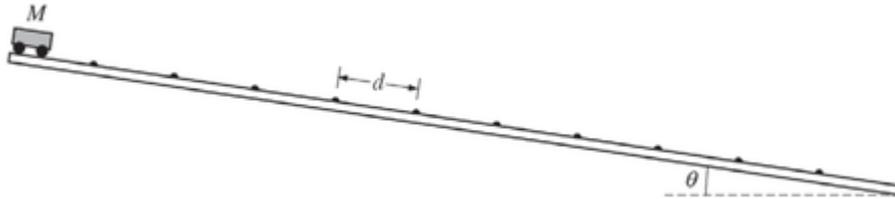


Free Response 4

1.



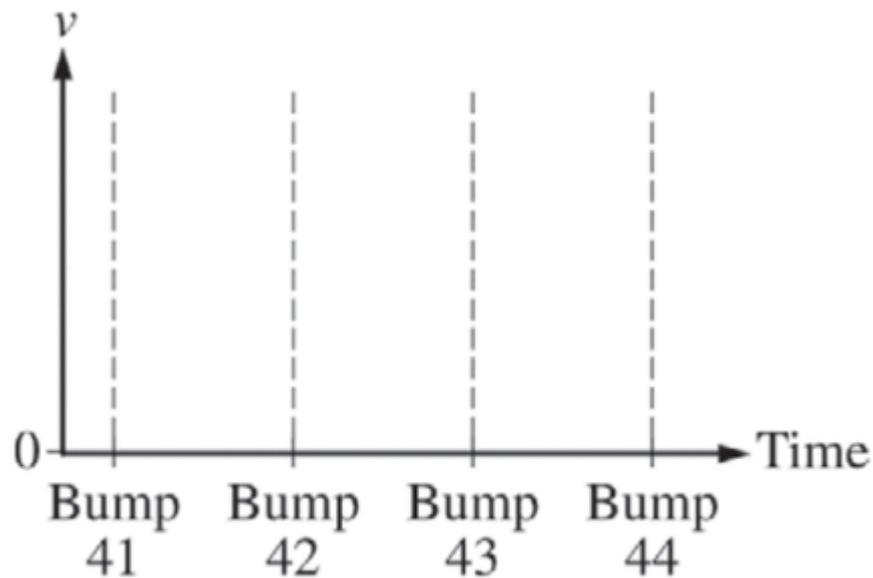
Note: Figure not drawn to scale.

A long track, inclined at an angle θ to the horizontal, has small speed bumps on it. The bumps are evenly spaced a distance d apart, as shown in the figure above. The track is actually much longer than shown, with over 100 bumps. A cart of mass M is released from rest at the top of the track. A student notices that after reaching the 40th bump the cart's average speed between successive bumps no longer increases, reaching a maximum value v_{avg} . This means the time interval taken to move from one bump to the next bump becomes constant.

- a. Consider the cart's motion between bump 41 and bump 44.
 - i. In the figure below, sketch a graph of the cart's velocity v as a function of time from the moment it reaches bump 41 until the moment it reaches bump 44.
 - ii. Over the same time interval, draw a dashed horizontal line at $v = v_{\text{avg}}$. Label the line " v_{avg} ".



Free Response 4



- b. Suppose the distance between the bumps is increased but everything else stays the same. Is the maximum speed of the cart now greater than, less than, or the same as it was with the bumps closer together?

___ Greater than ___ Less than ___ The same as

Briefly explain your reasoning.

- c. With the bumps returned to the original spacing, the track is tilted to a greater ramp angle θ . Is the maximum speed of the cart greater than, less than, or the same as it was when the ramp angle was smaller?

___ Greater than ___ Less than ___ The same as

Briefly explain your reasoning.

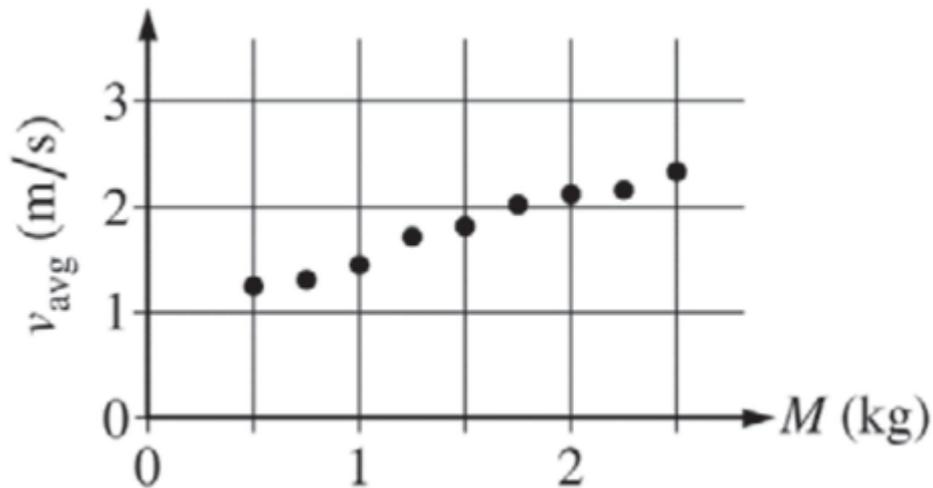
- d. Before deriving an equation for a quantity such as v_{avg} , it can be useful to come up with an equation that is intuitively expected to be true. That way, the derivation can be checked later to see if it makes sense physically. A student comes up with the following equation for the cart's maximum average speed: $v_{\text{avg}} = C \frac{Mg \sin \theta}{d}$, where C is a positive constant.

- i. To test the equation, the student rolls a cart down the long track with speed bumps



Free Response 4

many times in front of a motion detector. The student varies the mass M of the cart with each trial but keeps everything else the same. The graph shown below is the student's plot of the data for v_{avg} as a function of M .



Are these data consistent with the student's equation?

Yes No

Briefly explain your reasoning.

- ii. Another student suggests that whether or not the data above are consistent with the equation, the equation could be incorrect for other reasons. Does the equation make physical sense?

Yes No

Briefly explain your reasoning.

The text box below should be used for notes only and not your final response.



Please respond on separate paper, following directions from your teacher.

Upload your final response here.

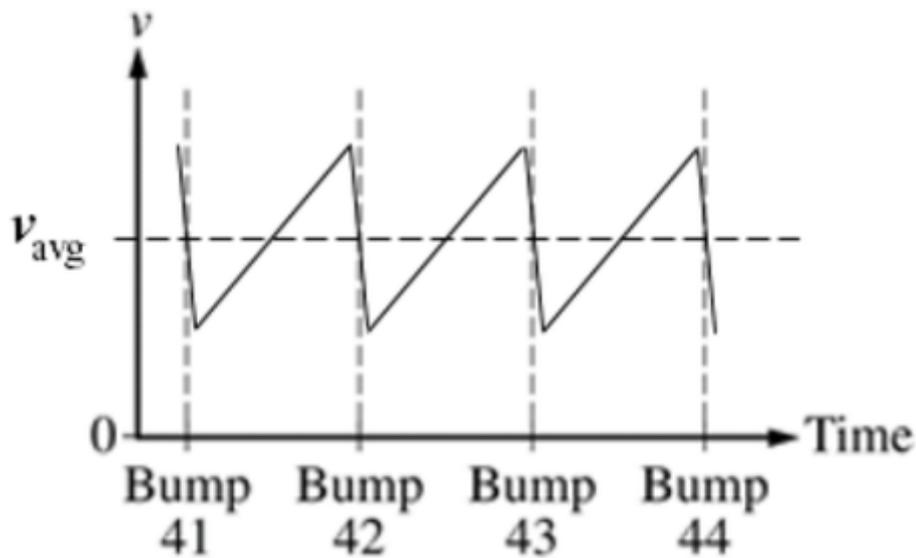


Free Response 4

 Please respond on separate paper, following directions from your teacher.

Part A

Part(i)



1 point is earned: For having a constant upward slope in each segment between bumps

1 point is earned: For having the velocity “reset” abruptly at each bump (i.e., as a sawtooth function, not a sinusoidal curve) to a minimum positive value that is the same for each bump

1 point is earned: For having the same maximum value in each cycle that occurs near the bump times (This point can be earned for a sinusoidal curve with peaks at the bump times.)

Part(ii)

1 point is earned: For drawing a v_{avg} line that is horizontal and consistent with the graph drawn, even if that graph is wrong



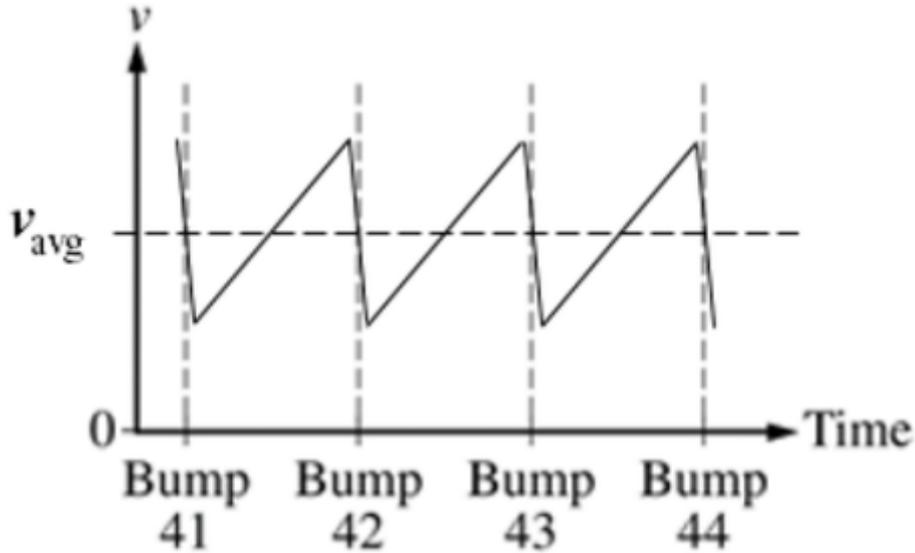
0	1	2	3	4
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Free Response 4

Student response earns 4 of the following 4 point(s)

Part(i)



1 point is earned: For having a constant upward slope in each segment between bumps

1 point is earned: For having the velocity “reset” abruptly at each bump (i.e., as a sawtooth function, not a sinusoidal curve) to a minimum positive value that is the same for each bump

1 point is earned: For having the same maximum value in each cycle that occurs near the bump times (This point can be earned for a sinusoidal curve with peaks at the bump times.)

Part(ii)

1 point is earned: For drawing a v_{avg} line that is horizontal and consistent with the graph drawn, even if that graph is wrong

Part B

Correct answer: Greater than

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.



Free Response 4

No points are earned if the wrong answer is selected.

1 point is earned: For indicating that there is more time between bumps

1 point is earned: For connecting that the cart has more time or more distance to accelerate between bumps

Example: The maximum speed is greater because the cart has more space (or time) to accelerate (build up speed) between bumps.

For indicating that the potential energy difference increases due to increased height between successive bumps

For relating the increase in potential energy to an increase in kinetic energy

✓

0	1	2
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Student response earns 2 of the following 2 point(s)

Correct answer: Greater than

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

No points are earned if the wrong answer is selected.

1 point is earned: For indicating that there is more time between bumps

1 point is earned: For connecting that the cart has more time or more distance to accelerate between bumps

Example: The maximum speed is greater because the cart has more space (or time) to accelerate (build up speed) between bumps.

For indicating that the potential energy difference increases due to increased height between successive bumps

For relating the increase in potential energy to an increase in kinetic energy



Free Response 4

Part C

Correct answer: Greater than

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

No points are earned if the wrong answer is selected.

1 point is earned: For indicating that the acceleration is greater

1 point is earned: For indicating that the component of the gravitational force increases

Example: The maximum speed is greater because the cart has more space (or time) to accelerate (build up speed) between bumps.

For indicating that the potential energy difference increases due to increased height between successive bumps

For relating the increase in potential energy to an increase in kinetic energy



0	1	2
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Student response earns 2 of the following 2 point(s)

Correct answer: Greater than

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

No points are earned if the wrong answer is selected.

1 point is earned: For indicating that the acceleration is greater

1 point is earned: For indicating that the component of the gravitational force increases

Example: The maximum speed is greater because the cart has more space (or time) to accelerate (build up speed) between bumps.



Free Response 4

For indicating that the potential energy difference increases due to increased height between successive bumps

For relating the increase in potential energy to an increase in kinetic energy

Part D

Part (i)

Correct answer: No

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

1 point is earned: For indicating that v_{avg} is not proportional to M

1 point is earned: For connecting the equation to the data

Examples:

The y-intercept of the graph is not zero, but the equation indicates that it should be zero.

Doubling the mass from the graph does not double v_{avg} , but the equation indicates that it should double.

If “yes” is selected, one point may be earned for explaining that v_{avg} increases with M or that v_{avg} looks (approximately) proportional to M for a limited portion of the data (e.g., the points at $M = 1.0 \text{ kg}$ and $M = 2.0 \text{ kg}$).

Part (ii)

Correct answer: No

1 point is earned: For indicating that the distance dependency is incorrect

1 point is earned: For indicating that, according to the equation, greater d leads to a smaller v_{avg} ,

OR for stating or implying the contradiction between this inverse relation and the reasoning of part (b)

Credit is earned for any answer that is consistent with reasoning in part (b).



Free Response 4

Example:

According to the equation, a larger d corresponds to a smaller v_{avg} , because they are inversely proportional. But according to the reasoning of part (b), a bigger distance d between the bumps leads to a larger maximum average speed, showing that the equation is implausible.

If “yes” is selected, one point can be earned for indicating that an increase in the angle increases v_{avg} .

0	1	2	3	4
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✓

Student response earns 4 of the following 4 point(s)

Part (i)

Correct answer: No

No points are earned if the correct answer is selected, but the explanation is completely incorrect or there is no explanation.

1 point is earned: For indicating that v_{avg} is not proportional to M

1 point is earned: For connecting the equation to the data

Examples:

The y-intercept of the graph is not zero, but the equation indicates that it should be zero.

Doubling the mass from the graph does not double v_{avg} , but the equation indicates that it should double.

If “yes” is selected, one point may be earned for explaining that v_{avg} increases with M or that v_{avg} looks (approximately) proportional to M for a limited portion of the data (e.g., the points at $M = 1.0 \text{ kg}$ and $M = 2.0 \text{ kg}$).

Part (ii)



Free Response 4

Correct answer: No

1 point is earned: For indicating that the distance dependency is incorrect

1 point is earned: For indicating that, according to the equation, greater d leads to a smaller v_{avg} ,

OR for stating or implying the contradiction between this inverse relation and the reasoning of part (b)

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Example:

According to the equation, a larger d corresponds to a smaller v_{avg} , because they are inversely proportional. But according to the reasoning of part (b), a bigger distance d between the bumps leads to a larger maximum average speed, showing that the equation is implausible.

If “yes” is selected, one point can be earned for indicating that an increase in the angle increases v_{avg} .
