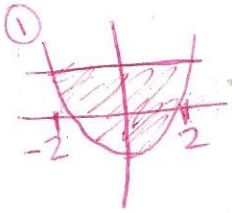
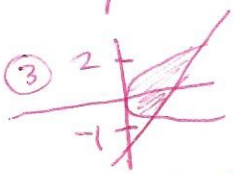
Areas and Volumes



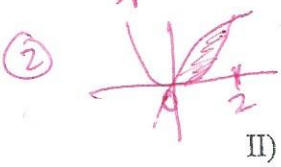
I) Find the areas of the regions bounded by the two curves below

1) $y = x^2 - 2$ and $y = 2$ $\int_{-2}^{2} 2 - x^2 + 2 = \int_{-2}^{2} 4 - x^2 = 10\frac{2}{3}$

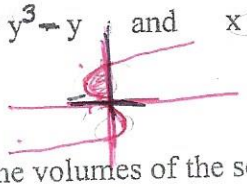
2) $y = x^2$ and $y = 4x - x^2$ $\int_{0}^{2} 4x - x^2 - x^2 = \int_{0}^{2} 4x - 2x^2 = 2\frac{2}{3}$



3) $x = y^2$ and the line $x = y + 2$ $\int_{-1}^{2} y + 2 - y^2 = 4.5$



4) $x = y^3 - y$ and $x = 0$ $\int_{0}^{1} y^3 - y = \frac{1}{4}$



II) Find the volumes of the solids formed

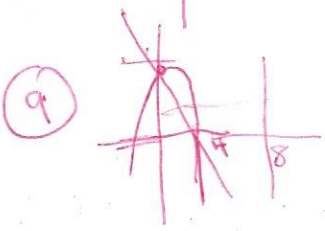
5) When $y = \sqrt{9 - x^2}$ is revolved around the x-axis $\pi \int_{-3}^{3} 9 - x^2 = 36\pi$

6) When region bounded by $y = \sec x$ and the x-axis from $x = \frac{-\pi}{4}$ to $x = \frac{\pi}{4}$ is revolved around the x-axis $\pi \int_{-\pi/4}^{\pi/4} \sec^2 x = 2\pi$

7) When the area bounded by $x = 1 - y^2$ and y-axis is revolved around the y-axis. $\pi \int_{-1}^{1} (1 - y^2)^2 dy = \frac{16}{15}\pi$

8) When the region bounded by $y = x^3$, $x = 2$ and x-axis is revolved around the line $x = 2$. $\pi \int_{0}^{8} (2 - y^{1/3})^2 dx =$

9) When the region bounded by $y = 16 - 4x$ and $y = 16 - x^2$ is rotated around the x-axis. $\pi \int_{0}^{4} (16 - x^2)^2 - (16 - 4x)^2 = 643.3982$



10) When the same two graphs as in # 9 are rotated about the line $x = 8$.

$R_{OUT} = 8 - \left(\frac{16 - y}{4}\right)$ $x = \sqrt{16 - y}$

$R_{IN} = 8 - \sqrt{16 - x^2}$

$\pi \int_{0}^{16} \left(8 - \frac{(16 - y)}{4}\right)^2 - \left(8 - \sqrt{16 - x^2}\right)^2 = 128\pi$