Each student has their own slide in a shared presentation deck where they share their work, thinking, and/or representations. Then, during the ‘walk’, students are free to view and comment on each other’s slides. This can be followed by a whole class discussion on what students learned from one another’s work, what they found interesting, what questions they have, etc. These creative uses of technology allow students opportunities to learn and participate in a multitude of ways, which is key to an effective learning environment.

Regardless of format, best practices of centering student thinking and collaboration remain key for mathematics learning. Teachers should limit their talking or telling to short instances (each lasting five minutes or less), stopping frequently to allow students to reflect on what’s been said either by recording their ideas in a shared document, discussing in small groups or online in breakout rooms, or creating other representations of their curiosities and understandings.

Teachers can also incorporate technology in ways that prepare students for their tech-enabled world. Many mathematics technology programs are available as applications that students can download for free on their phones. For example, students can engage with the collection, organization, cleaning, analysis, and visualization of data (Wilkerson et al., 2018). With augmented reality apps, students can explore measurement, shape, and modelling of three-dimensional items in the home or classroom. And with graphing tools students can explore the behavior and patterns of a variety of functions and mathematical models or even data they’ve collected about themselves or their community. With the help of technology, the upcoming school year can be an opportunity to update classrooms both in terms of pedagogy and content.
Technology – Recommendations

**T1. Utilize technology to make student thinking visible.**

Students can record video and/or audio of their thinking and reasoning. These recordings can be used as ways to support asynchronous student-to-student interaction or as formative assessment tools.

**T2. Create shared digital spaces that allow for physically-distanced collaboration.**

Whether students are in person or working together online, shared digital spaces can allow students to simultaneously document their ideas and engage in collaborative problem solving. Consider tools like digital or in person white boards, shared presentation slides, or shared documents where students can incorporate drawings, pictures, symbols and more.

**T3. Facilitate virtual gallery walks.**

Students can share their work with one another and give comments and feedback on one another’s work in shared presentation slides. Each student or student group prepares a slide in a shared deck, then groups virtually view each other’s work and provide feedback.

**T4. Explore and discover patterns and characteristics of data and functions using graphing technology.**

Many free websites and apps allow students to explore and manipulate the graphs of data gathered by students, equations, inequalities, functions (including trigonometric functions, parametric functions), etc.

**T5. Share lesson slides with students to annotate with their own notes during online class time.**

Through online programs or simply making a copy of the teacher’s slides, students can add notes and annotations to what the teacher provides. This allows students to create digital reference points to mathematics they want to think more about.

**T6. Support students to utilize their personal device as a resource where appropriate.**

While a “no device” policy may be appropriate in some situations, students will also benefit from learning the appropriate use of the technology that is available to them and will continue to be available to them in the future. Allowing students to learn the mathematical capabilities of their personal devices not only empowers students with those tools but also provides practice for the appropriate use of those tools.
Assessment

One of the worries that educators may have when students return to school is that students will have “gaps” that need to be filled. In some cases, this might prompt educators to quickly begin with a battery of tests to determine what students do not know. Yet, there is a great deal of research that suggests that assessments that are embedded in instruction and are done “just in time” may be more meaningful, help to improve student learning, and provide students with a better opportunity to show what they do know and can do (Suurtamm et al., 2016; William, 2007). A variety of assessment strategies, such as collaborative problem-solving tasks, individual conversations or interviews with students, snapshots of student work, or videos of students explaining their thinking help to provide students with different opportunities to show what they know and can do and help teachers to clearly see student thinking and inform next steps (Earl & Temperley, 2014; Suurtamm & Arden, 2017).

Assessment can be incorporated strategically as new units of work are introduced so that the teacher can help to view students’ understandings of prior knowledge and skills and determine what prerequisite work may need to be incorporated in the learning.

Such types of assessments do not necessarily need to have the look and feel of tests. For instance, before beginning work focused on addition of fractions, a teacher might have students work in pairs to play a game using materials to compare and order fractions. As students play the game the teacher can circulate (in person or virtually) and listen to students’ understanding such as their use of benchmark numbers (e.g. ½) to compare numbers, or their use of terminology such as “same denominator”. The teacher might also notice which materials or representations seem most familiar to students so that the teacher can lean on these representations as they proceed with the unit. This kind of formative assessment can be conducted in person or online, in the latter case teachers would visit the break-out rooms in which students are working together.

Such formative assessment strategies prompt student recall and provide feedback to teachers and students. They help the teacher to determine instructional steps to connect prior knowledge to new learning (William, 2007). In this new COVID-induced setting, students will be returning to learning situations having had a variety of mathematical experiences which need to be both valued and built upon to move learning forward.
Assessment – Recommendations

A1. Provide assessment opportunities for students to show what they know and can do.

This process is based on a growth mindset for students and teachers. It celebrates students’ prior learning and experiences while allowing teachers to provide students with meaningful feedback on ways to expand their learning. It also helps teachers determine next steps for instruction based on students’ current understandings.

A2. Incorporate assessment opportunities within instructional tasks.

Assessment that is embedded in instruction and occurs “just in time” is meaningful, prompts prior learning and focuses on skills and understandings needed at the time. It also provides teachers the opportunity to hear student thinking.

A3. Use a variety of assessment strategies.

Students need multiple opportunities to demonstrate their learning and using a variety of assessment strategies provide different opportunities to students. For instance, some students may be able to explain their thinking in discussion with a teacher rather than in a written format. For more examples see Boaler (2016).

A4. Recognize and build on student experience and prior learning.

Students will be bringing a variety of learning experiences after having been apart from a formal learning setting. These might include learning measurement through baking, being taught different mathematics procedures from parents’ experiences with mathematics, or developing computation strategies from playing games. Assessment should provide opportunities to share and celebrate these different experiences.

A5. Make student thinking visible and heard.

If working remotely (online) consider ways to use technology and digital resources to gain insights into student thinking rather than rely solely on assessment strategies that focus on giving credit for correct answers. If working in person, provide opportunities to see and hear student thinking (Cirillo & Langer-Osuna, 2018).

A6. Develop an assessment plan as part of short-term and long-range instructional planning.

As teachers develop their instructional plans they should plan for assessment opportunities to include a variety of types of assessments and focus on the important mathematics that students should be learning.
Curriculum

When teachers and students return to school, whether the teaching is online or in person, they will probably be faced with missed and forgotten content. This may prompt teachers to return to methods of teaching that are ineffective – teaching mathematics as a list of methods through lecturing, enabling speedy but shallow “coverage”. This faulty approach may seem particularly appealing to teachers if lessons are online. But while a global pandemic has brought many challenges to the world and to our educators, it has also brought an opportunity for an important reset, and there may be few more important areas in need of change than the content that is taught to students and the ways it is taught.

Before the time lost by Covid, teachers of mathematics had struggled to teach deeply when faced with content standards that present mathematics as a set of methods, as is common in most countries in the world. The standards used in most systems not only show mathematics as a set of methods, they present too many methods for teaching to be effective. The solution to this long-standing problem is teachers being given time to plan differently. When teachers meet together and work out the big ideas of their curriculum – ideas that are conceptually important, and that have many connections – and then choose rich, deep activities that focus on the big ideas, students are more engaged and successful (Boaler, 2016, 2019). One of the most important insights for students is that mathematics is a set of big, connected ideas. When teachers engage students in these big ideas, many of the smaller “methods” are met. If smaller methods are not encountered they are probably not as important to learn. This mapping of curriculum into big and coherent ideas, rather than disconnected methods, seems particularly important now, as schools have less time and need to prioritize.

In our work with the California Department of Education, two of the big ideas we are sharing with teachers are number sense and data literacy. Data activities have many advantages – as students work with real data sets, asking questions, investigating relationships, they can feel personally involved and connected. For these and other reasons, data lessons are a perfect start to the new school year. A focus on data analysis and data science also provides opportunities for students to recognise the global pandemic and examine data that relates to it - if data is carefully chosen to show positive events such as the impact of social distancing, or mask wearing, or changed activities, rather than death rates.

The use of rich, deep conceptual activities is an excellent strategy for the first week of the school year, K-16. Teachers may create their own activities but resources are also available to support teachers. One example, youcubed - a Stanford center to promote high-quality mathematics teaching and learning - offers a free set of these materials. This is known as “the week of inspirational maths” and is used in about two thirds of the schools in the US and three quarters of the schools in Canada. The week includes growth mindset videos for students and tasks that engage students deeply. Teachers can create their own “playlist” where they choose tasks and videos that are best for their students.
Curriculum – Recommendations

C1. Plan a set of big ideas.
Teachers meet and plan in groups to work out the “big ideas” in their grade level or course - ideas that are conceptually rich with many connections to other content areas. These are amazing conversations that help teachers plan and be well informed.

C2. Choose conceptually rich activities.
Teachers choose rich deep activities that address each of their “big ideas” (C1). These are activities that will probably extend over lessons, that will engage students in conversations and exploration. Conceptually rich activities can be solved using a variety of strategies and have a “low floor and high ceiling”. This means that students can engage with the task from a broad range of background knowledge.

C3. Start the year with the Week of Inspirational Maths Activities.
Teachers can set the right tone by immersing students in math activities that are meaningful and relevant to the learner. Let them experience the joy of math and show their ability to inquire about and solve real world problems. [www.youcubed.org/week-inspirational-math](http://www.youcubed.org/week-inspirational-math)

C4. Prioritize content.
Which content is most important? It may not be possible to teach as much content as in previous years. One way to prioritize is to teach the big ideas (C1 & 2). Another is to consider which content is most important. Two candidates for this list are number sense and data literacy.

C5. Utilize resources to enhance curriculum.
Teachers can include the use of resources to enhance the richness of a task and increase student engagement. Resources include any tool that might increase student curiosity or support their thinking and communication of ideas. These might include data visualizations, cell phone apps, graphing tools, manipulatives (digital or in person), etc.
Pedagogy

Whether online or in person, good pedagogy starts with rich, open tasks that engage students in mathematical exploration and reasoning as part of a sense-making community (Stein, Engle, Smith, & Hughes, 2008). During planning, teachers choose tasks that target important concepts and connections, and anticipate students’ strategies and struggles. Teachers then launch tasks, support exploration, and close the lesson with whole-class discussions based on students’ work. At the center of this work is students’ thinking about mathematics: their strategies, conjectures, explanations, questions, and curiosities. By centering student thinking, each learner is positioned as contributors to the classroom community, which supports a sense of belonging and identification with mathematics (Turner, et al, 2013). Starting the year with activities such as “mathographies” can also support students’ understanding of their identities as learners and offer teachers important knowledge of students’ histories doing and learning mathematics.

Students can think together about mathematics even during times of social distancing and distance learning. In person, even as students might not be able to sit together, teachers can facilitate whole-class discussions. Online technologies such as breakout groups, shared documents, and other collaboration or discussion tools can facilitate small group conversations and shared thinking among students.

Fostering productive and inclusive collaboration among students requires more than simply grouping them together and asking them to complete a task, even a rich, open-ended task (Langer-Osuna, 2011). Teachers must also work with students to create a sense-making community where a broad range of student thinking is not only acceptable, but is framed as an important part of doing and learning mathematics (Gresalfi & Cobb, 2006). For example, when teachers explicitly ask for strategies rather than answers, promote the sharing of different strategies, or frame errors as opportunities for reflection and sense-making, students are offered more ways to contribute to mathematical discussions and the stakes of participation are lowered. Students also benefit from structured group work, whether through the use of roles, turn-taking routines such as round robins, or routines that support peer listening and mutual understanding, such as revoicing routines. For example, teachers can model to students how to revoice a peer’s thinking by offering language stems such as, “I hear you say _____. Is that right?” Students can practice these routines as a class before moving into online breakout groups for discussion.

Strategies that support language learners and students with disabilities also help create a positive learning environment for all students (Lambert & Sugita, 2016; Moschkovich, 1999). Incorporating gestures, artifacts, and multiple forms of communication as part of classroom discourse, attending to and revoicing students’ mathematical ideas, allowing students time to rehearse and prepare for whole-class discussions, not limiting the use of resources, and using follow-up questions to help students complete or extend their explanations support the participation of all learners.

By centering student thinking, each learner is positioned as a contributor to the classroom community, which supports a sense of belonging and identification with mathematics.  

Pedagogy – Recommendations

**P1. Launch tasks to support exploration.**
Whether online or in person, launch tasks in ways that set the problem context, clarify expectations, and allow students to share what they already know that is relevant to the task.

**P2. Attend to and be curious about students’ mathematical thinking.**
Teachers should attend to the mathematical ideas being expressed rather than focusing on correcting vocabulary. By revoicing students’ statements, students can also confirm the correctness of the teachers’ interpretation.

**P3. Position students as mathematical thinkers.**
Teachers can position students as mathematical thinkers by widening possibilities for participation. For example, by making errors a normal, and often productive, part of doing mathematics, teachers lower the stakes of participation, normalize revision as part of what it means to learn and do mathematics, turn mistakes into opportunities for sense-making, and position students who struggle as important contributors to mathematics discourse that all students benefit from.

**P4. Elicit student thinking in a variety of ways.**
Teachers can ask open questions and elicit student conjectures or curiosities using online collaboration tools such as shared documents, or have students take pictures of drawings or other representations of their mathematical ideas.

**P5. Provide space for students to prepare share outs.**
All students benefit from opportunities to practice communicating their ideas to one another. Students working with paraprofessionals can rehearse sharing their strategies in preparation for whole-class discussion. This functions similarly to doing a think-pair-share before inviting student contributions to the whole-class.

**P6. Provide regular and open access to resources.**
Teachers can help create a classroom culture where students can access resources – like math notebooks and manipulatives – whenever they need them. Students with disabilities may use these resources more often or for longer amounts of time and benefit from being able to draw on them as necessary.
Meaningful learning can only occur when student and teacher well-being is ensured. Many people’s lives have been disrupted during recent months leading all of us left to navigate a new normal. Developing a learning community where one is allowed to take risks to explore mathematical and pedagogical ideas will go a long way in supporting high-quality mathematics teaching and learning.

An important way in which math educators can offer social and emotional support to students is by acknowledging each student’s identity and experiences. Teachers can begin the school year by asking students to create their math history, usually a short write up of the student’s past experiences in mathematics and how they feel about the subject. This gives students the opportunity to tell their story as a math learner and for the teacher to learn important insights to what that student might need or the ways they learn best. Teachers can also provide social and emotional support through engaging lessons that allow students to connect with the ideas being taught. Traditional lessons that have taught mathematics as a set of procedures to follow have resulted in widespread disengagement as students see no relevance for their lives. This is particularly harmful for students of color and for girls – who receive additional harmful messages that mathematics is not for them.

Walton and colleagues have shown through repeated studies that many students, particularly girls and students of color, do not feel that they ‘belong’ in certain disciplines (Walton et al., 2015). This is often due to a history of negative and off-putting messages communicated to students about who can and cannot learn mathematics (Chestnut et al, 2018). Other studies have shown that different topics and teaching approaches can lead to feelings of belonging or not belonging (Boaler, 2019; Boaler, Cordero & Dieckmann, 2019). In this white paper we have set out an approach that we propose will increase feelings of student belonging and connection with the content.

For many of the recommendations in this report, policy changes are either required or would be extremely helpful, particularly when trying to scale the improvements to math teaching and learning throughout the educational jurisdiction. Policy changes are an integral part of what Michael Fullan refers to as “pressure and support” (Fullan, 2001). Clear policy statements allow those who are trying to move math practice forward in their classrooms and schools to act with the jurisdiction’s seal of approval. This allows educators at all levels to confidently open their doors and invite others to learn and grow with them.

One example of a positive curriculum policy change is the new Ontario Mathematics Curriculum: Grades 1 to 8 (Ontario Ministry of Education, 2020). The policy now requires teaching across 6 strands: social-emotional learning skills, number, algebra, data, spatial sense, and financial literacy. It mandates the teaching of mathematics processes including: problem-solving, reasoning and proving, reflecting,
connecting, communicating, representing, and selecting tools and strategies. The policy provides clear direction for math content and math processes to all teachers in a system responsible for the education of 2 million school-aged children. Policy changes can set the stage for scalability if combined with the right support. Teachers should be empowered to take risks and make changes in their classrooms with the support of their administrators.

More than ever, teachers will need resources and tools to support them in providing high-quality math instruction to their students.

A variety of digital tools can help teachers shift to an online learning environment and maintain a collaborative inquiry approach even while students and teachers cannot be working side-by-side.

Digital resources can help engage students during teacher-led learning as well as when parents are supervising or guiding their learning. Jurisdictions, districts and schools should do what they can to ensure that all students and teachers have access to reliable internet connections and web-enabled devices.

It goes without saying that budgets will need to be prioritized for these resources and the professional learning required alongside.

Teachers can’t do this alone. They need to be given time to individually and collaboratively plan their teaching, to work out the “big ideas” in their grade level, to choose rich tasks, to share assessment ideas and to discuss student work.

Many jurisdictions moved to virtual meetings for professional learning during the recent period of school closures. We can and should learn from this approach. Surveying teachers will be useful to find out what worked with virtual professional learning and to see how these successful elements can be embedded in professional learning models when schools return to “the new normal” at some point in the future. It is essential that teachers continue to feel part of a valued community of learners, whether face to face or online. Like their students, teachers need a safe space to try new things. The concept of “anytime, anywhere” professional learning can be inherently liberating and hopeful, if developed with and for teachers.

Foundational to the work of supporting high-quality math teaching is equity. We know that significant inequities existed for groups of students prior to COVID. The inequities will undoubtedly have grown throughout the period of disrupted learning. As we return to learning situations, it is important to remember that all students deserve access to high-quality mathematics teaching. All students have the right to engage in rich tasks, be challenged with meaningful curriculum, be assessed in ways that highlight what they know and can do, and have access to technology to support their learning.

All students have the right to have their mathematical thinking respected and heard and for teachers to build on the learning that students bring to the table.

When reading through the list of recommendations, we hope that every teacher and educational leader will prioritize based on their specific context, focusing on the recommendations that will “raise the bar and narrow the gap” amongst the students with the greatest needs in their classrooms and schools.
Summary
Recommendations

**Technology**

**T1**
Utilize technology to make student thinking visible.

**T2**
Create shared digital spaces that allow for physically-distanced collaboration.

**T3**
Facilitate virtual gallery walks.

**T4**
Explore and discover patterns and characteristics of data and functions using graphing technology.

**T5**
Share lesson slides with students to annotate with their own notes during online class time.

**T6**
Support students to utilize their personal device as a resource where appropriate.

**Assessment**

**A1**
Provide assessment opportunities for students to show what they know and can do.

**A2**
Incorporate assessment opportunities within instructional tasks.

**A3**
Use a variety of assessment strategies.

**A4**
Recognize and build on student experience and prior learning.

**A5**
Make student thinking visible and heard.

**A6**
Develop an assessment plan as part of short-term and long-range instructional planning.
**Curriculum**

- **C1** Plan a set of big ideas.
- **C2** Choose conceptually rich activities.
- **C3** Start the year with the Week of Inspirational Maths Activities.
- **C4** Prioritize content.
- **C5** Utilize resources to enhance curriculum.

**Pedagogy**

- **P1** Launch tasks to support exploration.
- **P2** Attend to and be curious about students’ mathematical thinking.
- **P3** Position students as mathematical thinkers.
- **P4** Elicit student thinking in a variety of ways.
- **P5** Provide space for students to prepare share outs.
- **P6** Provide regular and open access to resources.
References


