IB MATHEMATICS HL Class of 2020
Year Two Summer Assignment

1. <u>Solve some problems</u>

- Mathematics is a language, a skill set, and a way of thinking. It's good to take a break from it, but it's also essential to stay in practice.
- Last summer I gave you way too many boring problems. Sorry! This year there are only <u>15 problems total</u>.
- Hopefully you'll find them to be interesting, a little challenging, but accessible and not too many!
- They are attached below. Solve them to exercise your mathematical knowledge and prepare for year two.
- Bring your solutions to school the first day of classes. (I hope we'll be back!!)

2. <u>Begin considering ideas for your mathematical exploration (IA)</u>

The IB Math Internal Assessment is a mathematical exploration. It's not about finding especially advanced mathematics. Rather it's about exploring something that is personally interesting to you (either pure mathematics or an application), then communicating your work in a report (12-20 pages, including diagrams and graphs). Your mathematical explanation should be at a level your peers would find accessible, interesting, and informative. Full details and criteria are in the course guide starting on p.78 (SL and HL have the same criteria except Criterion E).

<u>Here are some additional notes from the IB:</u>

- Students should be advised to try and present a personal example and try to obtain a solution rather than reproduce general information found in sources.
- The mathematics used need only be what is required to support the development of the exploration. This could be a few small topics or even a single topic from the syllabus. It will be better to do a few things well, rather than a lot of things not so well.
- The student should always explain why every step is done and how results are obtained.
- If the mathematics used is relevant to the topic being explored, commensurate with the course, and understood by the student, then it can achieve a high level.
- Mathematical understanding must be *demonstrated*. Obtaining a correct answer is not sufficient to demonstrate understanding.
- The exploration must meet the stated aim, and be concise and complete.

<u>Think about what you might be interested in exploring and writing on for your Math IA in the fall.</u>

- If you google IA topics, one of first hits is this mind map of ideas for explorations. What would a mind map look like based on your own interests?
- You don't need to turn anything in. Just reflect and explore ideas that interest you.
- The IA expectations have changed very little from the previous IB math courses. Here are examples of real IAs with Criterion Marking and Comments from previous IB Math SL and HL students.

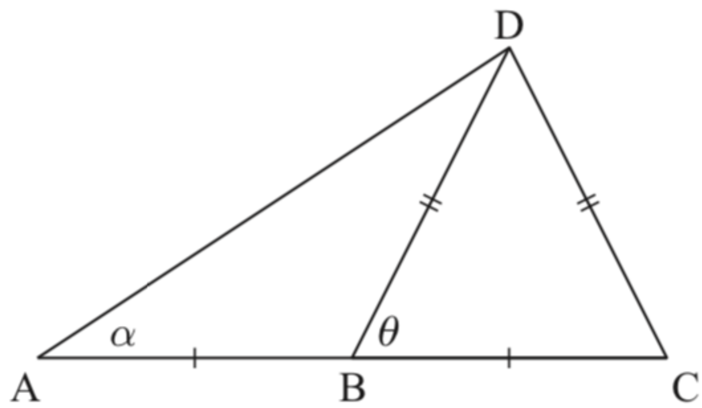
3. <u>Take some time off from school completely.</u>

I wish you an excellent summer, and I look forward to a successful Year Two of IB Math HL!

Ashton Betancourt (betancourta@asf.edu.mx)

Summer Practice Problems for Math Analysis HL:

1. AB = BC. Show that tan(θ) = 3tan(α)



2. For this question, you may assume that the domain of f is restricted to $x > 0$.

The function $f(x)$ has the following properties: $f'(x) > 0$ and $f''(x) < 0$ for all x, $f(2) = 1$, and $f'(2) = 2$.

a Find the equation of the tangent to $f(x)$ at $x = 2$, and sketch it on a graph.

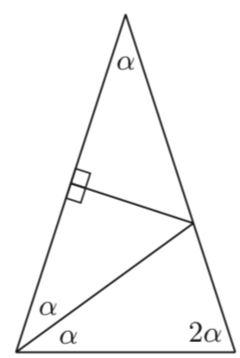
b Sketch a possible graph of $y = f(x)$ on the same set of axes.

c Explain why $f(x)$ has exactly one zero.

d Estimate an interval in which the zero of $f(x)$ lies.

3.

Use the figure alongside to show that $\cos 36° = \dfrac{1+\sqrt{5}}{4}$.



4.

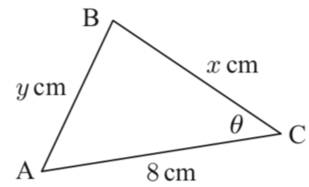
If $x^2 + y^2 = 52xy$, and $0 < y < x$, show that $\log\left(\dfrac{x-y}{5}\right) = \frac{1}{2}(\log x + \log 2y)$.

5.

$x^n + ax^2 - 6$ leaves a remainder of -3 when divided by $(x-1)$, and a remainder of -15 when divided by $(x+3)$. Find the values of a and n.

6.

Triangle ABC has perimeter 20 cm.



a Find y^2 in terms of x and θ and hence write $\cos\theta$ in terms of x only.

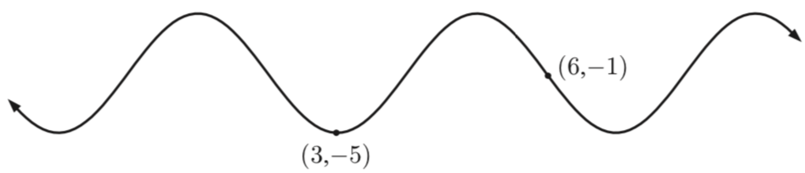
b If the triangle has area A, show that $A^2 = -20(x^2 - 12x + 20)$.

c Find the maximum area of the triangle. Comment on the triangle's shape when its area is a maximum.

7.

Find a trigonometric equation of the form $y = a\sin(b(x + c)) + d$ that represents the following graph with the information given below.

You may assume that $(3, -5)$ is a minimum point and $(6, -1)$ lies on the principal axis.



8.

If $f(2x + 3) = 5x - 7$, find $f^{-1}(x)$.

9.

Find exact solutions for the following:

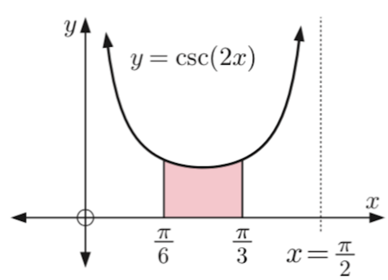
a $|1 - 4x| > \frac{1}{3}|2x - 1|$

b $\dfrac{x-2}{6 - 5x - x^2} \leqslant 0$

10. When you take the derivative of $y = \ln(\tan x)$, you should get $\sin x \cos x$ in the denominator. Part (a) of this problem asks you to use a trigonometric identity to change the form of your answer.

a Given that $y = \ln(\tan x)$, $x \in \,]0, \frac{\pi}{2}[$, show that $\dfrac{dy}{dx} = k\csc(2x)$ for some constant k.

b The graph of $y = \csc(2x)$ is illustrated on the interval $]0, \frac{\pi}{2}[$.
Find the area of the shaded region. Give your answer in the form $a\ln b$ where $a \in \mathbb{Q}$ and $b \in \mathbb{Z}^+$.



11.

a Show that $\sqrt{14 - 4\sqrt{6}}$ cannot be written in the form $a + b\sqrt{6}$ where $a, b \in \mathbb{Z}$.

b Write $\sqrt{14 - 4\sqrt{6}}$ in the form $a\sqrt{m} + b\sqrt{n}$ where $a, b, m, n \in \mathbb{Z}$.

12. Feel free to leave your answer in whichever form you like.

The real polynomial $P(z)$ of degree 4 has one complex zero of the form $1-2i$, and another of the form ai, where $a \neq 0$, $a \in \mathbb{R}$.

Find $P(z)$ if $P(0)=10$ and the coefficient of z^4 is 1. Leave your answer in factorised form.

13. In triangle ABC,

$$3\sin B + 4\cos C = 6 \text{ and}$$
$$4\sin C + 3\cos B = 1.$$

(a) Show that $\sin(B+C) = \frac{1}{2}$. *[6]*

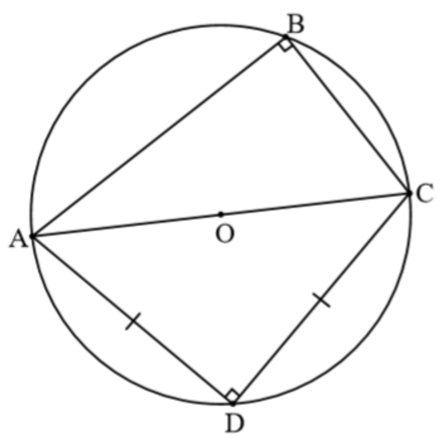
Robert conjectures that $\mathrm{C\hat{A}B}$ can have two possible values.

(b) Show that Robert's conjecture is incorrect by proving that $\mathrm{C\hat{A}B}$ has only one possible value. *[5]*

14.

(a) Given that $\cos 75° = q$, show that $\cos 105° = -q$. [1]

In the following diagram, the points A, B, C and D are on the circumference of a circle with centre O and radius r. [AC] is a diameter of the circle. $\mathrm{BC} = r$, $\mathrm{AD} = \mathrm{CD}$ and $\mathrm{A\hat{B}C} = \mathrm{A\hat{D}C} = 90°$.



(b) Show that $\mathrm{B\hat{A}D} = 75°$. [3]

(c) (i) By considering triangle ABD, show that $\mathrm{BD}^2 = 5r^2 - 2r^2 q\sqrt{6}$.

(ii) By considering triangle CBD, find another expression for BD^2 in terms of r and q. [7]

(d) Use your answers to part (c) to show that $\cos 75° = \dfrac{1}{\sqrt{6}+\sqrt{2}}$. [3]

15. Two sticks have length a and b with a > b. You break the longer stick at a random point. What is the probability that the resulting three sticks form a triangle? (Hint: Try some examples, perhaps start with a=20cm and b=10cm.)