Brockton High School
Content: Grade 11 Pre-Calculus
Week of: June 15 to June 24, 2020

Here is the lesson plan for this week:

Goals for this week
Learning Objectives: Review Quadratic Functions Use different forms of quadratic functions to identify features of their graphs such as zeros, vertex, and maximum/minimum values Transform quadratic functions – shift, scale, and reflect – and observe effects on their equations **Common Core Math Standards: (MA prerequisite content standards)** F.IF.B.4, F.IF.B.6, F.IF.C.7a
Literacy Objectives: Students will be able to: 1. create, interpret and explain a table, chart, or graph 2. explain and make a prediction based on a pattern 3. use evidence to support one's thinking *(https://www.bpsma.org/schools/brockton-high-school/about-us/mission-literacy-charts)*

This assignment is for the week, use your time wisely. You do not have to complete this in one sitting.

Step 1: Log on to CLEVER, then use the link to access Khan Academy. See detailed login instructions below.

Once you have logged in to Khan Academy through CLEVER, copy and paste the link below on your browser to access this week's material. Select course: **Math II** (found under high school math by grade) Lesson: **Quadratic Functions and Equations**

OR

Copy and paste this link on your browser : https://www.khanacademy.org/math/high-school-math/math2/xe2ae2386aa2e13d6:quad-2

Complete all practice problems associated with each topic, then complete Quiz 5. Watch the accompanying videos for guidance within each topic if needed.

Topic 1: Features and Forms of Quadratic Functions

Topic 2: Transforming Quadratic Functions

Log-in Information

1. Log-in to Clever
   Username: 6-digit BHS school ID # @bpsma.org
   Password: Date of birth bps 1920
   Example: Student (Michael) with ID #:123456
   Date of birth: January 1st, 2000
   Username: 123456@bpsma.org
   Password: 01012000bps1920

   Once you have logged in to CLEVER, click on the Khan Academy icon to log in to your Khan Academy account. You will find this icon on your math teacher's page, or under Enrichment. If you do not have a Khan Academy account, you can create one here.

Additional Support
Email: • Please email your math teacher with specific questions.
Office Hours: • Here is a list of math teachers' office hours. Your teacher is available to help you during their scheduled office hours. BHS Math Department Office Hours

Printable Resources (Optional)

5-1 Using Transformations to Graph Quadratic Functions

Objectives

Transform quadratic functions.

Describe the effects of changes in the coefficients of $y = a(x - h)^2 + k$.

Vocabulary

quadratic function
parabola
vertex of a parabola
vertex form

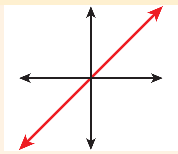
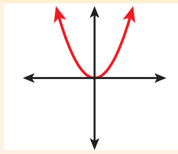
Why learn this?

You can use transformations of quadratic functions to analyze changes in braking distance. (See Example 5.)

In Chapters 2 and 3, you studied linear functions of the form $f(x) = mx + b$. A **quadratic function** is a function that can be written in the form $f(x) = a(x - h)^2 + k$ $(a \neq 0)$. In a quadratic function, the variable is always squared. The table shows the linear and quadratic parent functions.

Know it! Note

Linear and Quadratic Parent Functions

ALGEBRA	NUMBERS	GRAPH
Linear Parent Function $f(x) = x$	<table><tr><td>x</td><td>–2</td><td>–1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$f(x) = x$</td><td>–2</td><td>–1</td><td>0</td><td>1</td><td>2</td></tr></table>	
Quadratic Parent Function $f(x) = x^2$	<table><tr><td>x</td><td>–2</td><td>–1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$f(x) = x^2$</td><td>4</td><td>1</td><td>0</td><td>1</td><td>4</td></tr></table>	

Notice that the graph of the parent function $f(x) = x^2$ is a U-shaped curve called a **parabola**. As with other functions, you can graph a quadratic function by plotting points with coordinates that make the equation true.

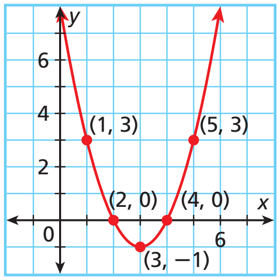
EXAMPLE 1

Graphing Quadratic Functions Using a Table

Graph $f(x) = x^2 - 6x + 8$ by using a table.

Make a table. Plot enough ordered pairs to see both sides of the curve.

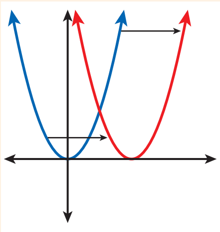
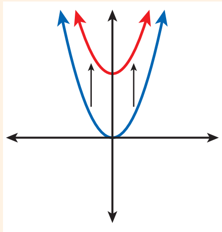
x	$f(x) = x^2 - 6x + 8$	$(x, f(x))$
1	$f(1) = 1^2 - 6(1) + 8 = 3$	$(1, 3)$
2	$f(2) = 2^2 - 6(2) + 8 = 0$	$(2, 0)$
3	$f(3) = 3^2 - 6(3) + 8 = -1$	$(3, -1)$
4	$f(4) = 4^2 - 6(4) + 8 = 0$	$(4, 0)$
5	$f(5) = 5^2 - 6(5) + 8 = 3$	$(5, 3)$



1. Graph $g(x) = -x^2 + 6x - 8$ by using a table.

You can also graph quadratic functions by applying transformations to the parent function $f(x) = x^2$. Transforming quadratic functions is similar to transforming linear functions (Lesson 2-6).

Translations of Quadratic Functions

Horizontal Translations	Vertical Translations
Horizontal Shift of $\lvert h \rvert$ Units	**Vertical Shift of $\lvert k \rvert$ Units**
 $f(x) = x^2$ $f(x - h) = (x - h)^2$ Moves left for $h < 0$ Moves right for $h > 0$	 $f(x) = x^2$ $f(x) + k = x^2 + k$ Moves down for $k < 0$ Moves up for $k > 0$

EXAMPLE 2 Translating Quadratic Functions

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

A $g(x) = (x + 3)^2 + 1$

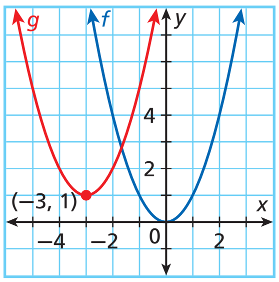
Identify h and k.

$$g(x) = \left(x - \underbrace{(-3)}_{h}\right)^2 + \underbrace{1}_{k}$$

Because $h = -3$, the graph is translated **3 units left**.

Because $k = 1$, the graph is translated **1 unit up**.

Therefore, g is f translated 3 units left and 1 unit up.



B $g(x) = (x - 2)^2 - 1$

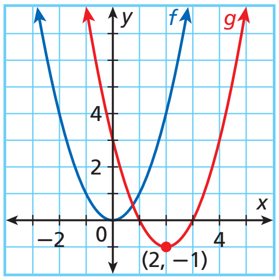
Identify h and k.

$$g(x) = (x - \underbrace{2}_{h})^2 + (\underbrace{-1}_{k})$$

Because $h = 2$, the graph is translated **2 units right**.

Because $k = -1$, the graph is translated **1 unit down**.

Therefore, g is f translated 2 units right and 1 unit down.



Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

2a. $g(x) = x^2 - 5$

2b. $g(x) = (x + 3)^2 - 2$

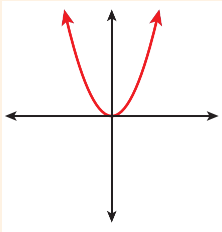
Recall that functions can also be reflected, stretched, or compressed.

Know it! Note

Reflections, Stretches, and Compressions of Quadratic Functions

Reflections

Reflection Across *y*-axis



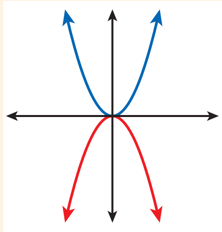
Input values change.

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

The function $f(x) = x^2$ is its own reflection across the *y*-axis.

Reflection Across *x*-axis



Output values change.

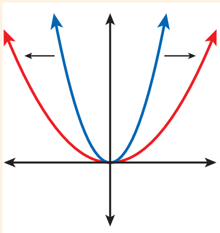
$f(x) = x^2$

$-f(x) = -(x^2)$
$= -x^2$

The function is flipped across the *x*-axis.

Stretches and Compressions

Horizontal Stretch/Compression by a Factor of $|b|$



Input values change.

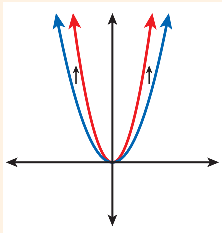
$f(x) = x^2$

$f\left(\frac{1}{b}x\right) = \left(\frac{1}{b}x\right)^2$

$|b| > 1$ stretches away from the *y*-axis.

$0 < |b| < 1$ compresses toward the *y*-axis.

Vertical Stretch/Compression by a Factor of $|a|$



Output values change.

$f(x) = x^2$

$a \cdot f(x) = ax^2$

$|a| > 1$ stretches away from the *x*-axis.

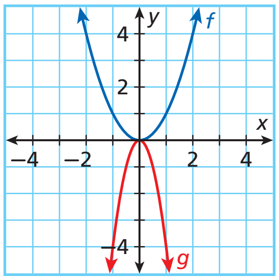
$0 < |a| < 1$ compresses toward the *x*-axis.

EXAMPLE 3 Reflecting, Stretching, and Compressing Quadratic Functions

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

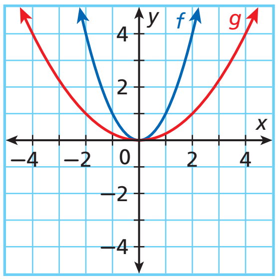
A $g(x) = -4x^2$

Because a is negative, g is a reflection of f across the x-axis. Because $|a| = 4$, g is a vertical stretch of f by a factor of 4.



B $g(x) = \left(\frac{1}{2}x\right)^2$

Because $b = 2$, g is a horizontal stretch of f by a factor of 2.



CHECK IT OUT! **Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.**

3a. $g(x) = (2x)^2$

3b. $g(x) = -\frac{1}{2}x^2$

If a parabola opens upward, it has a lowest point. If a parabola opens downward, it has a highest point. This lowest or highest point is the **vertex of a parabola**.

The parent function $f(x) = x^2$ has its vertex at the origin. You can identify the vertex of other quadratic functions by analyzing the function in *vertex form*. The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$, where a, h, and k are constants.

Know it! Note

Vertex Form of a Quadratic Function

$$f(x) = a(x - h)^2 + k$$

a indicates a reflection across the *x*-axis and/or a vertical stretch or compression.

h indicates a horizontal translation.

k indicates a vertical translation.

Because the vertex is translated h horizontal units and k vertical units from the origin, the vertex of the parabola is at (h, k).

EXAMPLE 4 Writing Transformed Quadratic Functions

Use the description to write the quadratic function in vertex form.

The parent function $f(x) = x^2$ is reflected across the x-axis, vertically stretched by a factor of 6, and translated 3 units left to create g.

Step 1 Identify how each transformation affects the constants in vertex form.

reflection across x-axis:	a is negative	$a = -6$
vertical stretch by 6:	$\lvert a \rvert = 6$	
translation left 3 units:	$h = -3$	

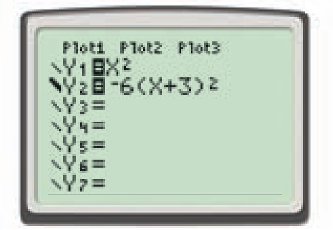
Step 2 Write the transformed function.

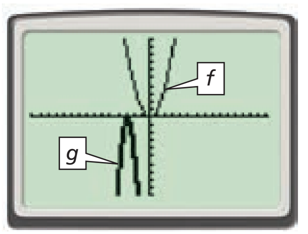
$g(x) = a(x - h)^2 + k$ — *Vertex form of a quadratic function*

$= -6(x - (-3))^2 + 0$ — *Substitute −6 for a, −3 for h, and 0 for k.*

$= -6(x + 3)^2$ — *Simplify.*

Check Graph both functions on a graphing calculator. Enter f as **Y1** and g as **Y2**. The graph indicates the identified transformations.





Helpful Hint

When the quadratic parent function $f(x) = x^2$ is written in vertex form, $y = a(x - h)^2 + k$, $a = 1$, $h = 0$, and $k = 0$.

CHECK IT OUT!

Use the description to write the quadratic function in vertex form.

4a. The parent function $f(x) = x^2$ is vertically compressed by a factor of $\frac{1}{3}$ and translated 2 units right and 4 units down to create g.

4b. The parent function $f(x) = x^2$ is reflected across the x-axis and translated 5 units left and 1 unit up to create g.

EXAMPLE 5 *Automotive Application*

The minimum braking distance d in feet for a vehicle on dry concrete is approximated by the function $d(v) = 0.045v^2$, where v is the vehicle's speed in miles per hour. If the vehicle's tires are in poor condition, the braking-distance function is $d_p(v) = 0.068v^2$. What kind of transformation describes this change, and what does the transformation mean?

Examine both functions in vertex form.

$$d(v) = 0.045(v - 0)^2 + 0 \qquad d_p(v) = 0.068(v - 0)^2 + 0$$

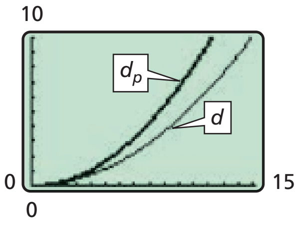
The value of a has increased from 0.045 to 0.068. The increase indicates a vertical stretch.

Find the stretch factor by comparing the new a-value to the old a-value:

$$\frac{a \text{ from } d_p(v)}{a \text{ from } d(v)} = \frac{0.068}{0.045} \approx 1.5$$

The function d_p represents a vertical stretch of d by a factor of approximately 1.5. Because the value of each function approximates braking distance, a vehicle with tires in poor condition takes about 1.5 times as many feet to stop as a vehicle with good tires does.

Check Graph both functions on a graphing calculator. The graph of d_p appears to be vertically stretched compared with the graph of d.



Use the information above to answer the following.

5. The minimum braking distance d_n in feet for a vehicle with new tires at optimal inflation is $d_n(v) = 0.039v^2$, where v is the vehicle's speed in miles per hour. What kind of transformation describes this change from $d(v) = 0.045v^2$, and what does this transformation mean?

THINK AND DISCUSS

1. Explain how the values of a, h, and k in the vertex form of a quadratic function affect the function's graph.

2. Explain how to determine which of two quadratic functions expressed in vertex form has a narrower graph.

3. GET ORGANIZED Copy and complete the graphic organizer. In each row, write an equation that represents the indicated transformation of the quadratic parent function, and show its graph.

Transformation	Equation	Graph
Vertical translation		
Horizontal translation		
Reflection		
Vertical stretch		
Vertical compression		

5-1 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MB7 5-1
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** The highest or lowest point on the graph of a quadratic function is the __?__. (*vertex* or *parabola*)

SEE EXAMPLE 1 p. 315

Graph each function by using a table.

2. $f(x) = -2x^2 - 4$
3. $g(x) = -x^2 + 3x - 2$
4. $h(x) = x^2 + 2x$

SEE EXAMPLE 2 p. 316

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

5. $d(x) = (x - 4)^2$
6. $g(x) = (x - 3)^2 + 2$
7. $h(x) = (x + 1)^2 - 3$

SEE EXAMPLE 3 p. 317

8. $g(x) = 3x^2$
9. $h(x) = \left(\frac{1}{8}x\right)^2$
10. $p(x) = 0.25x^2$
11. $h(x) = -(5x)^2$
12. $g(x) = 4.2x^2$
13. $d(x) = -\frac{2}{3}x^2$

SEE EXAMPLE 4 p. 318

Use the description to write each quadratic function in vertex form.

14. The parent function $f(x) = x^2$ is vertically stretched by a factor of 2 and translated 3 units left to create g.
15. The parent function $f(x) = x^2$ is reflected across the x-axis and translated 6 units down to create h.

SEE EXAMPLE 5 p. 319

16. **Physics** The safe working load L in pounds for a natural rope can be estimated by $L(r) = 5920r^2$, where r is the radius of the rope in inches. For an old rope, the function $L_o(r) = 4150r^2$ is used to estimate its safe working load. What kind of transformation describes this change, and what does this transformation mean?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–19	1
20–25	2
26–28	3
29–30	4
31	5

Extra Practice

Skills Practice p. S12
Application Practice p. S36

Graph each function by using a table.

17. $f(x) = -x^2 + 4$
18. $g(x) = x^2 - 2x + 1$
19. $h(x) = 2x^2 + 4x - 1$

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

20. $g(x) = x^2 - 2$
21. $h(x) = (x + 5)^2$
22. $j(x) = (x - 1)^2$
23. $g(x) = (x + 4)^2 - 3$
24. $h(x) = (x + 2)^2 + 2$
25. $j(x) = (x - 4)^2 - 9$
26. $g(x) = \frac{4}{7}x^2$
27. $h(x) = -20x^2$
28. $j(x) = \left(\frac{1}{3}x\right)^2$

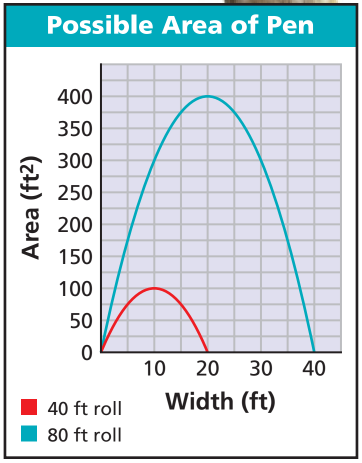
Use the description to write each quadratic function in vertex form.

29. The parent function $f(x) = x^2$ is reflected across the x-axis, vertically compressed by a factor of $\frac{1}{2}$, and translated 1 unit right to create g.
30. The parent function $f(x) = x^2$ is vertically stretched by a factor of 2.5 and translated 2 units left and 1 unit up to create h.
31. **Consumer Economics** The average gas mileage m in miles per gallon for a compact car is modeled by $m(s) = -0.015(s - 47)^2 + 33$, where s is the car's speed in miles per hour. The average gas mileage for an SUV is modeled by $m_u(s) = -0.015(s - 47)^2 + 15$. What kind of transformation describes this change, and what does this transformation mean?

32. Pets Keille is building a rectangular pen for a pet rabbit. She can buy wire fencing in a roll of 40 ft or a roll of 80 ft. The graph shows the area of pens she can build with each type of roll.

a. Describe the function for an 80 ft roll of fencing as a transformation of the function for a 40 ft roll of fencing.

b. Is the largest pen Keille can build with an 80 ft roll of fencing twice as large as the largest pen she can build with a 40 ft roll of fencing? Explain.



Using $f(x) = x^2$ as a guide, describe the transformations for each function.

33. $p(x) = -(x - 4)^2$

34. $g(x) = 8(x + 2)^2$

35. $h(x) = 4x^2 - 2$

36. $p(x) = \frac{1}{4}x^2 + 2$

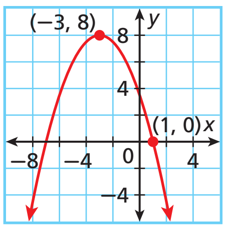
37. $g(x) = (3x)^2 + 1$

38. $h(x) = -\left(\frac{1}{3}x\right)^2$

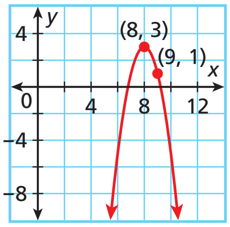
Match each graph with one of the following functions.

A. $a(x) = 4(x + 8)^2 - 3$ **B.** $b(x) = -2(x - 8)^2 + 3$ **C.** $c(x) = -\frac{1}{2}(x + 3)^2 + 8$

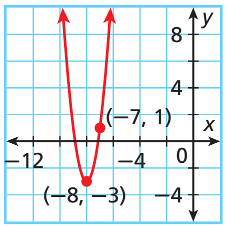
39.



40.



41.

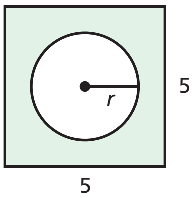


42. Geometry The area A of the circle in the figure can be represented by $A(r) = \pi r^2$, where r is the radius.

a. Write a function B in terms of r that represents the area of the shaded portion of the figure.

b. Describe B as a transformation of A.

c. What are the reasonable domain and range for each function? Explain.



43. Critical Thinking What type of graph would a function of the form $f(x) = a(x - h)^2 + k$ have if $a = 0$? What type of function would it be?

44. Write About It Describe the graph of $f(x) = 999{,}999(x + 5)^2 + 5$ without graphing it.

MULTI-STEP TEST PREP

45. This problem will prepare you for the Multi-Step Test Prep on page 364.

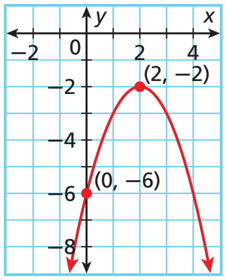
The height h in feet of a baseball on Earth after t seconds can be modeled by the function $h(t) = -16(t - 1.5)^2 + 36$, where -16 is a constant in ft/s^2 due to Earth's gravity.

a. What if...? The gravity on Mars is only 0.38 times that on Earth. If the same baseball were thrown on Mars, it would reach a maximum height 59 feet higher and 2.5 seconds later than on Earth. Describe the transformations that must be applied to make the function model the height of the baseball on Mars.

b. Write a height function for the baseball thrown on Mars.

TEST PREP

Use the graph for Exercises 46 and 47.



46. Which best describes how the graph of the function $y = -x^2$ was transformed to produce the graph shown?

Ⓐ Translation 2 units right and 2 units up
Ⓑ Translation 2 units right and 2 units down
Ⓒ Translation 2 units left and 2 units up
Ⓓ Translation 2 units left and 2 units down

47. Which gives the function rule for the parabola shown?

Ⓕ $f(x) = (x+2)^2 - 2$
Ⓗ $f(x) = (x-2)^2 - 2$
Ⓖ $f(x) = -(x+2)^2 - 2$
Ⓙ $f(x) = -(x-2)^2 - 2$

48. Which shows the functions below in order from widest to narrowest of their corresponding graphs?

$m(x) = \frac{1}{6}x^2$ $\quad n(x) = 4x^2$ $\quad p(x) = 6x^2$ $\quad q(x) = -\frac{1}{2}x^2$

Ⓐ m, n, p, q
Ⓒ m, q, n, p
Ⓑ q, m, n, p
Ⓓ q, p, n, m

49. Which of the following functions has its vertex below the x-axis?

Ⓕ $f(x) = (x-7)^2$
Ⓗ $f(x) = -2x^2$
Ⓖ $f(x) = x^2 - 8$
Ⓙ $f(x) = -(x+3)^2$

50. **Gridded Response** What is the y-coordinate of the vertex of the graph of $f(x) = -3(x-1)^2 + 5$?

CHALLENGE AND EXTEND

51. Identify the transformations of the graph of $f(x) = -3(x+3)^2 - 3$ that would cause the graph's image to have a vertex at $(3, 3)$. Then write the transformed function.

52. Consider the functions $f(x) = (2x)^2 - 2$ and $g(x) = 4x^2 - 2$.

a. Describe each function as a transformation of the quadratic parent function.
b. Graph both functions on the coordinate plane.
c. Make a conjecture about the relationship between the two functions.
d. Write the rule for a horizontal compression of the parent function that would give the same graph as $f(x) = 9x^2$.

SPIRAL REVIEW

53. **Packaging** Peanuts are packaged in cylindrical containers. A small container is 7 in. tall and has a radius of 2 in. A large container is 5.5 in. tall and has a radius twice that of the small container. The price of the large container is three times the price of the small container. Is this price justified? Explain. *(Previous course)*

Identify the parent function for g from its function rule. *(Lesson 1-9)*

54. $g(x) = 4x + \sqrt{3}$

55. $g(x) = 3\sqrt{x+4}$

Write each function in slope-intercept form. Then graph the function. *(Lesson 2-3)*

56. $2y + 5x = 14$

57. $x - \frac{1}{2}y + 4 = -1$