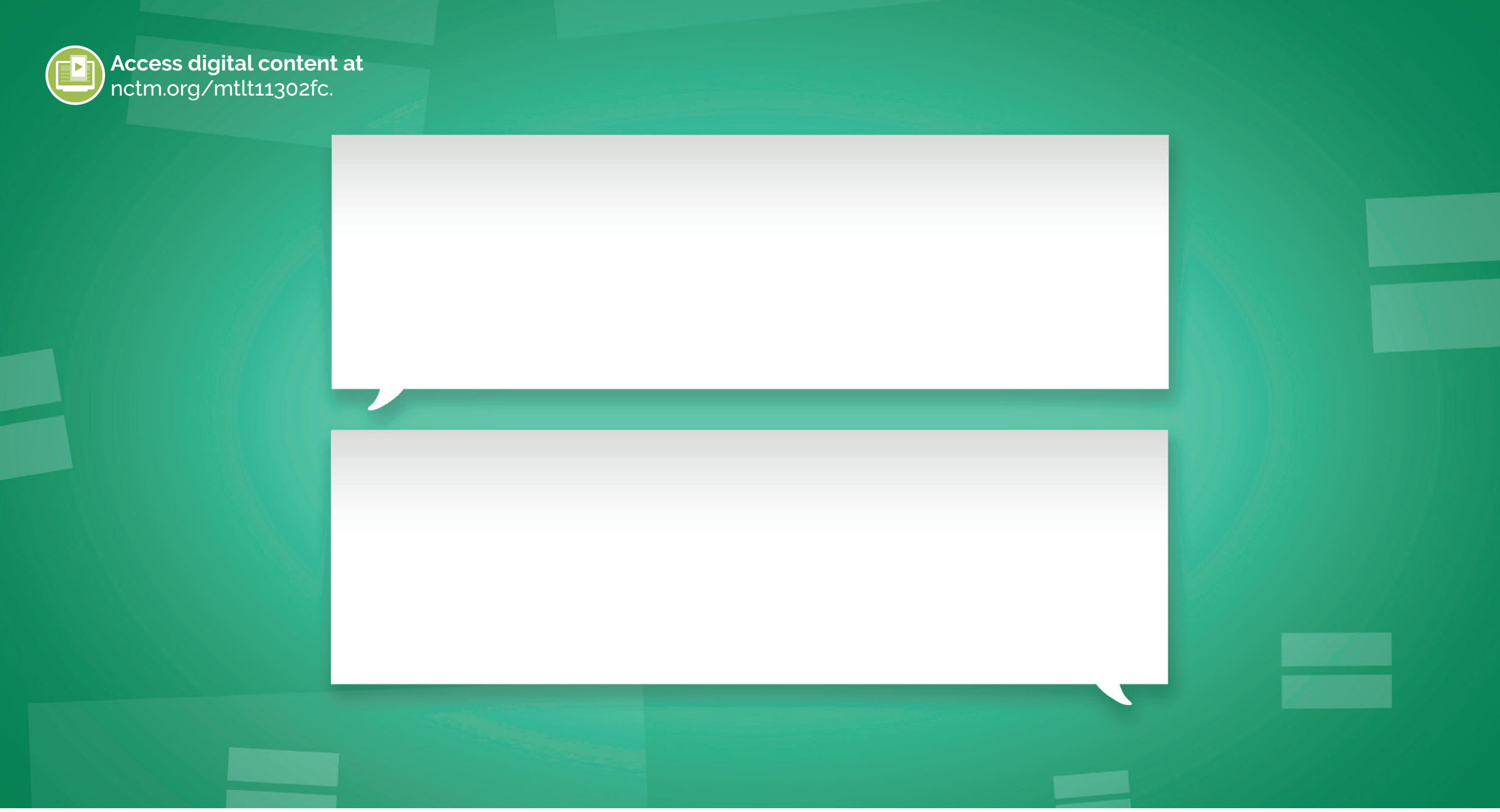
Access digital content at nctm.org/mtlt11302fc.



Justification as an **Equity Practice**

See how to give students equitable access to mathematics, develop their agency, and support their engagement.

Kristen N. Bieda and Megan Staples

Despite strong efforts, and some progress, our profession still has great strides to make in organizing classrooms to produce more equitable outcomes in mathematics. Teaching that supports *all* students to learn mathematics for understanding is not a stance or an ideology. Equity is a fundamental guiding principle for school mathematics teaching:

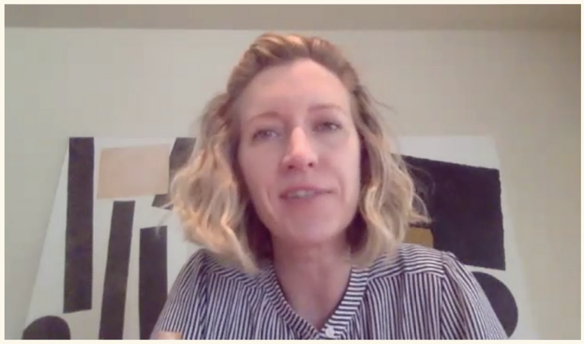
> The question is not whether all students can succeed in mathematics but whether the adults organizing mathematics learning opportunities can alter traditional beliefs and practices to promote success for all. (NCTM 2014, p. 60)

Justification, an essential mathematical practice, is well-known for its role in promoting rigor and developing mathematical understanding. Equally powerful is its role as a discursive practice that provides students with *access* to mathematical thinking and reasoning (the mathematics behind the answers) and promotes student *agency* with respect to mathematics. These two aspects—access and agency—link the mathematical practice of justification to teaching that aims at achieving more equitable outcomes. Our goal is to demonstrate this connection so that readers can appreciate, understand, and ultimately enact an expanded role of justification in their classrooms. To accomplish this, we first offer definitions of these important terms—*justification*, *equity*, *access*, and *agency*. We then explain the role of student access and agency in achieving equity outcomes, illustrating how engaging students in justification can present opportunities to provide access and develop agency, supported by examples from classroom practice. The examples we share are grounded in prealgebra content, but the principles are applicable across grade levels because justification at any level promotes access and agency. We also encourage you to watch the reflection video (video 1) that accompanies this article, in which we share stories from our own classroom experiences that shaped our understanding of justification as a lever for equity.

Justification at any level promotes access and agency.

Mathematical justification is the process of supporting your mathematical claims and choices when solving problems or explaining why your claim or answer makes sense. Justification encompasses mathematical argumentation as expressed in the third of the Common Core State Standards for Mathematical Practice (SMP 3): *Construct viable arguments and critique the reasoning of others* (NGA Center and CCSSO 2010).

Video 1 Why Justification for Equity?



Watch the full video online.

Equity is a fundamental guiding principle for school mathematics teaching (NCTM 2014). What does it mean to teach for equity? Gutiérrez (2002) argues that equity is achieved, in part, when we are unable to predict students' participation and achievement in mathematics solely on demographic characteristics or proficiency in English. This notion of equity offers a powerful vision for what is possible. It means all students have ample support to achieve in mathematics classes. But what might that look like in daily practice? Following Gutiérrez (2002), and connecting with justification, we focus on two principles of students' engagement in mathematics classrooms: (1) providing access and (2) developing agency.

Kristen N. Bieda, kbieda@msu.edu, is an associate professor of teacher education and mathematics education at Michigan State University in East Lansing. She teaches courses for future secondary mathematics teachers and graduate students in mathematics and teacher education. Her research interests include students' engagement in justification and proof in secondary mathematics classrooms, as well as the role of field experiences in secondary mathematics teacher preparation.

Megan Staples, megan.staples@uconn.edu, is an associate professor of mathematics education in the Neag School of Education, University of Connecticut in Storrs. Her teaching focuses on the preparation of secondary mathematics teachers. Her research focuses on how teachers organize classroom environments that support powerful practices such as collaboration, justification, and argumentation.

doi:10.5951/MTLT.2019.0148

Access refers to each student in a class having opportunities to engage in rigorous mathematics and to learn mathematics that is meaningful. *Agency* is when students develop a sense that they can do mathematics and generate mathematical ideas; each student sees mathematics as a tool to use.

These two principles—providing access and developing agency—are intertwined. Students' agency with mathematics develops in relation to the opportunities they are provided (access) and the responses they experience as a result of participating (Martin 2000). Students' access to mathematics and rigor is influenced by their sense of agency, which shapes how they engage, take up challenges, and see themselves as capable of learning new ideas. To demonstrate this point further, here is a brief excerpt to illustrate what commonly happens when teachers open up access for students to think about important mathematics but students do not feel they have the agency to do so. This excerpt comes from a class of lower-attaining ninth-grade students, although this kind of exchange is likely familiar to teachers across all grade levels.

Ms. F: [*Encouraging the group to engage in the problem*] You can even create three cards, assign each person a—

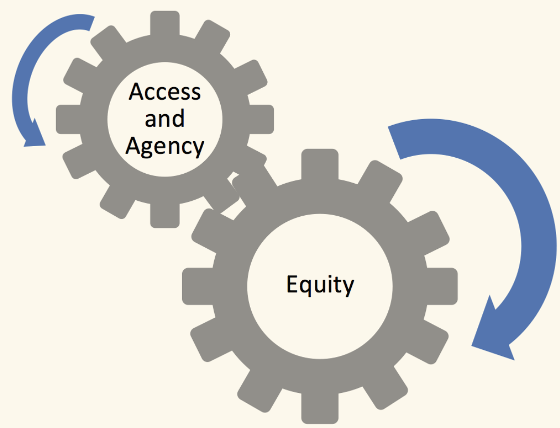
Dontay: We're not gonna—

Ms. F: Yeah, you are. What are you thinkin', Dontay?

Dontay: We're thinking we're gonna be stuck on this 'til you say it's time to go on to something else.

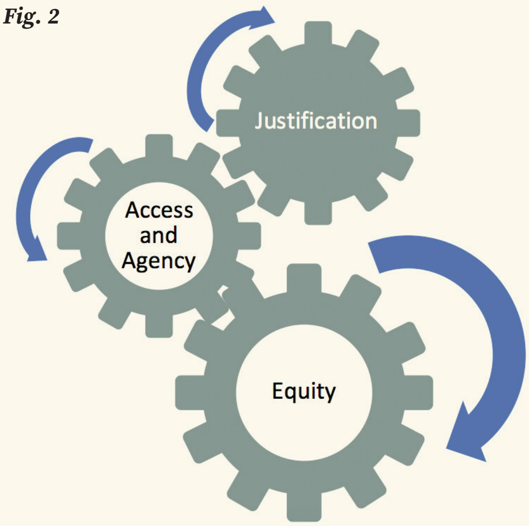
Dontay's response in the excerpt suggests that the access he has had to working through the problem-solving process in mathematics class has shaped his agency for tackling challenges in mathematics class, in this case, in a not-so-productive way. Classrooms that offer access and develop agency help achieve more equitable outcomes (Gutiérrez 2002). Figure 1 offers a visual representation for this relationship. As the gears illustrate, access and agency work together to promote equity. Teachers, of course, play a crucial role in creating these opportunities and shaping these experiences.

Fig. 1



Access and agency work together to promote equitable outcomes.

Fig. 2



Researchers have found that because justification promotes agency and access, it can lead to increasingly equitable outcomes.

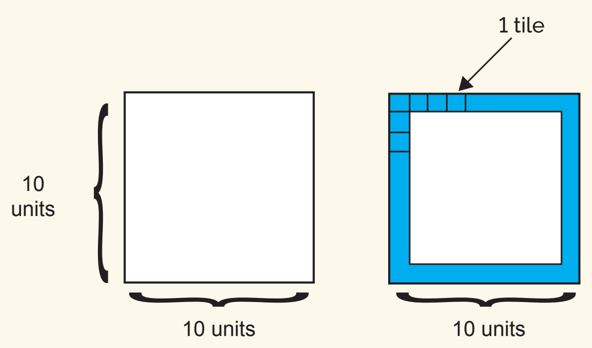
TEACHING WITH JUSTIFICATION TO PROMOTE EQUITY

Some researchers have found that justification promotes agency and access (see figure 2), and thus more equitable outcomes (e.g., Boaler 2008; Boaler and Staples 2008; Hiebert et al. 1997). In the next sections, we use two classroom vignettes to envision how justification can contribute to key principles of providing access and developing agency. We present actual and composite classroom vignettes, followed by discussions that connect these principles to equitable outcomes.

Providing Access by Asking Why?

The three-turn exchange on the next page represents a fairly typical interaction in mathematics classrooms. (The transcripts—including names—in vignette 1,

Fig. 3



The Border problem has students find the number of square tiles needed to place a border inside various sizes of squares.

although fictitious, are based on actual classroom episodes.) The excerpt includes how the student arrived at his answer, but the excerpt does not involve justification of *why* the student knows the answer is correct. We then revamp it to show how justification can open up access.

Vignette 1: Take One

Students are working on the Border problem (see figure 3) in which they find the number of square tiles needed to place a border inside various sizes of squares. In this case, the side length is 10 units.

Ms. T: OK, let's look at the Border problem. How many tiles are needed for the border if the side has 10 units, and how did you get it?
Joel: Thirty-six, 'cause I did 10 plus 10 plus 8 plus 8.
Ms. T: OK. Good. Anybody get something else? [*Pausing*] No? OK. Any questions? [*Pausing*] OK. Let's look at the next one.

In the exchange, we see the teacher productively encouraging student input, elaboration, and alternate solutions and questions. Some students could have their answer affirmed, and perhaps their process as well, if their approach was the same as Joel's. Notice, however, that the exchange provided no access to students in the class to make sense of the mathematics that Joel shared. The teacher had Joel share *how* he got his answer, offering some access to the procedural aspects of his work. However, students did not learn *why* he added, or why he added those particular numbers, to solve this problem. As a result, students had minimal opportunity to access mathematical sense making and reflect on their own thinking process.

The question "Why?" can offer an opportunity for students to justify and thus can open access for students to think about mathematical concepts. Let's consider "take two" for this vignette, where more intentional prompting for justification opens up access to student thinking and, thus, important mathematics. This vignette was inspired by a video clip of the Border problem (Boaler and Humphreys 2005) and other enactments we have seen of generalization tasks.

Vignette 1: Take Two

Ms. T: OK, let's look at number 5. How many tiles are needed for the border if the side has 10 squares, and how did you get it?
Joel: Thirty-six, 'cause I did 10 plus 10 plus 8 plus 8.
Ms. T: OK. Say a little more about why you added 10 and 10 and 8 and 8.
Joel: I did 10 plus 10 because I counted two sides of tiles fully for 10 and then counted the other two sides, but not the corners, for 8 each. So 36.
Ms. T: Thanks, Joel. OK, so Joel said 36 and offered his reasoning for why it's 36. And we can see what he did. Who has comments on Joel's strategy?
Miana: [*Her hand shoots up*] Oh!
Ms. T: Yes, Miana.
Miana: I just figured something out!
Ms. T: Yes?
Miana: I did 10 times 4, but I just realized I should do 10 times 4 minus 4.
Ms. T: OK, where are your 10s? And what's your revised answer?
Miana: The 10s were each full side. So I'll get 40 minus 4, which is 36. You can first . . .

In this revised excerpt, having Joel elaborate on his reasoning about where his numbers came from—that is, his justification—offered Miana access to his mathematics, which provided an opportunity for her to think differently about the problem and reflect on her own reasoning. Joel's reasoning revealed that he had thought about how to avoid double counting the corner tiles, which potentially was the key idea that helped Miana revise her approach.

Asking Why: Building Agency

All children have a deep capacity for doing mathematics and using mathematics to figure out the world. However, some schooling practices may diminish their curiousity and willingness to take a risk with an idea, which in turn contributes to students failing to develop a sense of agency in mathematics classrooms. If their classroom experiences have largely involved learning about rules to be practiced and applied in exercises, they have learned that their role is to practice doing someone else's mathematics and not actually generate mathematical ideas and make sense of them for themselves. We can begin to build students' agency in elementary school, even with foundational content, which some may perceive as having little room for sense making. Consider the following classroom exchange about understanding the structure of whole number multiplication (adapted from Koestler et al. 2013):

Ms. Gray: Let's review. So, if I told you to find 3 times 5, what would you do? Raoul?
Raoul: You would add 3 copies of 5: 5 plus 5 plus 5.
Ms. Gray: OK, what if I asked you to find 1 times 5? What would you do? Melissa?
Melissa: You add 1 copy of 5, so 5 plus 5.
Amira: Wait! Two times 5 is 5 plus 5.
Joseph: I'm confused.
Ricky: Maybe 1 times 5 and 2 times 5 are the same. We just have to remember they are the same.

When Ricky says, "We just have to remember they are the same," his use of *we* suggests that he sees the students in the class as recipients of information, not creators of knowledge. Students become repeaters and receivers, not doers and thinkers or agents for creating and discovering mathematics.

We offer the next excerpt as a way to consider how to productively build agency through providing students with opportunities to justify their reasoning in mathematics classrooms. This exchange is excerpted from a ninth-grade classroom of lower-attaining students taught by an experienced teacher. This takes place at the beginning of the academic year, so there is much work to do to help students see themselves as capable mathematical thinkers. Students are working on the In-Out table (functions) shown in figure 4. To complete the table, students must infer a pattern in the In-Out pairs and fill in the missing information. Again, note that although this is a high school classroom, this content—looking for consistent patterns—is a common mathematical activity at all grade levels. We encourage the reader to pause to think about how to fill in the missing letters and words.

The function represented in the In-Out chart can be explained in this way: The Out is the second vowel of the In word. Thus, *e* would be the Out for *mathematics*. For the In of *Can't Do*, any word with only one vowel works, such as *stretch* or *add*.

Ms. N is the main classroom teacher, and Ms. F is a supporting special education teacher. As you read the excerpt, notice how the student portrays himself, and notice how Ms. N's request for justification creates an opportunity for the teachers to help the student see himself differently—as a doer of mathematics—on the basis of his reasoning and contributions.

Vignette 2: Part One

The teacher has assigned groups to fill in a set of In-Out tables. After a few minutes, the teacher writes some of the tables on the board and asks for students to share their ideas.

Ms. N: Come on up, Alex. Go ahead.
Alex: OK [*picking up the marker and looking at his paper*]. Oh, there it is. Um, XXX about that. [*He looks at the board.*] I have, I have no clue anymore.
Ms. N: You have no clue anymore.
Alex: Nope. [*He puts down the marker and sits down.*]
Ms. N: [*Looking to the class*] OK, someone who hasn't been up.

Another student exchange ensues, and it comes to light that students did not understand the *Can't Do* phrase, so this is clarified.

Alex: Well, wait a minute—I do get it!
Ms. N: Well, go back up then.
Ms. F: Well, go back up.
Alex: Alright! I'm not stupid!
Ms. N: Nobody ever said that you were.
Alex: My dad said I was. Um, OK. [*He looks at his paper. Then he writes H in the Out for mathematics.*]
Ms. N: Is that an *H*?
Alex: Yeah. I know I have really bad writing. [*He writes in an A for the IN of "Can't Do"* (see figure 5).]

Let's pause here. What might the teacher do at this point? Alex has shared a response, but it includes what seems to be an incorrect answer with an *H* for mathematics. His sharing comes on the heels of commenting about his father's view of his smartness. How might you sensitively and productively handle this situation in a way that promotes agency as well as access? What role

Fig. 4

IN	OUT
division	I
ever	E
opportunity	O
toast	A
safe	E
people	O
mathematics	_____
_____	Can't do

Students must find a pattern in the In-Out pairs and fill in the missing information.

Fig. 5

IN	OUT
division	I
ever	E
opportunity	O
toast	A
safe	E
people	O
mathematics	__*h*____
____*A*____	Can't Do

Asking Alex for his rule allowed him to justify his answers, giving everyone (Alex included) the chance to see him as a mathematical thinker.

might justification play? Note that the teacher is working in the moment, but this interaction will have meaning beyond this moment for how this child sees himself mathematically.

We continue with this vignette as it unfolded in class. The teacher chose to ask Alex for his rule, which allowed him to provide the justification and reasoning behind his answers and why they made sense. Notice how that request created the opportunity for Alex (and his classmates) to see himself as a mathematical thinker (develop agency), despite the "incorrect" answer.

Vignette 2: Part Two

Ms. N: So, what do you think the rule is?

Alex: OK. OK, I'm not too sure how to explain it, but there's a pattern. See, that's the fourth letter; that's an *I*. [*He points to the board as he talks.*] That's the third letter; that's an *E*. Fourth letter, *O*. Third letter, *A*. Fourth letter, *E*. Third letter, *O*. Fourth letter, *H*. One letter [*pointing to his A*], and it can't be done.

Ms. N: Oh, now, very interesting pattern. That wasn't the pattern that some of us have, but that's a very interesting pattern.

Now, when we're doing these, the same rule has to apply for all of them. What yours does is you have two rules [*she approaches the board*], which says, "If it's the first word in the pattern, we use the fourth letter [*she points to the board*]; if it's the second word in the pattern, we use the third letter."

Alex: Oh.

After a little more discussion of the rule, Ms. N makes the following comments: "And you know what [*speaking to the class*]? I am so happy that he went up here and did this because if he hadn't done this, we wouldn't have been able to stress this point that we have to do one rule for all of them and not two. So, I want to thank you very much for coming up here."

In this continuation of the excerpt, the teacher's request for Alex's rule allowed Alex to be seen as a mathematical thinker even though his response did not describe a function for that In-Out table. (Note that one *could* consider Alex's rule to be a function if the input is a string of two words. The function then would take ordered pairs of words as the input, and would return as the output the third letter of the first word and fourth letter of the second word.) The teacher's response in the last three lines of the vignette model a way to position a student publicly as a thinker and doer of mathematics, even when the student has provided the wrong answer. Alex had the opportunity to see the productive nature of his response and how it related to functions. He was further praised for contributing to the class's learning opportunities (access for the class) and clarification of a key point. Demonstrating the value of a student's contribution in such a public and specific way resonates with Cohen and Lotan's (1995) work on the value and importance of *assigning competence* to students in heterogenous group settings, which has been connected to more equitable participation across groups of students and positive outcomes.

THE JUSTIFICATION-EQUITY LINK

These vignettes offer examples of how justification can promote access and agency. The justification moves created opportunities to access important mathematical ideas and positioned students in ways that highlighted their capacities as thinkers and doers of mathematics. These types of opportunities—repeated daily for students—move us toward Gutiérrez's (2002) vision of classrooms that support more equitable outcomes. Not all children enter school feeling entitled to agency for, or access to, being successful in mathematics. The verbal and nonverbal discourse that teachers employ, supported by rich tasks, is fundamental for developing students' agency for doing powerful mathematics. This is possibly the most important reason justification should be an essential part of everyday practice in mathematics classrooms.

The verbal and nonverbal discourse that teachers employ, supported by rich tasks, is fundamental for developing students' agency for doing powerful mathematics. This is possibly the most important reason justification should be an essential part of everyday practice.

Learning More

We hope this article inspires and empowers you to focus on creating more opportunities for students to engage in mathematical justification as a way to promote equitable outcomes for students in your class. The focus on equity implications of justification is new, so we encourage you to learn with us as we explore, as a field, how to implement teaching practices that encourage justification to build students' access and agency in mathematics. Some resources to support learning about the role of justification in equitable classrooms can be found in the supplemental online materials.

REFERENCES

Boaler, Jo. 2008. "Promoting 'Relational Equity' and High Mathematics Achievement through an Innovative Mixed-Ability Approach." *British Educational Research Journal* 34, no. 2 (April): 167–94.

Boaler, Jo, and Cathy Humphreys. 2005. *Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning*. Portsmouth, NH: Heinemann.

Boaler, Jo, and Megan Staples. 2008. "Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School." *Teachers College Record* 110, no. 3 (March): 608–45.

Carpenter, Thomas P., Elizabeth Fennema, Karen C. Fuson, James Hiebert, Hanlie Murray, and Diana Wearne. 1997. *Making Sense: Teaching and Learning Mathematics with Understanding*. 1st ed. Portsmouth, NH: Heinemann.

Cohen, Elizabeth G., and Rachel A. Lotan. 1995. "Producing Equal-Status Interactions in the Heterogeneous Classroom." *American Educational Research Journal* 32, no. 1 (Spring): 99–120.

Gutiérrez, Rochelle. 2002. "Enabling the Practice of Mathematics Teachers in Context: Toward a New Equity Research Agenda." *Mathematical Thinking and Learning* 4, nos. 2–3 (July): 145–87.

Koestler, Courtney, Mathew D. Felton, Kristen N. Bieda, and Samuel Otten. 2013. *Connecting the NCTM Process Standards and the CCSSM Practices*. Reston, VA: National Council of Teachers of Mathematics.

Martin, Danny Bernard. 2000. *Mathematics Success and Failure among African-American Youth: The Roles of Sociohistorical Context, Community Forces, School Influence, and Individual Agency*. 1st ed. London, UK: Routledge.

National Council of Teachers of Mathematics (NCTM). 2014. *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). 2010. *Common Core State Standards for Mathematics*. Washington, DC: NGA Center and CCSSO. http://www.corestandards.org.