Brockton High School
Content: Math III
Week of: June 15 to June 24, 2020

Greetings Math III Students! We hope you are safe and well with your families!
This assignment is for the week, use your time wisely. You do not have to complete this in one sitting. Here is the lesson plan for this week:

Goals for this week
Learning Objectives: Students will be able to: • Convert angle measures from radians to degrees/radians to degrees. • Calculate values on a sine and cosine function. • Model real-world problems using trigonometric functions. • Use Pythagorean identifies to evaluate sine and cosine, • Solve equations and evaluate using trig functions • Use normal distributions to understand probabilities and margin of error *(Standards A.REI.1, A.SSE.2, F.IF.7, F.TF.1/2/3/4/7, S.ID.4,S.MD.6/7)*
Literacy Objectives: Students will be able to: 1. to create, interpret and explain a table, chart or graph 2. to compute, interpret and explain numbers 3. to interpret and present statistics that support an argument or hypothesis 4. to identify, explain and make a prediction based on a pattern *(https://www.bpsma.org/schools/brockton-high-school/about-us/mission-literacy-charts)*

Instructional Video Links: Please watch at least one of these videos to help guide you.

- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:trig/x2ec2f6f830c9fb89:radians/v/radian-and-degree-conversion-practice
- https://www.youtube.com/watch?v=V5ArB_GFGYQ
- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:trig/x2ec2f6f830c9fb89:unit-circle/v/unit-circle-definition-of-trig-functions-1
- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:trig/x2ec2f6f830c9fb89:pythagorean-id/v/pythagorean-trig-identity-from-unit-circle
- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:trig/x2ec2f6f830c9fb89:pythagorean-id/v/using-the-pythagorean-trig-identity
- https://www.youtube.com/watch?v=TpADRvW8zm8
- https://www.khanacademy.org/math/ap-statistics/density-curves-normal-distribution-ap/stats-normal-distributions/v/ck12-org-normal-distribution-problems-empirical-rule
- https://www.khanacademy.org/math/ap-statistics/density-curves-normal-distribution-ap/measuring-position/v/z-score-introduction

Your teacher also has a list of additional instructional videos. Feel free to also search for your own tutorial videos to guide you.

Key Terms

- **unit circle –** a circle whose radius is one unit of distance.
- **trigonometric function –** used to find angle and side measures of right triangles; includes sine, cosine, tangent, secant, cosecant, cotangent, Arcsine, Arccosine, Arctangent, Arcsecant, Arccosecant, Arccotangent.
- **radian –** a unit that is used to measure angles. One radian is equal to $\frac{180}{\Pi}$ degrees or approximately 57.30 degrees.
- **maximum/minimum point-** of a graph of a function is the ordered pair on the graph with the greatest/least y-coordinate.
- **sine of an angle –** in a right triangle, the sine of an angle is equal to the ratio of the length of the side opposite the angle to the length of the hypotenuse.
- **cosine of an angle -** in a right triangle, the cosine of an angle is equal to the ratio of the length of the side adjacent the angle to the length of the hypotenuse.
- **arcsine function –** the inverse of the sine function.
- **arccosine function –** the inverse of the cosine function.
- **tangent of an angle -** in a right triangle, the tangent of an angle is equal to the ratio of the length of the side opposite the angle to the length of the side adjacent to the angle
- **arctangent function –** the inverse of the tangent function.
- **standard deviation –** a measure of the variation of the values in a data set from the mean of the data.

Practice Activities:
On-Line:
All students now have access to an on-line program called **Mathia (Carnegie Learning)**

- If you are **already in Mathia**, please continue to work in the program. The suggested pacing is to complete Module 6 by the end of the week.
- If you are **new to Mathia**: Please see the log-in information below and begin.

Practice Activities:
Printable resources: Optional resources if technology is not available

Math 3 Printable Resources: What Goes Around pages M4-49 through M4-62

Extension Activities: Complete the STRECH activity om page M4-63 and return to your teacher

Log-in Information

1. Log-in to Clever(if you need to sign in follow number 2 then 3; if you automatically log in go to 3)
2. If you need to sign in to Clever: Username: 6-digit BHS school ID # @bpsma.org
   Password: Date of birth bps 1920
   Example: Student with ID #:123456
   Date of birth: January 1st, 2000
   Username: 123456@bpsma.org
   Password: 01012000bps1920
3. Under Math (in the left- hand column) click on Carnegie Learning (MATHia)
4. Pick up where you left off in Mathia

Additional Support

- Please email your math teacher with specific questions.
- Here is a list of math teachers' office hours:
- https://brocktonpublicschools-my.sharepoint.com/:x:/g/personal/danielcorbett_bpsma_org/EWk_ij9UwjpPtRAHBUkEpS4B3vue-_IG8VYz0AwG9ovJjQ?e=4%3arkcL1r&at=9&CT=1588698277992&OR=OWA-NT&CID=8d0078f2-9a27-460f-e632-64578875ee60

4

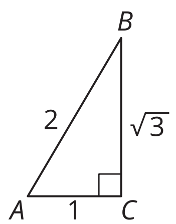
What Goes Around

The Sine and Cosine Functions

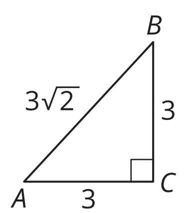
Warm Up

Determine the sine ratio and cosine ratio of $\angle A$ in each triangle.

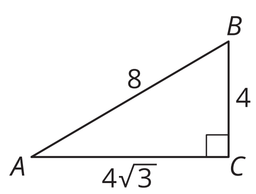
1.



2.



3.



Learning Goals

- Define the sine and cosine functions.
- Calculate values for the sine and cosine of reference angles.
- Define the sine and cosine of an angle as a coordinate of a point on the unit circle.
- Graph and compare the sine and cosine functions.

Key Terms

- sine function
- cosine function
- trigonometric function
- periodicity identity

You have previously explored the relationship of the side lengths in special right triangles, and you know how to determine the sine and cosine ratios of angles in a right triangle. How can these relationships on a unit circle be represented as functions on a coordinate plane?

GETTING STARTED

The Right Triangle Connection

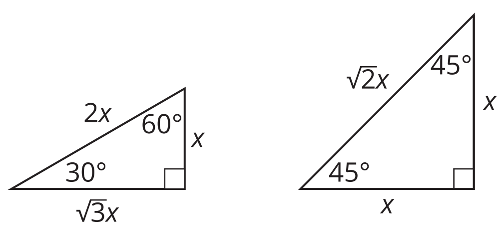
Recall that the sine ratio (sin), given a reference angle, θ, is the ratio of the length of the opposite side to the length of the hypotenuse in a right triangle.

$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

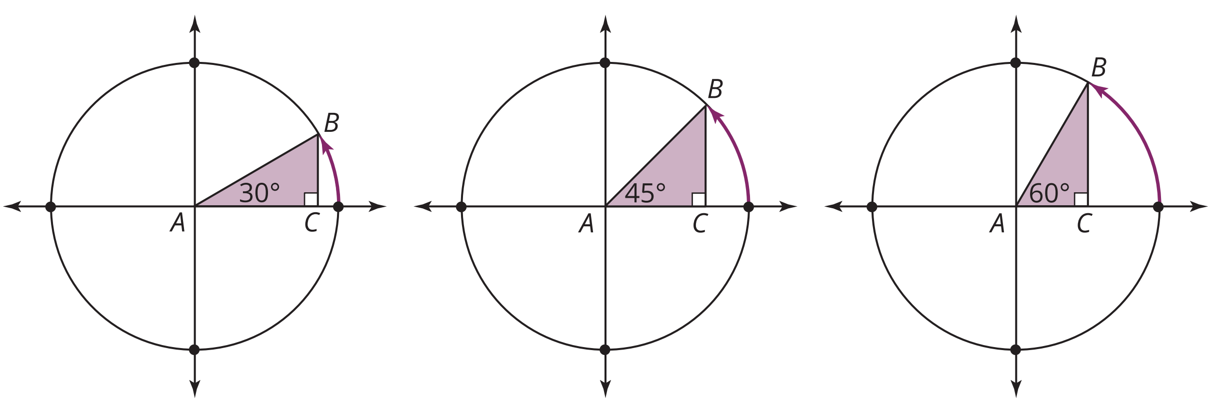
The cosine ratio (cos), given a reference angle, θ, is the ratio of the length of the adjacent side to the length of the hypotenuse.

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

The side-length relationships for a 30° -60° -90° triangle and a 45° -45° -90° triangle are shown.



The diagram shows a right triangle *ABC* placed on a unit circle centered at the origin. The central angle measures $\theta = 30°$, $\theta = 45°$, and $\theta = 60°$ are shown.



1. What is the length of the hypotenuse *c* in each circle? Label the measures on each triangle.

2. **Label the side lengths of the triangles in each diagram in radical form.**

3. **The hypotenuse of each right triangle represents the terminal ray of a central angle that intersects the unit circle at point *B*.**

   a. **Complete the table to record the sine and cosine of each angle measure, θ, and the coordinates of the point where the terminal ray intersects the unit circle. Explain your reasoning.**

θ	cos θ	sin θ	Coordinates of Point *B*, (Intersection of Terminal Ray and Unit Circle)
30°			
45°			
60°			

   b. **Write the coordinates of the intersection of the terminal ray and the unit circle at 0°.**

   c. **Write the coordinates of the intersection of the terminal ray and the unit circle at 90°.**

4. **Jorge conjectured that the coordinates of the point where the terminal ray of a central angle θ intersects the unit circle can always be written as (cos θ, sin θ). Do you think Jorge's conjecture is correct? Explain your reasoning.**

ACTIVITY

4.1 Unit Circle Coordinates in Quadrant I

In the unit circle, you will label the first quadrant now. Then you will be able to determine the coordinates of the points in the other quadrants in the next activity.

Use the unit circle located at the end of this lesson and your answers to the questions in the Getting Started to complete this activity.

1. Determine the coordinates of the points in the first quadrant on the unit circle. Label the coordinates.

2. Use the unit circle to evaluate each measure.

$\sin(\frac{\pi}{6}\text{ radian}) =$ ______________________

$\cos(\frac{\pi}{6}\text{ radian}) =$ ______________________

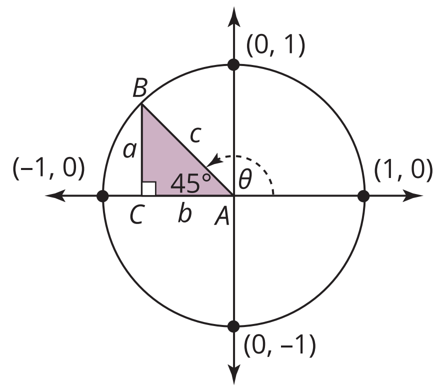
$\sin(\frac{\pi}{4}\text{ radian}) =$ ______________________

$\cos(\frac{\pi}{4}\text{ radian}) =$ ______________________

3. For each angle measure in Question 2, evaluate the sine and cosine of the complement. Explain your reasoning.

ACTIVITY 4.2 Unit Circle Coordinates Beyond Quadrant I

Now that you have identified values of sine and cosine in the first quadrant, how can you use that knowledge to identify values in other quadrants?



1. The diagram shows a 45° central angle positioned in the second quadrant on the unit circle.

a. State the measure of *θ* in degrees and in radians. Explain how you determined your answer.

b. Identify the coordinates of the point at which the terminal ray of the angle intercepts the circle. Explain how you determined your answer.

Remember:

When creating a triangle using a terminal ray, the right triangle drawn must always be bound to the *x*-axis.

c. What do you notice about the coordinates of this point and the coordinates of the symmetrical point in the first quadrant?

2. Use what you know about symmetry to label the coordinates of the remaining points on the unit circle located at the end of the lesson.

Think about:

Does the cosine have a negative or a positive value? Does the sine have a negative or positive value?

3. Look back at Jorge's conjecture in the Getting Started. Is his conjecture correct? Explain your reasoning.

4. **Describe when the values of cosine and sine are positive and negative in the unit circle. Label this information on the unit circle at the end of the lesson.**

5. **Ray makes this conclusion.**

> Ray
> Many different central angle measures have the same sine or cosine values.

Provide examples to support Ray's conclusion.

ACTIVITY

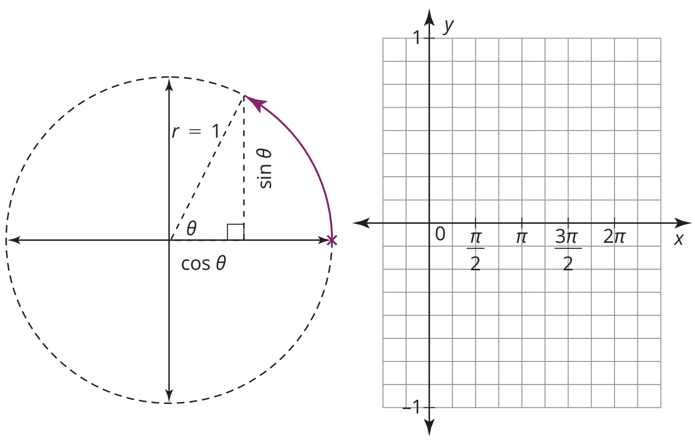
4.3 Sine and Cosine Functions

Let's consider how to represent the values from your unit circle as functions on a coordinate plane.

1. Use your completed unit circle to graph the function $y = \sin x$.

a. As the terminal ray traverses the unit circle counterclockwise in standard position, plot the output value, $\sin \theta$, that corresponds to the input value, θ, which is the radian measure of the central angle, from 0 to 2π radians.

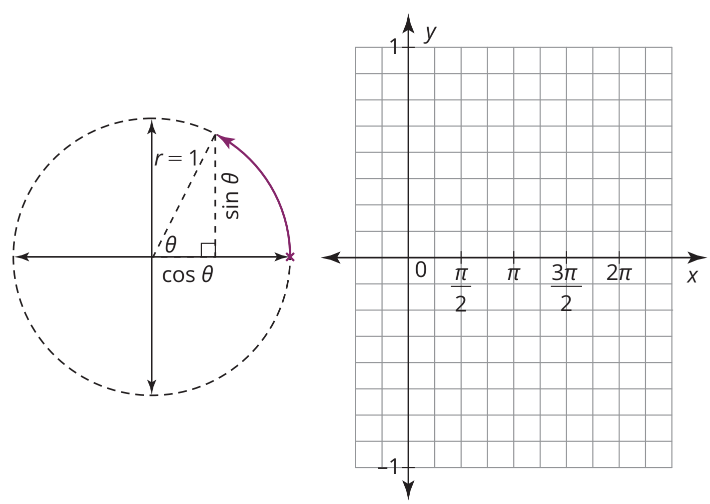
Plot the output values from left to right on the graph as you move counterclockwise around the unit circle.



b. What coordinate values on the unit circle did you use to create the graph of $y = \sin x$?

2. Use your completed unit circle to graph the function $y = \cos x$.

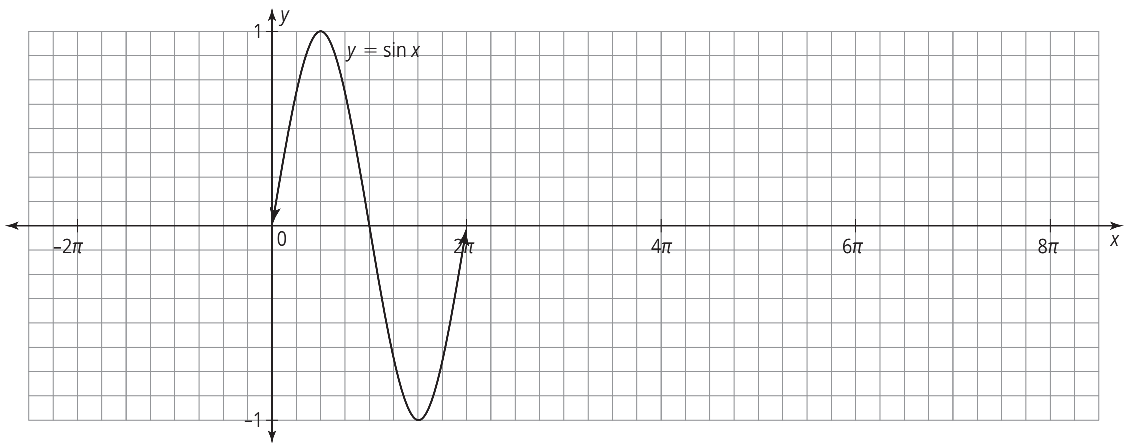
a. As the terminal ray traverses the unit circle counterclockwise in standard position, plot the output value, $\cos \theta$, that corresponds to the input value, θ, which is the radian measure of the central angle, from 0 to 2π radians.

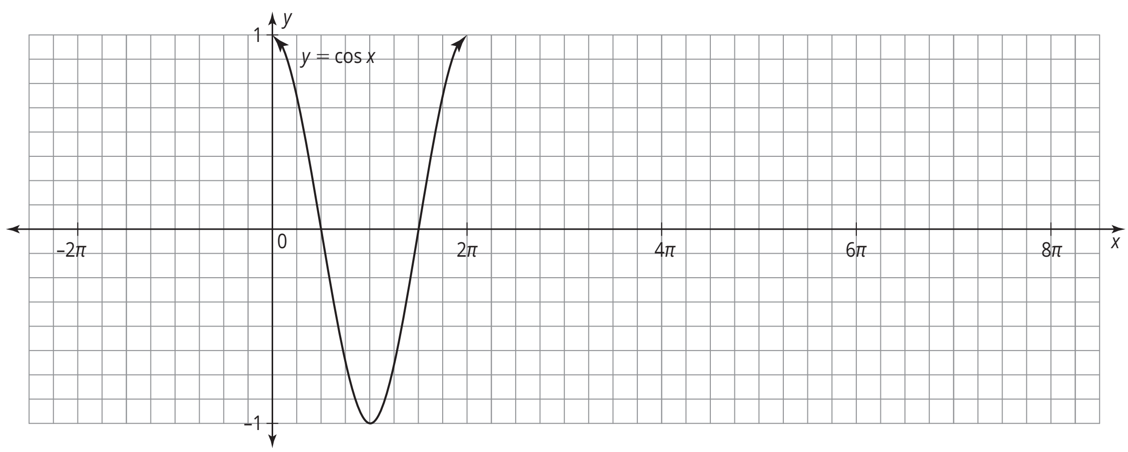


b. What coordinate values on the unit circle did you use to create the graph of $y = \cos x$?

You have graphed the *sine function* and *cosine function*. The **sine function** and **cosine function** are periodic **trigonometric functions**. Each of these trigonometric functions takes angle measures (θ values) as inputs and outputs real number values, which correspond to coordinates of points on the unit circle.

3. Consider the functions $y = \sin x$ and $y = \cos x$.





a. Extend the graphs of the functions $y = \sin x$ and $y = \cos x$ over the domain $0 \le x \le 8\pi$.

b. **Determine the values of sin x and cos x at 4π, 6π, and 8π radians.**

c. **Describe how you can determine each value from part (b) on the unit circle for each function.**

4. **Now consider a domain of $-2\pi \leq x \leq 8\pi$ for the functions $y = \sin x$ and $y = \cos x$.**

a. **Extend the graphs of the functions $y = \sin x$ and $y = \cos x$ in Question 3 through $x = -2\pi$.**

b. **Determine each sine value.**

$\sin(-\frac{\pi}{2}) =$ ______________________

$\sin(-\pi) =$ ______________________

$\sin(-\frac{3\pi}{2}) =$ ______________________

$\sin(-2\pi) =$ ______________________

c. **Determine each cosine value.**

$\cos(-\frac{\pi}{2}) =$ ______________________

$\cos(-\pi) =$ ______________________

$\cos(-\frac{3\pi}{2}) =$ ______________________

$\cos(-2\pi) =$ ______________________

Ask yourself:

Can the sine and cosine functions output any real number, given any angle measure input?

5. Consider the values of sin($x + 2\pi$). How do these values compare to the values of sin x?

6. Consider the values of cos($x + 2\pi$). How do these values compare to the values of cos x?

The period of the sine function is 2π radians, and the period of the cosine function is 2π radians. Thus, you can write two *periodicity identities*:

- $\sin(x + 2\pi) = \sin x$
- $\cos(x + 2\pi) = \cos x$

Each of these is called a **periodicity identity** because they are each based on the period of the function, 2π.

NOTES

TALK the TALK

Comes Around

1. Complete the table.

Angle Measure (θ)		$\cos\theta$	$\sin\theta$
radians	degrees		
0	$0°$	1	0
$\frac{\pi}{6}$	$30°$		
$\frac{\pi}{4}$	$45°$		
$\frac{\pi}{3}$	$60°$		
$\frac{\pi}{2}$	$90°$		
$\frac{2\pi}{3}$	$120°$		
$\frac{3\pi}{4}$	$135°$		
$\frac{5\pi}{6}$	$150°$		
π	$180°$		

Angle Measure (θ)		$\cos\theta$	$\sin\theta$
radians	degrees		
$\frac{7\pi}{6}$	$210°$		
$\frac{5\pi}{4}$	$225°$		
$\frac{4\pi}{3}$	$240°$		
$\frac{3\pi}{2}$	$270°$		
$\frac{5\pi}{3}$	$300°$		
$\frac{7\pi}{4}$	$315°$		
$\frac{11\pi}{6}$	$330°$		
2π	$360°$		

2. Compare and contrast the functions $y = \sin x$ and $y = \cos x$. Describe the similarities and differences between the two functions.

NOTES

3. **Identify each of the characteristics for $y = \sin x$ and $y = \cos x$.**

	$y = \sin x$	$y = \cos x$
y-intercept(s)		
Domain		
Range		
Period		
Minimum Output Value		
Maximum Output Value		
Amplitude		
Midline		

4. **Describe the intervals of increase and decrease for both the sine and cosine functions. Explain your reasoning.**

5. **Identify the x-intercepts for each function.**

   a. **x-intercepts for $y = \sin x$**

   b. **x-intercepts for $y = \cos x$**

6. **Use the language of transformations to explain how the sine and cosine functions are related.**

Ask yourself:

How can you write this relationship mathematically?

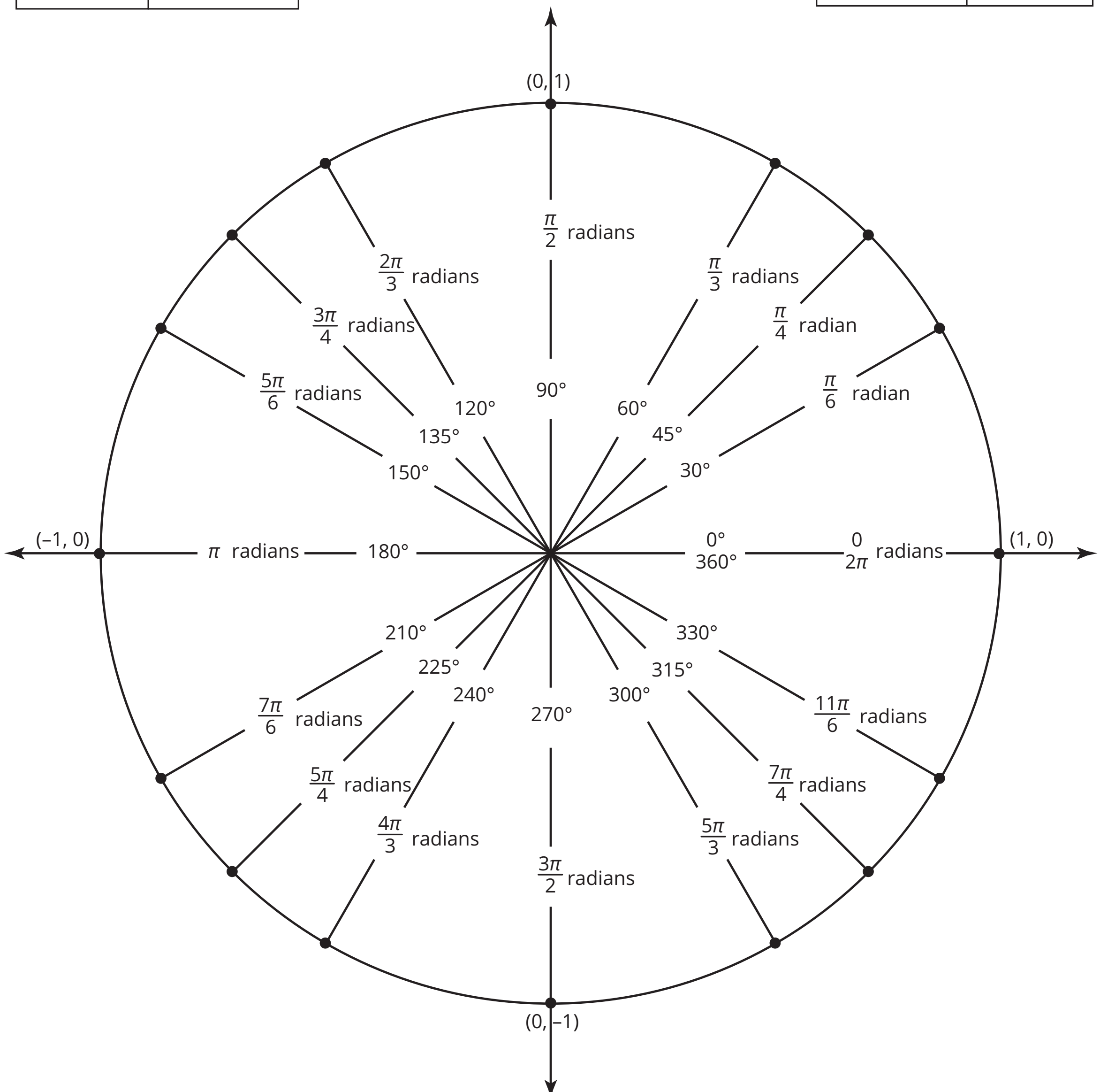
Sine and Cosine on the Unit Circle

Quadrant II

sin θ	
cos θ	

Quadrant I

sin θ	
cos θ	



Quadrant III

sin θ	
cos θ	

Quadrant IV

sin θ	
cos θ	

Assignment

Write

Write a definition for each term in your own words.

1. sine function
2. cosine function
3. trigonometric function
4. periodicity identity

Remember

The cosine of the central angle measure of a unit circle is the x-coordinate of the point where the terminal ray intersects the unit circle and the sine of the same central angle measure is the y-coordinate of the same point.

The sine function, $y = \sin x$, and cosine function, $y = \cos x$, are periodic trigonometric functions that take angle measures (θ values) as inputs and outputs real number values, which correspond to coordinates of points on the unit circle. The period of each function is 2π radians, therefore $\sin(x + 2\pi) = \sin x$ and $\cos(x + 2\pi) = \cos x$.

Practice

1. Determine θ and $\cos\theta$ when $\sin\theta = \frac{\sqrt{3}}{2}$ and $\cos\theta$ is negative. Restrict values for θ such that $0 \le \theta \le 2\pi$.
2. Determine θ and $\sin\theta$ when $\cos\theta = -\frac{\sqrt{2}}{2}$ and $\sin\theta$ is negative. Restrict values for θ such that $0 \le \theta \le 2\pi$.
3. Determine 3 values for θ such that $\sin\theta = -\frac{\sqrt{3}}{2}$.
4. Determine 3 values for θ such that $\cos\theta = \frac{\sqrt{2}}{2}$.
5. Determine 3 values for θ such that $\cos\theta = 0$.
6. Determine the value of each ratio.

   a. $\sin\left(\frac{15\pi}{4}\right)$

   b. $\cos\left(\frac{17\pi}{6}\right)$

   c. $\sin\left(\frac{25\pi}{6}\right)$

   d. $\cos\left(\frac{19\pi}{4}\right)$

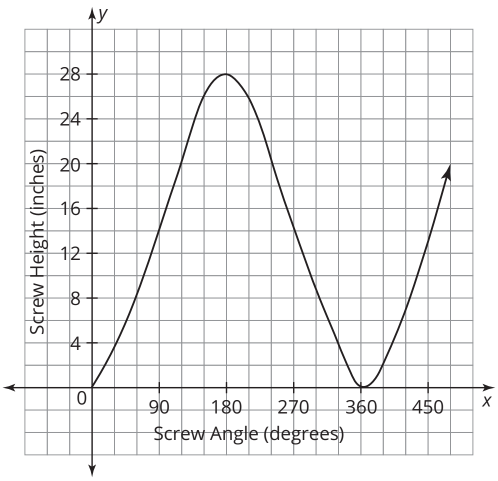
Stretch

1. Determine θ and $\sin\theta$ when $\cos\left(\frac{\theta}{2}\right) = -\frac{\sqrt{3}}{2}$. Restrict values for θ such that $0 \le \theta \le 2\pi$.
2. Complete the table of values for the functions shown.

		Function			
		$\sin\theta$	$2\sin\theta$	$\sin\theta + 1$	$\sin(\theta + \pi)$
Angle Measure (θ)	0				
	$\frac{\pi}{6}$				
	$\frac{\pi}{4}$				
	$\frac{\pi}{3}$				
	$\frac{\pi}{2}$				

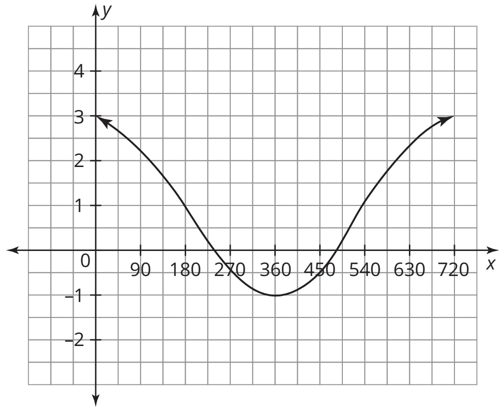
Review

1. Ahmed is riding his bike. The tires on the bike have a diameter of 28 inches. He runs over a screw, but is able to keep riding the bike. Assume the tire rotates clockwise and the screw is at an angle of 0° when it is at ground level. The graph shows the height of the screw above the ground as a function of the angle of the screw.



   a. Determine the amplitude of the function.
   b. Determine the period of the function.
   c. Determine the height of the screw when the screw angle is 630°.

2. Consider the periodic function shown, with x in degrees.



   a. Determine the amplitude of the function.
   b. Determine the period of the function.
   c. Determine the value of the function when $x = 900°$.

3. Write the equations of the two relations used to create this bird in terms of the function $f(x) = x^2$. Include any restrictions on the domains.