AP Calculus BC Summer Assignment Name__________________

Due Date: First Day of School

Welcome to AP Calculus BC! This is an exciting, challenging, fast paced course that is taught at the college level. We have a lot of material to cover before the AP exam in May and, as such, we cannot spend a lot of time in class reviewing pre-requisite material. This mandatory packet is designed to review much of that material for you. Spend time working on this over the summer. If you struggle with specific areas, use your old notes, previous textbook(s), and/or the internet as a resource. You can also email me if you have questions (bmilks@solebury.org). Do not wait until the last minute to complete this assignment and do not email with me with questions during the week that school starts. That is too late...you should work on this sooner. The majority of the packet should be completed without a calculator, although there are few sections that indicate calculator use is expected. I have an answer key for this packet that you are welcome to look at during registration week. A couple of items that you should be aware of:

1. We will be using a TI-89 graphing calculator for the course. Please feel free to buy one this summer to familiarize yourself with its features. I can/will sign out calculators during registration week or on the first day of school if you wish to borrow one for the year.

2. Know your trig!! You should know your unit circle COLD. Trigonometry will often show up in the middle of a problem and the faster you are with your facts, the better equiped you will be to solve problems. If you want to practice filling out a unit circle (maybe a few times throughout the summer) go to **Embedded Math** (http://www.embeddedmath.com/) and click on worksheets. You can find a blank unit circle to print. This is important!

3. Know your basics from exponential and logarithmic functions (know that $\ln e = 1$, $\ln 1 = 0$, and that *log* has an understood base of 10, etc.).

4. Back to Trig!!! Know your double angle identities for sine and cosine. Know your (3) trig pythagorean identities, and know your reciprocal identities.

The greatest skill you can have to be successful in this course is to be a consistent worker. Being good at math will only take you so far. Being willing to work will get you to the finish line! You need to complete homework when it is assigned, pay attention in class, and meet with me outside of class as needed. I encourage you to form a small study group to work on homework and to prepare for assessments.

I look forward to a successful year of AP Calculus!

Britta

Section 1: Algebra Review

1. Solve $xy + 2x + 1 = y \ \ for\ y$

2. Factor completely: $x^2(x-1) - 4(x-1)$

3. Solve $\ln(y-1) - lny = x + lnx \ \ for\ y$

4. Factor $3x^{\frac{3}{2}} - 9x^{\frac{1}{2}} + 6x^{-\frac{1}{2}}$

Simplify each expression.

5. $\dfrac{\left(x^2\right)^3 x}{x^7}$

6. $\sqrt{x} \cdot \sqrt[3]{x} \cdot x^{\frac{1}{6}}$

7. $\dfrac{5(x+h)^2 - 5x^2}{h}$

8. $\dfrac{\frac{1}{x} + \frac{4}{x^2}}{3 - \frac{1}{x}}$

Simplify, by factoring first. Leave answers in factored form.

Example:

$$\frac{(x+1)^3(4x-9)-(16x+9)(x+1)^2}{(x-6)(x+1)} = \frac{(x+1)^2[(x+1)(4x-9)-(16x+9)]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2[4x^2-5x-9-16x-9]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2[4x^2-21x-18]}{(x-6)(x+1)}$$

$$= \frac{(x+1)^2[(4x+3)(x-6)]}{(x-6)(x+1)}$$

$$= (x+1)(4x+3)$$

9. $(x-1)^3(2x-3)-(2x+12)(x-1)^2$

10. $\dfrac{(x-1)^2(3x-1)-2(x-1)}{(x-1)^4}$

Simplify by rationalizing the numerator.

Example:

$$\frac{\sqrt{x+4}-2}{x} = \frac{\sqrt{x+4}-2}{x} \bullet \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \frac{x+4-4}{x\left(\sqrt{x+4}+2\right)} = \frac{x}{x\left(\sqrt{x+4}+2\right)} = \frac{1}{\sqrt{x+4}+2}$$

11. $\dfrac{\sqrt{x+9}-3}{x}$

12. $\dfrac{\sqrt{x+h}-\sqrt{x}}{h}$

Solve each equation or inequality for x over the set of real numbers.

13. $2x^4 + 3x^4 - 2x^2 = 0$

14. $\frac{2x-7}{x+1} = \frac{2x}{x+4}$

15. $\sqrt{x^2 - 9} = x - 1$

16. $|2x - 3| = 14$

17. $x^2 - 2x - 8 < 0$ (answer in interval notation)

18. $\frac{3x+5}{(x-1)(x^4+7)} = 0$

Solve each system of equations algebraically and graphically.

19. $\begin{cases} x + y = 8 \\ 2x - y = 7 \end{cases}$

20. $\begin{cases} y = x^2 - 3x \\ y = 2x - 6 \end{cases}$

Section 2: Trigonometry Review

21. Use your knowledge of the unit circle, to evaluate each of the following. You MUST know your unit circle. Leave answers in radical form. Do NOT use your calculator.

a) $sin 30°$

b) $cos\frac{2\pi}{3}$

c) $tan 45°$

d) $sin\left(-\frac{\pi}{6}\right)$

e) $tan\pi$

f) $cos\frac{5\pi}{6}$

g) $\cos(90°)$

h) $cos\frac{3\pi}{4}$

i) $cot\frac{\pi}{6}$

j) $\cos^{-1}\left(\frac{1}{2}\right)$

k) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

l) $\tan^{-1}(1)$

Solve each trigonometric equation for $0 \leq x \leq 2\pi$.

22. $sinx = \frac{\sqrt{3}}{2}$

23. $\tan^2 x = 1$

24. $cos\frac{x}{2} = \frac{\sqrt{2}}{2}$

25. $2\sin^2 x + sinx - 1 = 0$

26. $3cosx + 3 = 2\sin^2 x$

Solve each exponential or logarithmic equation.

27. $5^x = 125$

28. $8^{x+1} = 16^x$

29. $81^{\frac{3}{4}} = x$

30. $8^{-\frac{2}{3}} = x$

31. $\log_2 32 = x$

32. $\log_x \frac{1}{9} = -2$

33. $\log_4 x = 3$

34. $\log_5(x-6) = 1 - \log_5(x-2)$

35. $\log x - \log(x-3) = 2$

Expand each of the following using the law of logs.

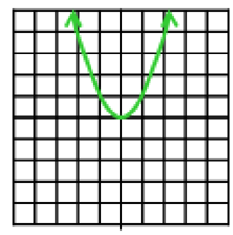
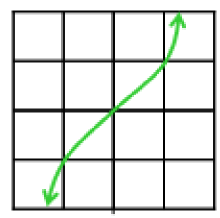
36. $\log_3 5x^3$

37. $\ln \frac{5x}{y^2}$

38. $2ln\sqrt{y} - \frac{1}{2} lny^4 + ln2y$

Section 3: Graphing Review

I. Symmetry - Even/Odd Functions

Quick Review		
Even Function	Symmetric about the y axis $f(-x) = f(x)$ for all x	Example: $y = x^2$ 
Odd Function	Symmetric about the origin (equivalent to a rotation of 180 degrees) $f(-x) = -f(x)$ for all x	Example: $y = x^3$ 

To determine algebraically if a function is even, odd, or neither, find $f(-x)$ and determine if it is equal to $f(x)$, $-f(x)$, or neither.

Example: Determine if $f(x) = \frac{4x}{x^2+1}$ is even or odd.

$f(-x) = \frac{4(-x)}{(-x)^2+1} = \frac{-4x}{x^2+1} = -\frac{4x}{x^2+1} = -f(x)$ Therefore, $f(x)$ is an odd function.

Determine if the following functions are even, odd, or neither.

39. $f(x) = \frac{(x^2)}{x^4+3}$

40. $f(x) = \frac{x}{x+1}$

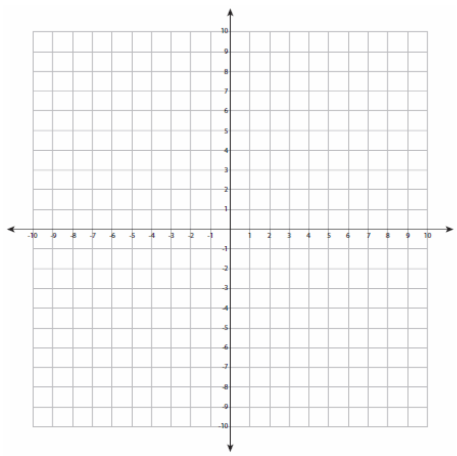
41. $f(x) = 1 + 3x^2 + 3x^4$

42. $f(x) = 1 + 3x^3 + 3x^5$

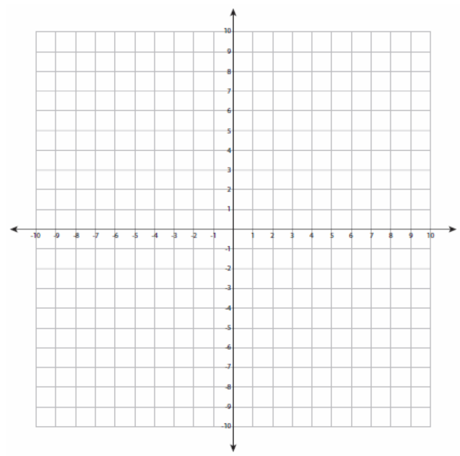
II. Essential Graphs

Sketch each graph. You should know the graphs of these functions.

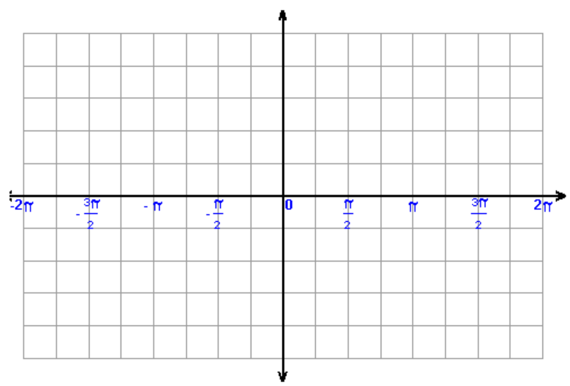
43. $f(x) = \sqrt{x}$



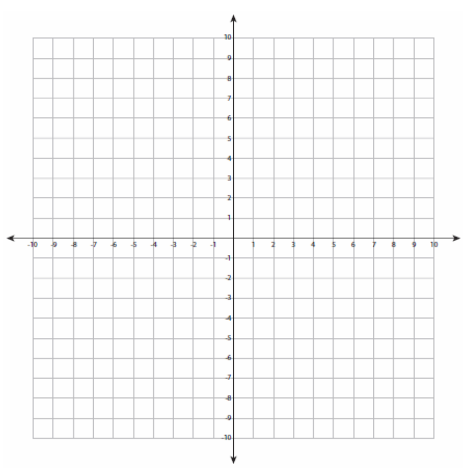
44. $f(x) = x^3$



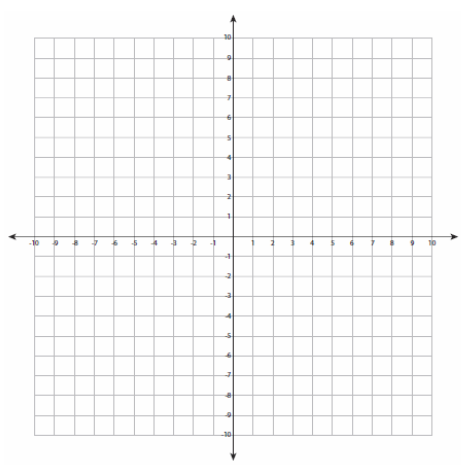
45. $f(x) = sinx$



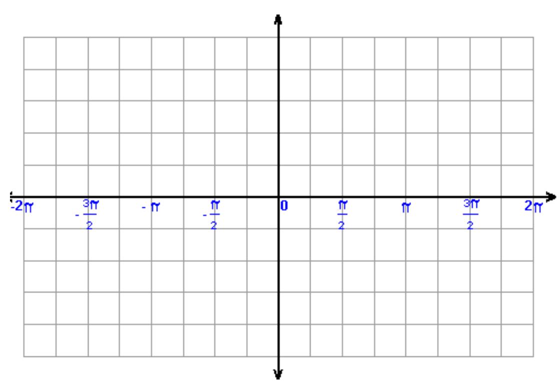
46. $f(x) = e^x$



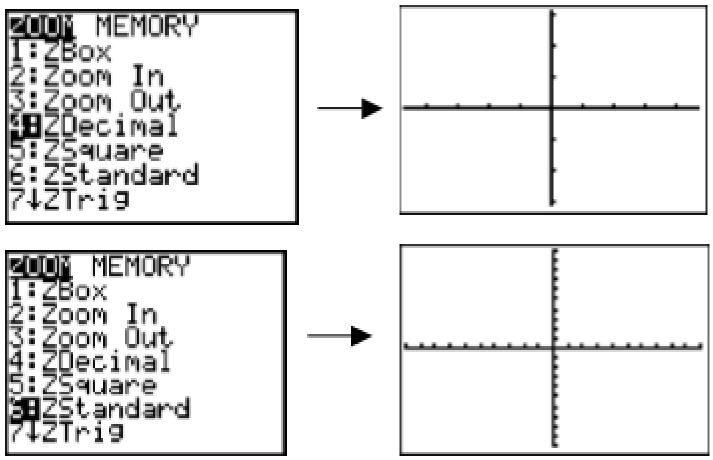
47. $f(x) = lnx$



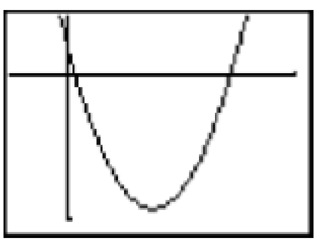
48. $f(x) = cosx$



Graphing Calculator Skill #1: You should be able to graph a function in a viewing window that shows the important features. You should be familiar with the built-in zoom options for setting the window such as zoom-decimal and zoom-standard. You should also be able to set the window conditions to values you choose.



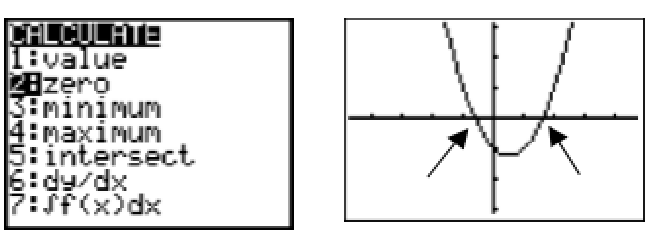
49. Find the appropriate viewing window to see the intercepts and the vertex defined by $y = x^2 - 11x + 10$. Use the window editor to enter the x and y-values.



Window: Xmin = _______
Xmax = _______
Xscl = _______
Ymin = _______
Ymax = _______
Yscl = _______

Graphing Calculator Skill #2: You should be able to graph a function in a viewing window that shows the x-intercepts (also called roots and zeros). You should be able to accurately estimate the x-intercepts to 3 decimal places. Use the built-in zero command on your graphing calculator.

50. Find the x-intercepts of $y = x^2 - x - 1$. $Window$ $[-4.7,4,7] \times [-3.1,3.1]$



(Write intercepts as points)

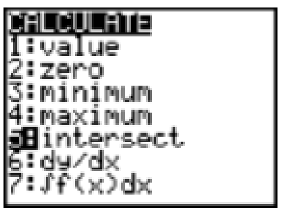
x-intercepts: _______________

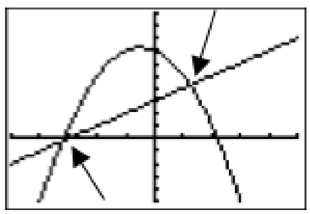
51. Find the x-intercepts of $y = x^3 - 2x - 1$ ____________________________

Graphing Calculator Skill #3: You should be able to graph two functions in a viewing window that shows the intersection points. Sometimes it is impossible to see all points of intersection in the same viewing window. You should be able accurately estimate the coordinates of the intersection points to 3 decimal places. Use the built-in intersection command.

52. Find the coordinates of the intersection points for the functions:

$$f(x) = x + 3 \;\; and \;\; g(x) = -x^2 - x + 7$$



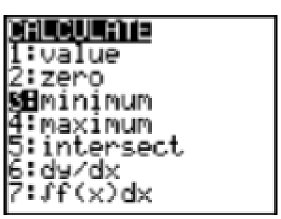


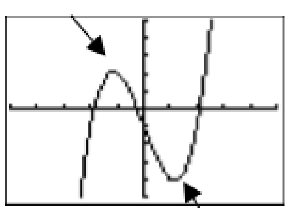
Intersection points: _______________

53. Find the coordinates of the intersection points of $f(x) = 4x^2 \; and \; g(x) = 2^x$

Graphing Calculator Skill #4: You should be able to graph a function and estimate the local maximum and minimum values to 3 decimals. Use the built-in max/min command.

54. Find the maximum and minimum values of the function $y = x^3 - 4x - 1$





(Value means the y-value)

Minimum value: __________

Maximum value: __________

55. Find the maximum and minimum values of the function $y = x^3 - 4x^2 + 4x$

56. Find the x-intercept(s), y-intercept, relative maximum, and relative minimum of

$y = x^3 + 2x^2 - 1$

57. Find the coordinates for the points of intersection points for $f(x) = 2x^2 + x - 9$ and $g(x) = -\frac{3}{4}x + 3$

Section 4: Linear Equations

58. Write the equation for the line in both forms given a slope and a point:

a) $m = \frac{2}{3}$ $and\ P(3{,}5)$

Point-Slope:

Slope-Intercept:

b) $m = -\frac{4}{5}$ $and\ P(1, 2)$

Point-Slope:

Slope-Intercept:

59. Write the equation of the passing through the given points:

a) P(2, 2) and Q(4, 2)

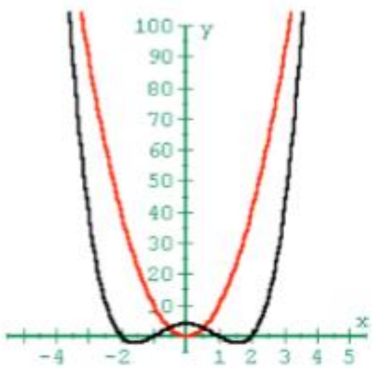
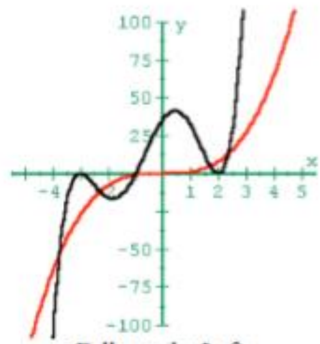
b) P(3, -2) and Q(3, 7)

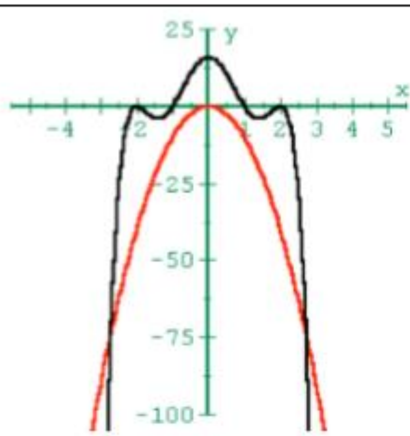
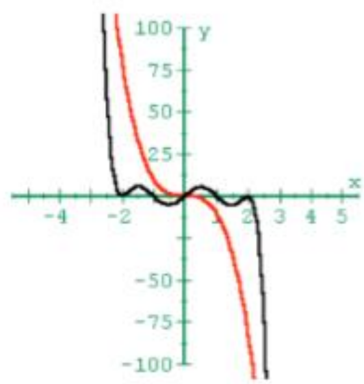
60. The slope of a line is $-\frac{1}{2}$ and the line passes through the points $(2, 5)\ and\ (-4, y)$. Find y.

Section 5: Polynomial Functions

Polynomials : $f(x) = a_n x^n + \ldots + a_2 x^2 + a_1 x + a_0$

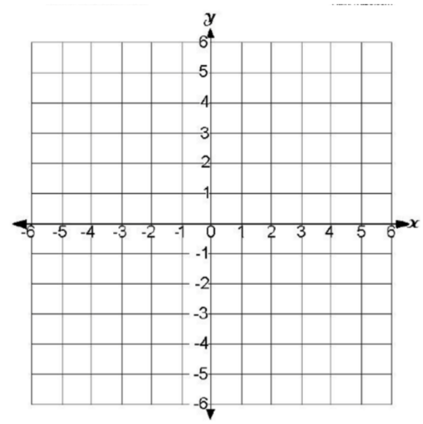
Studying End Behavior

Even Degree	**Odd Degree**
 Rises on the Left Rises on the Right $f(x) = x^4 - 5x^2 + 4$ $a_n > 0$	 Falls on the Left Rises on the Right $f(x) = x^5 + 3x^4 - 9x^3 - 23x^2 + 24x + 36$ $a_n > 0$

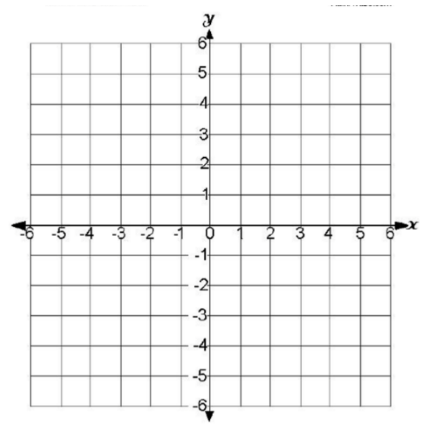
 Falls on the Left **Falls on the Right** $f(x) = -x^6 + 9x^4 - 24x^2 + 16$ $a_n < 0$	 Rises on the Left **Falls on the Right** $f(x) = -x^7 + 9x^5 - 24x^3 + 16x$ $a_n < 0$

61. Sketch a graph of the function without using a calculator. Identify the y-intercept (although it will probably not have drawn to scale on the given grid).

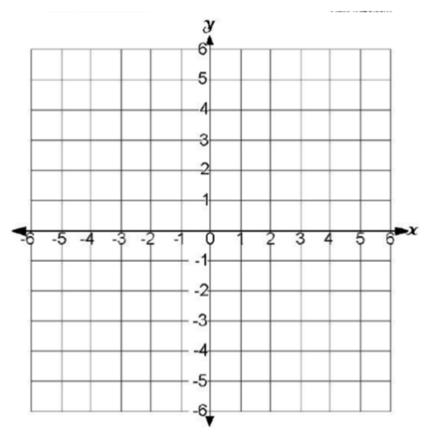
a) $f(x) = -(2x+7)^3(x-1)$



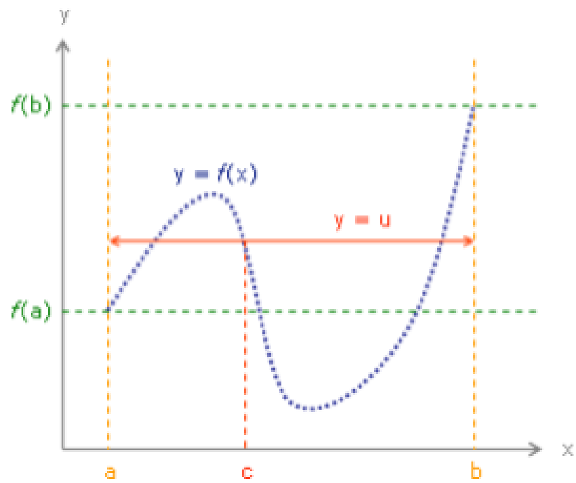
b) $g(x) = x^3(x+4)^2(2x-5)$



c) $f(x) = -x^2(x-5)^2(x+3)$



Section 6: The Intermediate value theorem (IVT) states the following: If the function *y = f(x)* is continuous on the interval [a, b], and *u* is a number between *f(a) and f(b),* then there is a $c \in [a, b]$ such that *f(c) = u.*



Example: Suppose we want to know if $f(x) = x^4 - 7x^3 - 4x + 8$ is ever zero.

Solution: Since this function is a polynomial, we know that it is continuous everywhere. At x = -1, we get *f(-1)=20*. At x = 1, we get *f(1) = -2*. So at the two endpoints of the interval

[-1, 1], the functions has values of 20 and -2. Therefore, *f(x)* must take on all values between -2 and 20 as x varies between - 1 and 1. In particular, *f(x)* must take on the value 0 for some x in [-1, 1]. The Intermediate Value Theorem (IVT) does not tell us exactly **where** *f(x)* equals 0, only that it **is** 0 somewhere on the interval [-1, 1].

62. Show that $p(x) = 2x^3 - 5x^2 - 10x + 5$ has a root somewhere between – 1 and 2.

63. Use the Intermediate Value Theorem to prove that the equation $x^3 = x + 8$ has at least one solution.

AP Multiple Choice Questions on IVT:

a) Let f be a continuous function on the closed interval $[-3,6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that:

a) $f(0) = 0$

b) $-1 \leq f(x) \leq 3$ for all x between -3 and 6.

c) $f(c) = 1$ for at least one c between -3 and 6.

d) f(c) = 0 for at least once c between -1 and 3.

b)

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval [0, 2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0, 2] if k =

a. 0

b. $\frac{1}{2}$

c. 1

d. 2

e. 3

c)

x	0	2	5	9	11
$g(x)$	1	2.8	1.7	1	3.4

The table above shows selcted values of a continuous function g. For $0 \leq x \leq 11$, what is the fewest possible number of times $g(x) = 2$?

(A) One (B) Two (C) Three (D) Four

Section 7: Average Rate of Change

Definition: Average Speed
Average speed is found by dividing the distance covered by the elapsed time.

$$\frac{\Delta y}{\Delta t} = \frac{\text{total distance traveled}}{\text{time elapsed}} = \frac{\text{final position} - \text{initial position}}{\text{final time} - \text{initial time}}$$

64. Find the average speed of a car that has traveled 350 miles in 7 hours.

65. Suppose $f(1)=2$ and the average rate of change of f between 1 and 5 is 3. Find $f(5)$.

66. The position $p(t)$, in meters, of an object at time t, in seconds, along a line is given by $p(t) = 3t^2 + 1$.

a) Find the change in position between times t = 1 and t = 3.

b) Find the average velocity of the object between times t = 1 and t = 4.

c) Find the average velocity of the object between any time t and another time $t + \Delta t$.

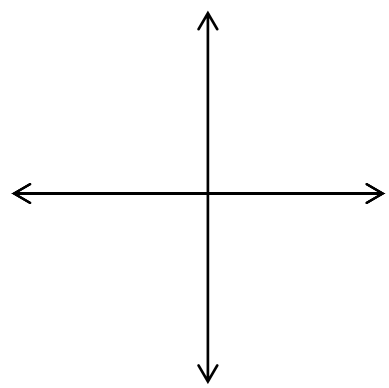
Section 8: Parametric Functions

Parametric equations are given below.

67. Complete the table and sketch the curve represented by the parametric equations (label the initial and terminal points as well as indicate the direction of the curve). Then eliminate the parameter and write the corresponding rectangular equation whose graph represents the curve. Be sure to define the portion of the graph of the rectangular equation traced by the parametric equations.

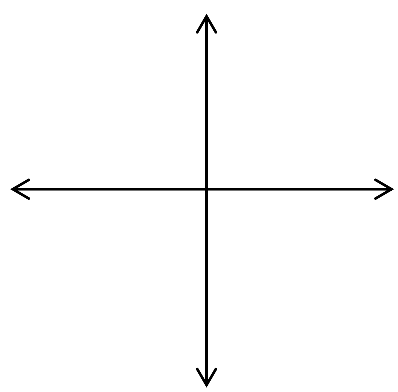
a). $x = 4\sin t, \quad y = 2\cos t, \quad 0 \le t \le 2\pi$

t	***0***	$\pi/4$	$\pi/2$	$3\pi/4$	π	$3\pi/2$	2π
x							
y							

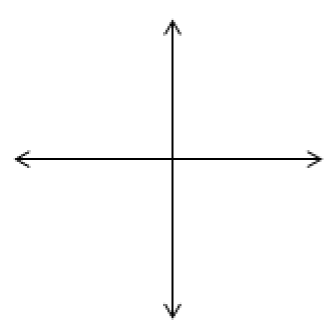


b) $x = 2t - 5, \quad y = 4t - 7, \quad -2 \le t \le 3$

t	***– 2***	***– 1***	***0***	***1***	***2***	***3***
x						
y						



c) $x = t^2, y = \sqrt{4 - t^2}$



Section 9: Inverse Functions

68. Algebraically find the inverse of $y = \frac{3}{x-2} - 1$

69. If $f(x) = x^3 - 1,$ find f^{-1} and verify that $f\left(f^{-1}(x)\right) = f^{-1}\left(f(x)\right) = x$

70. Discuss the relationship between the domain and range of a function and its inverse.

71. Given the one-to-one function *f*. The point *(a,c)* is on the graph of f. Give the coordinates of a point on the graph of f^{-1}.

Section 10: Partial Fraction Decomposition

Find the partial fraction decomposition of:

72. $\frac{5x+11}{3x^2-5x-2}$

73. $\frac{x^2+2}{(x-1)(x+2)(x-3)}$

74. $\frac{x^4+2x+7}{x^2+3x+2}$ *(hint: long division first…)*

Section 11: Polar Equations (you should do all of these questions without a calculator, although you can/should check your polar graph with a calculator when you are finished).

75. Plot the polar point $\left(3, -\frac{3\pi}{4}\right)$ and find three additional representations of this point on the interval $-2\pi < \theta < 2\pi$

76. Convert the given points from polar to rectangular form.

a) $\left(2, \frac{2\pi}{3}\right)$ b) $\left(-3, -\frac{3\pi}{4}\right)$ c) $\left(-2, \frac{5\pi}{6}\right)$ d) $\left(-3, -\frac{\pi}{2}\right)$

77. Convert the given points from rectangular to polar form. Give two ways to write each point in polar form.

a) $(0, 2)$ b) $\left(-1, \sqrt{3}\right)$ c) $\left(-\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ d) $\left(\sqrt{3}, -1\right)$

78. Convert the following polar equations into rectangular form. Describe the graphs.

a) r = 2 b) $\theta = \frac{\pi}{3}$ c) $r = sec\theta$ d) $r = 3cos\theta + 2sin\theta$

79. Convert the following rectangular equations into polar form.

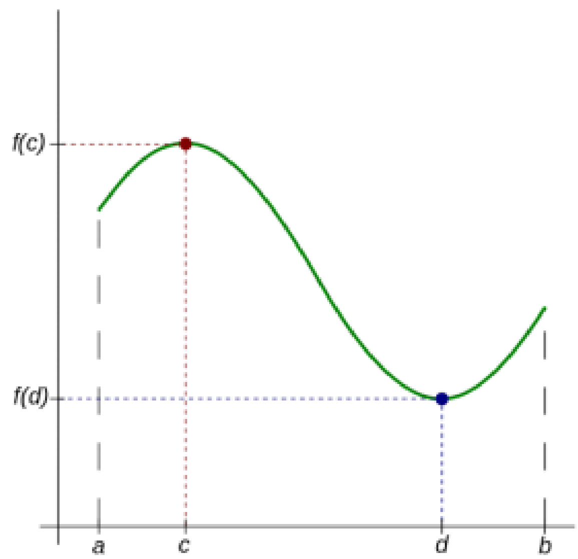
a) $y = x$ b) $x = 10$ c) $x^2 + y^2 = 4$ d) $x^2 - y^2 = 4x$

80. Sketch the graph of the following polar equation: $r = 3 + 2cos\theta$

Section 12: Extreme Value Theorem

The Extreme Value Theorem (EVT)

- ***Formal Statement:*** If a function f is continuous on a closed interval $[a, b]$, then:
    1. There exists a number c in $[a, b]$ such that $f(x) \leq f(c)$ for all x in $[a, b]$.
    2. There exists a number d in $[a, b]$ such that $f(x) \geq f(d)$ for all x in $[a, b]$.
- ***Translation:*** If a function f is continuous on a closed interval $[a, b]$, then f takes on a maximum and a minimum value on that interval.
- ***Picture:***



- ***Special Notes:***
    - A function may attain its maximum and minimum value more than once. For example, the maximum value of $y = \sin(x)$ is 1 and it reaches this value many, many times.
    - The extreme values often occur at the endpoint of the domain. That's why it's so important to check the endpoints of an interval when doing a maximization/minimization problem!
    - For a constant function, the maximum and minimum values are equal (in fact, all the values are equal).

In "easy language", the EVT says that if a function is continuous on an interval that you are looking at (no breaks/holes/gaps/asymptotes), then it must have an absolute maximum and minimum value on that interval.

81 – 82: Use a graphing calculator to find the maximum and minimum values of each of the functions.

81. a) $f(x) = x^3 - 3x^2 - 9x + 4$ on the interval $[-4, 2]$

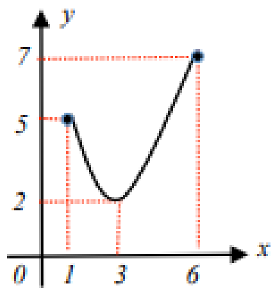
b) Consider the open interval (-4, 2). Would the result change?

82. a) $f(x) = x^2 + \frac{2}{x}$ on the interval [-1, 2].

b) Consider the closed interal [0.5, 2]. Would the result change?

83. Find the minimum and maximum of the following functions on the specific intervals.

a)

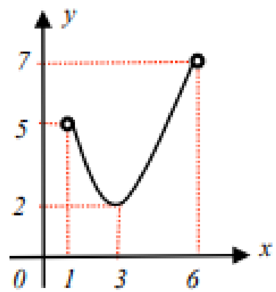


interval: [1, 6]

minimum = __________

maximum = _________

b)

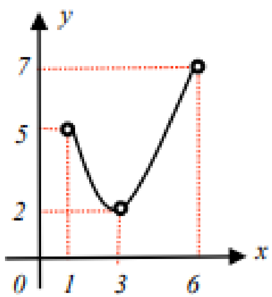


interval: (1, 6)

minimum = __________

maximum = _________

c)

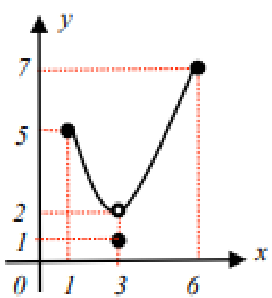


interval: (1, 6)

minimum = __________

maximum = _________

d)



interval: [1, 6]

minimum = __________

maximum = _________

Section 13: Limits and Continuity

84. Find the limits (algebraically), if they exist.

a) $\lim_{x\to 4} \frac{2x^3 - 7x^2 - 4x}{x - 4}$

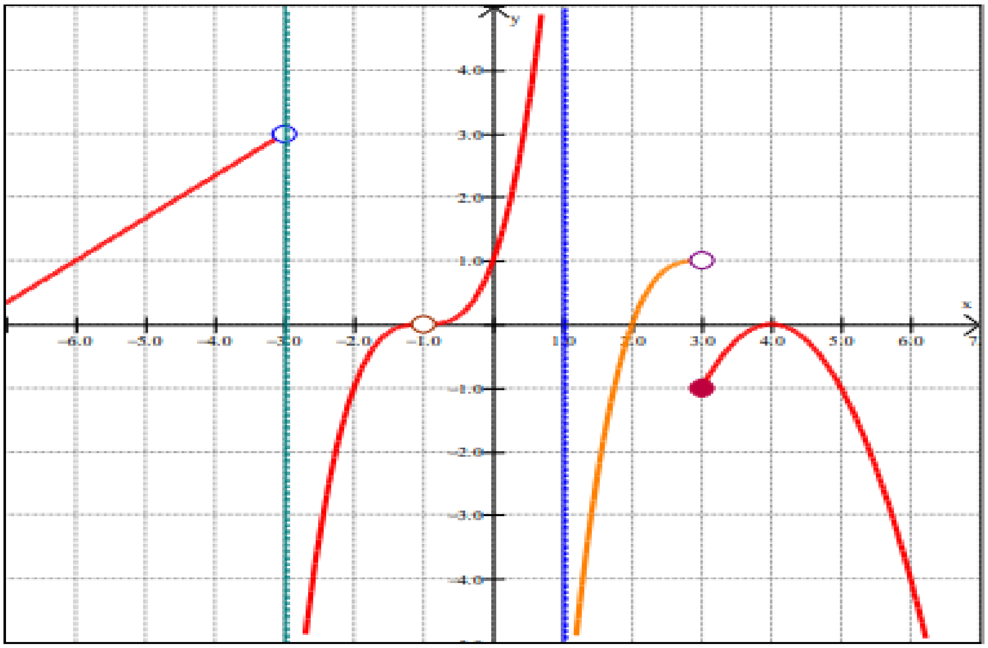
b) $\lim_{x\to 9} \frac{\sqrt{x} - 3}{9 - x}$

c) $\lim_{x\to 1} \frac{x^2 - 2x - 5}{x + 1}$

d) $\lim_{x\to \infty} \frac{2x - 3x^3 + 4x^2 - 1}{2x^3 - 3x + 7}$

85. Consider the function $f(x) = \begin{cases} x^2 + kx & x \leq 5 \\ 5\sin\left(\frac{\pi}{2}x\right) & x > 5 \end{cases}$ In order for the function to be continuous at x = 5, what must the value of k be?

86. Determine if the following limits exist based on the graph below of $p(x)$. If the limit exists, state the value. Note that x = -3 and and x =1 are vertical asymptotes.



a) $\lim_{x \to 1^+} p(x)$

b) $\lim_{x \to 1^-} p(x)$

c) $\lim_{x \to 1} p(x)$

d) $\lim_{x \to 3^-} p(x)$

e) $\lim_{x \to 3^+} p(x)$

f) $\lim_{x \to 3} p(x)$

g) $\lim_{x \to -1^+} p(x)$

h) $\lim_{x \to -1^-} p(x)$

i) $\lim_{x \to -1} p(x)$

j) $\lim_{x \to 2^+} p(x)$

k) $\lim_{x \to 2^-} p(x)$

l) $\lim_{x \to 2} p(x)$

87. Referring back to question 91,is $p(x)$ continuous at x = 1? x = 2? x = 3? x = -1? Explain why or why not for each x value.

88. Write the formulas for the area of a right triangle, an equilateral triangle, a circle, a semi-circle, and a trapezoid. Write the formulas for the volume of a right circular cylinder and a right circular cone. Draw a corresponding picture for each figure.

Section 14: Derivatives Practice

89. Find dy/dx for the following:

a) $y = \frac{1}{x^3} - 2x^4 + \sqrt{x}$

b) $y = 4\sqrt{x} - \frac{3}{\sqrt{x}} + 2x - 5$

c) $y = 2x^2 \sec x$

d) $y = 4xtan^2(3x^4)$

e) $f(x) = \frac{2x^2}{\sin(3x)}$

f) $y = \ln(5x^2 - 3)$

g) $y = 6^{\cot(2x)}$

h) $y = 3e^{2x}$

i) $y = \log_5 \cos(4x)$

90. Use implicit differentiation to find dy/dx: $2xy^2 + 5y - 4 = 8$

91. Find the equation of the line tangent and normal to the graph of $3xy^3 - 4x^2 + 2y = -25$ at the point (3, 1)

92. The lines tangent to the graphs of $g(x)$ and $h(x)$ are perpendicular to one another when x = 2.

If $g(x) = \ln(2x^3 + 5)$, find the slope for h(x) at x = 2.

93. Find the equation of the line tangent to $f(x) = 2x(3x^2 - 4)^2$ at x = 1.

94. Given $= \frac{5e^{3x}\sqrt{2x-5}}{\sin^2(2x)}$, fine dy/dx. Hint…do NOT use quotient rule. Take the log or natural log of both sides of equation. Expand the right hand side and then take the derivative!!!

95. Given $y = \mathrm{x}^{\tan x}$, find dy/dx. Again…take log or natural log of both sides first (to bring down exponent) and then take the derivative.

96. Find the derivative for the following by recognizing what function's derivative is being calculated and then use shortcut rules to find the answer.

a) $\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$

b) $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$

c) $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

d) $\lim_{x \to \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$

97. Given $f(x) = \sqrt{x+1}$.

a) Find $f'(3)$ and write the equation of the tangent line and normal line when x = 3. Provide a quick (but accurate) sketch of the curve and the tangent and normal lines.

b) Find $f'(3)$ using the limit definition of the derivative.

c) Find $f'(3)$ using the alternate definition of the derivative.