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2020 Summer Packet for Incoming Geometry Students

The math faculty at Shepaug Valley School would like to welcome you to 2019 - 2020 school year! We are looking forward to helping you achieve your greatest potential. We hope a quality education is one things you will value.

We have developed the attached review packet to help you prepare for the Geometry class you will be taking this fall. This packet includes material that students are expected to understand before beginning the Geometry curriculum. The topics covered by the packet are the foundational skills necessary to be successful in Geometry. High School Geometry teachers will be collecting the packet and giving an assessment within the first few days of school. The completed assignment will be collected the first day of school.

Students may use any resources available to them to complete this packet. Helpful websites include:

www.purplemath.com
www.math.com
www.khanacademy.com

Please spend the time needed to do a quality job on this packet. Show and organize your work for each problem. Use a calculator ***where indicated*** but write down your calculations and show all of your work!

Enjoy your summer vacation and keep your education moving forward during this break.

Geometry Summer Assignment

The following topics will begin your study of Geometry. These topics are considered to be a review of your previous math courses and will not be covered in length during the start of the school year.

Note: You should expect to purchase a scientific calculator for this course.

Section 1: Fractions

To multiply fractions:

- Multiply the numerators of the fractions
- Multiply the denominators of the fractions
- Place the product of the numerators over the product of the denominators
- Simplify the fraction

Example: Multiply $\frac{2}{9}$ and $\frac{3}{12}$

- Multiply the numerators (2*3=6)
- Multiply the denominators (9*12=108)
- Place the product of the numerators over the product of the denominators, $\frac{6}{108}$
- Simplify the fraction, $\frac{6}{108} = \frac{1}{18}$

To divide fractions:

- Invert (i.e. turn over) the 2nd fraction and multiply the fractions
- Multiply the numerators of the fractions
- Multiply the denominators of the fractions
- Place the product of the numerators over the product of the denominators
- Simplify the fraction

Example: Divide $\frac{2}{9}$ and $\frac{3}{12}$

- Invert the 2nd fraction and multiply, $\frac{2}{9} \div \frac{3}{12} = \frac{2}{9} * \frac{12}{3}$
- Multiply the numerators (2*12=24)
- Multiply the denominators (9*3=27)
- Place the product of the numerators over the product of the denominators, $\frac{24}{27}$
- Simplify the fraction, $\frac{24}{27} = \frac{8}{9}$

1) $12 \times \frac{3}{4} =$

2) $\frac{1}{5} \times \frac{10}{4} =$

3) $\frac{2}{7} \times \frac{21}{30} =$

4) $\frac{\frac{20}{3}}{4} =$

5) $\frac{1}{10} \div \frac{3}{5} =$

6) $\frac{2}{5} \div \frac{8}{10} =$

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Section 2: Simplifying Algebraic Expressions

The difference between an expression and an equation is that an expression doesn't have an equal sign. Expressions can only be simplified, not solved. Simplifying an expression often involves combining like terms. Terms are like **if and only if** they have the same variable and degree or if they are constants. Simplifying expressions also refers to substituting values to get a resultant value of the expression.

Simplify the following expressions by combining like terms.

7) $3 + 2y^2 - 7 - 5x - 4y^3 + 6x$

8) $x^2 + x^2 + x + x$

9) $4(3x - 2x^3 + 5) - 6x$

10) $8a - (7b - 4a) - 3(4a + 2b)$

Evaluate the following expressions by substituting the given values for the variables.

11) $6a^2 - 2b + 4ab - 5a$ $a = -3$ and $b = 4$

12) $-k^2 + 4m - 2km - (3k + 2m)$ $k = -2$ and $m = 3$

13) $3(4c - 2d) + d(dc^2 + 7)$ $c = -2$ and $d = 3$

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Section 3: Solving Equations

When solving an equation, remember to combine like terms first. Terms are like **if and only if** they have the same variable and degree or if they are constants. Then, take steps to isolate the variable by following the order of operations backwards and doing the inverse operation.

Solve each equation and check your answer.

14) 3n + 2 = 17

15) 4 – 2y = 8

16) $3(n - 4) = 15$

17) $6 - (3t + 4) = -17$

18) $5k + 2(k + 1) = 23$

19) $\frac{5}{7}p - 10 = 30$

20) $-\frac{1}{2}m - 3 = 1$

21) $(w + 5) - (2w + 5) = 5$

22) $7 = \frac{5}{x}$

23) $\frac{1}{3}(2x - 1) = \frac{3}{4}(x + 2)$

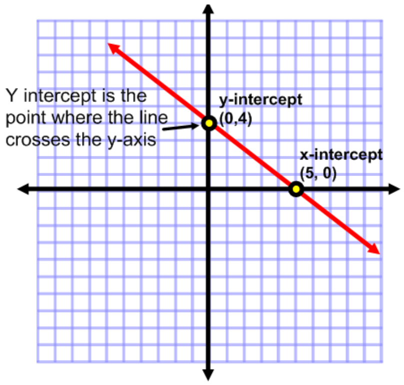
On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Section 4: Graphing Linear Equations

A linear function is a function where the highest power of x is 1. You have seen these functions in many forms. Some of the common forms are y = mx + b (slope-intercept form) and Ax + By = C (standard form). Notice in both forms that the exponent for x is 1.

Every linear function has an x and y intercept.

- x – intercept: Where a function crosses the x – axis. The coordinate is represented by (x, 0).
- y – intercept: Where a function crosses the y – axis. The coordinate is represented by (0, y).

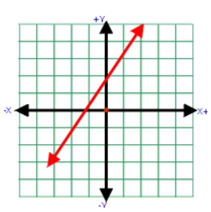


A key concept to consider when thinking of linear functions is slope.

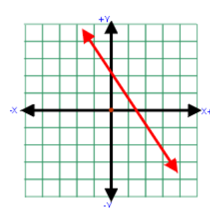
Slope is the "m" in the y = mx + b and is defined to be $\frac{A}{B}$ for standard form of a line. Here are some definitions of slope:

$$slope = m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

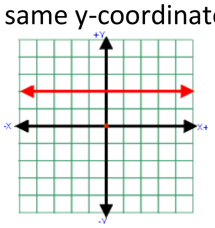
<u>Positive</u> slopes increase from left to right.



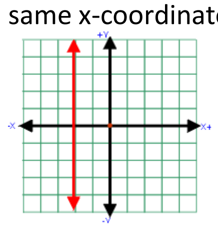
<u>Negative</u> slopes decrease from left to right.



If a line has a slope of <u>zero</u>, it is horizontal and the equation is y = b. All points on the line have the same y-coordinate.



If a line has an <u>undefined</u> slope, it is vertical and the equation is x = c. All points on the line have the same x-coordinate.



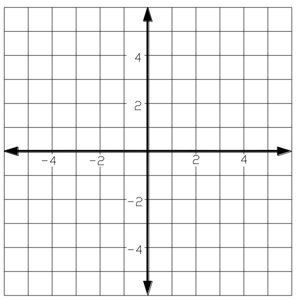
Formulas for equations of a line:

Slope-Intercept: $y = mx + b$

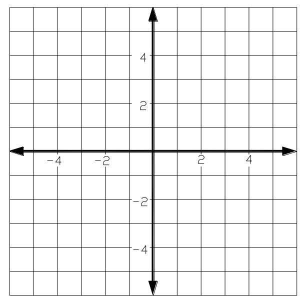
Point-Slope Form: $y - y_1 = m(x - x_1)$

Graph each linear equation. (Note: You may need to put the equation in slope-intercept form.)

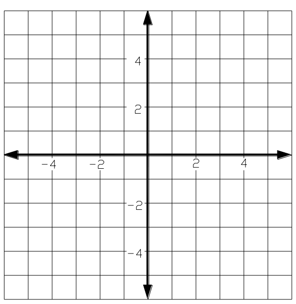
24) $y = 2x - 2$



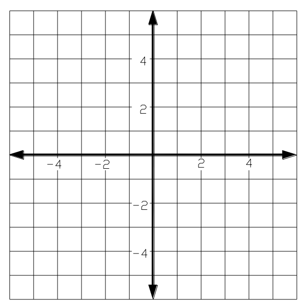
25) $y = -\frac{1}{3}x + 4$



26) $y - 4x = -5$



27) $4x + 3y = 12$



On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Section 5: Writing Equations of Lines

There are several ways to write an equation of a line. If you would like to review a video of this procedure, please click on the following link, http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/equation-of-a-line/v/equation-of-a-line-3. The example below is writing an equation for the line through the points, (2,3) and (–1, –5).

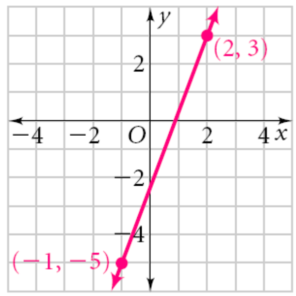
Write equations for the line in point-slope form and in slope-intercept form.

Step 1 Find the slope.

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

$$\frac{-5 - 3}{-1 - 2} = \frac{8}{3}$$

The slope is $\frac{8}{3}$.



Step 2 Use either point to write the equation in point-slope form.

Use (2, 3).

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{8}{3}(x - 2)$$

Step 3 Rewrite the equation from Step 2 in slope-intercept form.

$$y - 3 = \frac{8}{3}(x - 2)$$

$$y - 3 = \frac{8}{3}x - 5\frac{1}{3}$$

$$y = \frac{8}{3}x - 2\frac{1}{3}$$

Write an equation of a line given the following information in **slope-intercept fo**rm.

28) (3, –4) m = 6

29) (4,0) m = 1

30) (–2, –7) $m = -\frac{3}{2}$

31) (2,7) and (1, –4)

32) (–3, –4) and (3, –2)

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Section 6: Solving Systems of Equations

Solving systems of equations is used to solve a combination of equations with more than one variable. There are three methods to solving a system of equations: graphing, substitution, and elimination. We will only cover the last two in this review. When the lines intersect at exactly one point, the (x,y) values of that point are the solutions to the equation.

- **Substitution:** Here is an example of solving a system of equations by substitution.

Solve the system by substitution. $\begin{cases} 4x + 3y = 4 \\ 2x - y = 7 \end{cases}$

Step 1 Solve for one of the variables. Solving the second equation for y is easiest.

$$2x - y = 7$$
$$y = 2x - 7$$

Step 2 Substitute the expression for y into the other equation. Solve for x.

$$4x + 3y = 4$$
$$4x + 3(2x - 7) = 4 \quad \text{Substitute for } y.$$
$$4x + 6x - 21 = 4 \quad \text{Distributive Property}$$
$$4x + 6x = 25$$
$$x = 2.5$$

Step 3 Substitute the value of x into either equation. Solve for y.

$$y = 2x - 7$$
$$y = 2(2.5) - 7 \quad \text{Substitute for } x.$$
$$y = -2$$

The solution is $(2.5, -2)$.

- **Elimination:** Here is an example of solving a system of equations by eliminating a variable.

Solve the system below by elimination.

$$\begin{cases} 3x + 7y = 15 \\ 5x + 2y = -4 \end{cases}$$

To eliminate the y terms, make them additive inverses by multiplying.

① $3x + 7y = 15$ $\qquad 6x + 14y = 30$ Multiply ① by 2.

② $5x + 2y = -4$ $\qquad -35x - 14y = 28$ Multiply ② by −7.

$$-29x = 58 \quad \text{Add.}$$
$$x = -2 \quad \text{Solve for } x.$$

$$3x + 7y = 15 \quad \text{Choose an original equation.}$$
$$3(-2) + 7y = 15 \quad \text{Substitute the value of } x.$$
$$-6 + 7y = 15 \quad \text{Simplify.}$$
$$7y = 21$$
$$y = 3 \quad \text{Solve for } y.$$

The solution is $(-2, 3)$.

Solve the following systems of equations by substitution.

33) $x + 12y = 68$

$x = 8y - 12$

34) $3x - y = -1$

$y = 2x - 1$

35) $3x + 2y = 6$

$x - 2y = 10$

Solve the following systems of equations by elimination.

36) $10x + 6y = 0$

$-7x + 2y = 31$

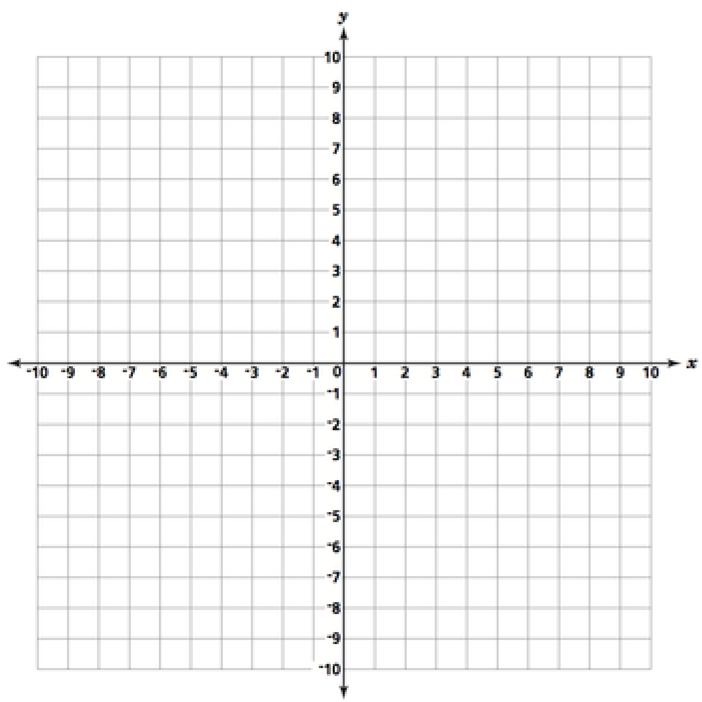
37) $2x + 5y = -4$

$3x - y = 11$

Solve the following systems of equations by graphing.

38) $3x + 2y = 14$

$3x - 2y = 10$



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Section 7: Multiplying Polynomials

FOIL Method

The FOIL method is a special case of a more general method for multiplying algebraic expressions using the distributive law.

- **F**irst (“first” terms of each binomial are multiplied together)
- **O**uter (“outside” terms are multiplied—that is, the first term of the first binomial and the second term of the second)
- **I**nner (“inside” terms are multiplied—second term of the first binomial and first term of the second)
- **L**ast (“last” terms of each binomial are multiplied)

The general form is:

$$(a+b)(c+d) = \underbrace{ac}_{\text{first}} + \underbrace{ad}_{\text{outside}} + \underbrace{bc}_{\text{inside}} + \underbrace{bd}_{\text{last}}$$

Once you have multiplied by using the FOIL method, you must combine any like terms.

- Use FOIL to multiply $(x + 3)(x + 2)$

  "first": $(x)(x) = x^2$
  "outer": $(x)(2) = 2x$
  "inner": $(3)(x) = 3x$
  "last": $(3)(2) = 6$

  So: $(x + 3)(x + 2) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6$

39) Multiply (x + 5) (x + 7)

40) Multiply (y – 3) (y – 5)

41) Multiply (4x + 2) (4x – 2)

42) Multiply (2a + 3) (3a – 4)

43) Multiply (a + 4) (a – 4)

44) Multiply $(5t + 4)^2$

45) Multiply $(3y - 2)(3y + 2)$

46) Multiply $(w^2 + 2)(w^2 - 2)$

47) Multiply $(2x - 5y)(3x + y)$

48) Multiply $(5x + 3)(-x + 2)$

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Distribution Method

Sometimes you will have to multiply one multi-term polynomial by another multi-term polynomial. This type of multiplication will use a form of distribution for one of the polynomials to the other polynomial.

- Simplify $(x + 3)(4x^2 - 4x - 7)$

$(x + 3)(4x^2 - 4x - 7)$
$= (x)(4x^2 - 4x - 7) + (3)(4x^2 - 4x - 7)$
$= 4x^2(x) - 4x(x) - 7(x) + 4x^2(3) - 4x(3) - 7(3)$
$= 4x^3 - 4x^2 - 7x + 12x^2 - 12x - 21$
$= 4x^3 - 4x^2 + 12x^2 - 7x - 12x - 21$
$= 4x^3 + 8x^2 - 19x - 21$

49) Multiply $(w + 2)(w^2 + 2w - 1)$

50) Multiply $(x - 1)(x^2 + x + 1)$

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

Factoring

Factoring is the process of "un-doing" a polynomial. Factors are numbers multiplied together to get a product.

Example 1:

$t^2 + 8t + 12$

1•12, 2•6, 3•4 are factors of 12	Identify the factors of the whole number.
2 and 6 can be added to get 8	Find the factors of 12 that add or subtract to equal 8.
$(t\ \ +\ \ )(t\ \ +\ \ )$	Identify the signs that fit into the factors. Use the following table as a reference. ____ + ____ + ____ = (+) (+) ____ – ____ + ____ = (–) (–) ____ – ____ – ____ = (–) (+) ____ + ____ – ____ = (–) (+)
$(t + 2)(t + 6)$	Substitute the numbers into the appropriate factors.

Example 2:

$x^2 - 6x + 8$

1•8, 2•4 are factors of 8	Identify the factors of the whole number.
2 and 4 can be added to get 6	Find the factors of 8 that add or subtract to equal 6.
$(x\ \ -\ \ )(x\ \ -\ \ )$	Identify the signs that fit into the factors. Use the following table as a reference. ____ + ____ + ____ = (+) (+) ____ – ____ + ____ = (–) (–) ____ – ____ – ____ = (–) (+) ____ + ____ – ____ = (–) (+)
$(x - 2)(x - 4)$	Substitute the numbers into the appropriate factors.

Example 3:

$p^2 - 3p - 40$

1•40, 2•20, 4•10, 5•8 are factors of 40	Identify the factors of the whole number.
5 and 8 can be subtracted to get 3	Find the factors of 40 that add or subtract to equal 3.
$(p\ \ -\ \ )(p\ \ +\ \ )$	Identify the signs that fit into the factors. Use the following table as a reference. ____ + ____ + ____ = (+) (+) ____ – ____ + ____ = (–) (–) ____ – ____ – ____ = (–) (+) ____ + ____ – ____ = (–) (+)
$(p - 8)(p + 5)$	Substitute the numbers into the appropriate factors.

Factor each polynomial.

51. $p^2 + 9p + 20$

52. $g^2 - 7g + 10$

53. $n^2 + 3n - 18$

54. $y^2 - 5y - 6$

55. $t^2 + 9t - 10$

56. $r^2 + 4r - 12$

57. $d^2 - 12d + 27$

58. $y^2 - 2y - 24$

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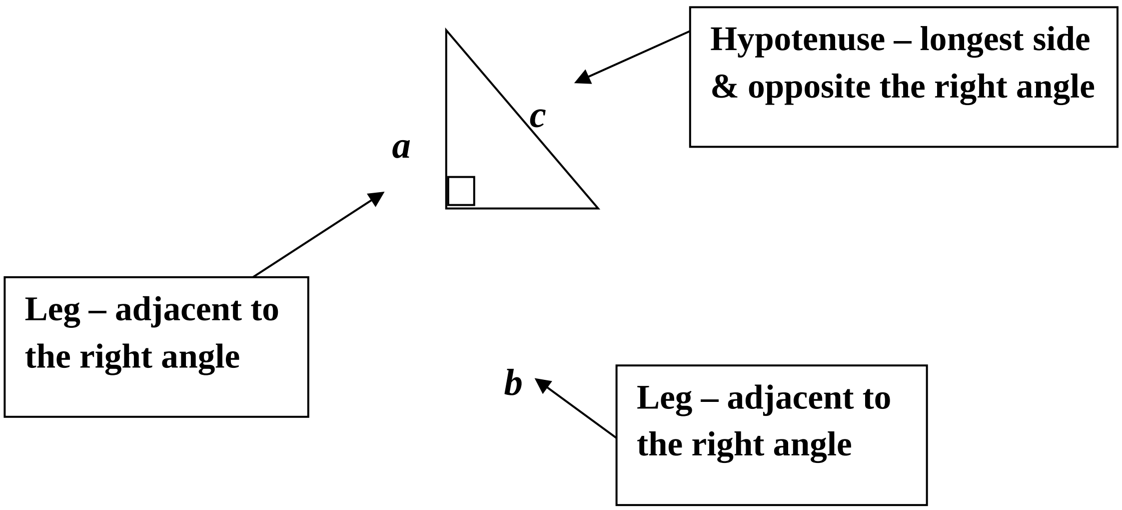
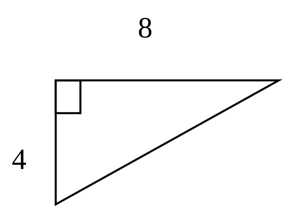
Pythagorean Theorem

The Pythagorean Theorem is a formula unique to only right triangles.



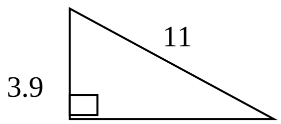
$$a^2 + b^2 = c^2$$

Example 1:



4 and 8 are the legs; hypotenuse is unknown	Identify the legs and the hypotenuse.
$4^2 + 8^2 = x^2$	Substitute the numbers into the equation.
$16 + 64 = x^2$ $80 = x^2$ $\sqrt{80} = \sqrt{x^2}$ $8.9 = x$	Solve using the rules of exponents and radicals. Round to the nearest tenth.

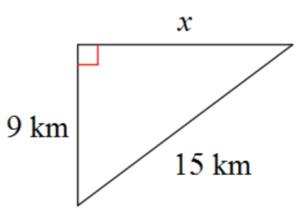
Example 2:



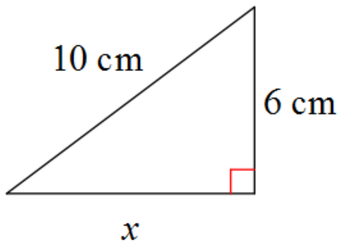
3.9 is a leg; 11 is the hypotenuse	Identify the legs and the hypotenuse.
$3.9^2 + x^2 = 11^2$	Substitute the numbers into the equation.
$15.21 + x^2 = 121$ $x^2 = 105.79$ $\sqrt{x^2} = \sqrt{105.79}$ $x = 10.3$	Solve using the rules of exponents and radicals. Round to the nearest tenth.

Solve for the missing length using the Pythagorean Theorem. Round to the nearest tenth when necessary.

59.



60.



61. An architect is making a floor plan for a rectangular gymnasium. If the gymnasium is 24 meters long and 18 meters wide, what will the distance be between opposite corners? Draw a diagram and show all your work.

62. A ladder is leaning against the side of a 10 meter house. If the base of the ladder is 3 meters away from the house, how tall is the ladder? Draw a diagram and show all your work.

On a scale of 1 – 5 (1: Weak, 5: Strong) rate yourself on this section of math: 1 2 3 4 5

*Revised from Shaler Area School District