Learning throughout AP Physics C: Mechanics

Every student in this course has a different history. There are differences in mathematics skills, science background, writing abilities, communication, independence, and personality. While you have some variance in starting point, there are several factors and considerations that will help to ensure your success in this course, no matter your background.

- Stay organized.
  - Keep a binder or folder for the material given in class.
  - Keep all of your work in digital and physical forms, especially laboratory work for review before the examination.
  - Read the material before we discuss it in class, and demonstrate your curiosity and engagement regularly during class discussions.
  - Purchase an AP Physics C: Mechanics review book as early as possible, and practice as often as you can stand.
- Be proactive.
  - Pre-read units. Do problems before they are assigned. This will make the course significantly easier. Review algebra regularly.
  - Attend extra help. By clarifying any issues you have early on, you can avoid catastrophe as the material and concepts accumulate. It is imperative that you have a handle on early material, as it is the foundation for the rest of the course.
  - **Check your school email regularly**, and make it a habit to check PowerSchool for grades, assignments, and supplemental material.
  - Work often with problems that you find difficult.
- Embrace mathematics.
  - The AP Physics C course is heavily focused on investigating physical phenomena through equations and mathematical expressions. You need to be able to navigate this with ease, so your fundamental skills must be strong. Focus heavily on the calculus course that you are taking. A comfort with calculus will be essential as this course progresses.
  - It is necessary to learn and practice a few basic skills prior to the start of the course. They will be used often. I also encourage you to preview the content of your upcoming Calculus courses, as we will be learning necessary material over about one week.

I know that you will have other demanding classes during the year. You must work hard to balance your time to ensure that you give enough effort to each course. Remain consistent in your pursuit of knowledge in this course. While the year will provide you with many distractions from college applications to vacations, it is essential that you stay on top of the schedule with which you will be provided.

I look forward to working with you as the year progresses. Stay engaged in and fascinated by the laws of nature, and be prepared for a rigorous, interesting investigation into how our world works.

AP Physics C: Mechanics Summer Work

All work is due upon return to school in the Fall and graded for the first quarter. Start early, but work gradually.

- **There will be a test on force concepts upon your return (these do not exceed an 11th grade understanding of forces and involve no mathematics).**

- You MUST visit apcentral.collegeboard.com to investigate the curriculum and the specifics of the AP Physics C: Mechanics test. **There will also be a quiz on this upon your return.**

- Complete videos and exercises on Khan Academy (to be posted to your accounts when you sign up)
  - Sign up for the class here: https://www.khanacademy.org/join/2QHUQY7R
  - Be sure you are logged in when you work on the assignments.

- **You should read through *"Surely You're Joking, Mr. Feynman": Adventures of a Curious Character* by Richard P. Feynman and Ralph Leighton.** Prepare for discussion soon after we return in the fall. Make note of sections you liked and parts you took issue with, and why.

- **While not mandatory, you should also read *Humble Pi: When Math Goes Wrong in the Real World* by Matt Parker (https://www.amazon.com/Humble-Pi-When-Wrong-World/dp/0593084683/ref=tmm_hrd_swatch_0?_encoding=UTF8&qid=1589201637&sr=8-1).**

- Submit a 1-2 page summary of current research in physics. This should be from an academic, not popular, source. Ideally, you will find articles on arxiv.org, but other sources are acceptable. Cite your sources. Ask me if you have questions about valid sources.

Typically, I assign textbook work, but in lieu of physical textbooks, I will be giving out a sheet of problems that you should complete by your return. This will be sent out in early June.

I will also set up two optional meetings during the summer to discuss summer work questions.

You should also spend time this summer working on your college essays, as this course will demand a large investment of effort throughout the year. It is unwise to leave the entire application process for the fall when you will have many other demands on your time.

AP Physics C: Mechanics
Summer Work – Problem Set

Mr. March-Steinman

Summer / 2020

Contents

1 Background Information

This problem set is designed to get you used to thinking about physics and mathematics in a different way to help prepare you for the upcoming year. We will be revisiting these ideas with more depth. I know there's a lot of writing, but I've tried to parse it down to the necessary bits and have worked to keep the language easier to understand. If you find it dense, just do one page per day. You're welcome to give feedback on confusing sections and mercilessly make fun of me for typos.

For each section, there will be two problems per section which may involve both calculations and derivations. These are not all introductory, so you might need to try out some basic problems using worksheets or videos you find online before attempting them. Calculations, while pleasant for engineers, are not particularly important for this course. We aim to derive new expressions to generalize relationships between elements in different kinds of problems. This lets us better see the edge cases and limiting factors of any situation.

Some of these problems may be easy, but some might take you a significant amount of time. I encourage you in these cases to take a step back and think about the problems before approaching them again. This process is more important than coming to an immediate answer; you will gain far more insight into a situation by ingesting the material in this manner.

I give a brief background to each concept prior to the problems with the aim of guiding your thinking for every section. Your answers to derivation problems must include only the variables listed in the end. While these are simple examples, they can get far more complex over time. Treat them like puzzles!

As we move through, I will try to be clear about any new notation you might encounter. There will be a lot this year, though there is significant crossover with mathematics.

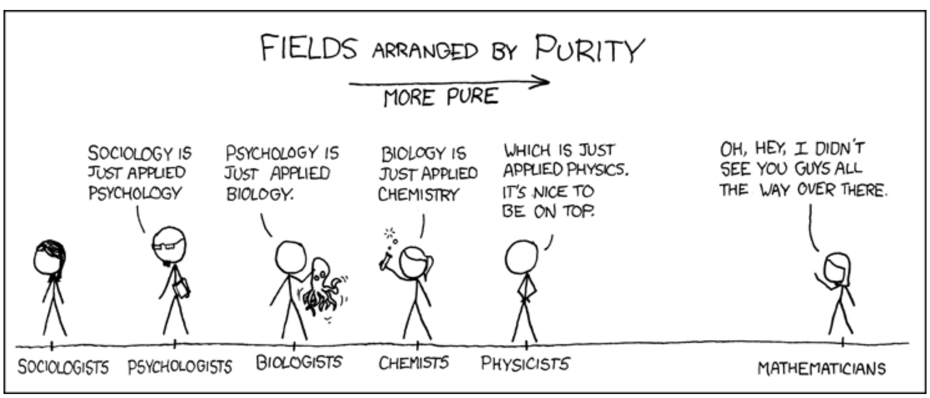

Figure 1: https://xkcd.com/435/

2 Spherical Cows

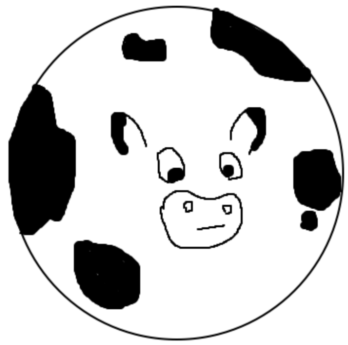

Figure 2: A spherical cow projected onto a 2-D surface.

The real world is stunningly complex and full of unfathomably numerous things and interactions. It is futile to try to account for all of this complexity at the level of this course (or, indeed, in most practical or theoretical matters). Much of what you'll see in this course is the result of approximations and simplifications. Often, this "good enough" approach is particularly useful for solving problems, but there are many cases where our answers will diverge from reality due to the choices that we make. We'll try to send some of these spherical cows to pasture as the year moves along and our sophistication increases, but you'll always have to keep some of them in mind when considering the limitations inherent in problems.

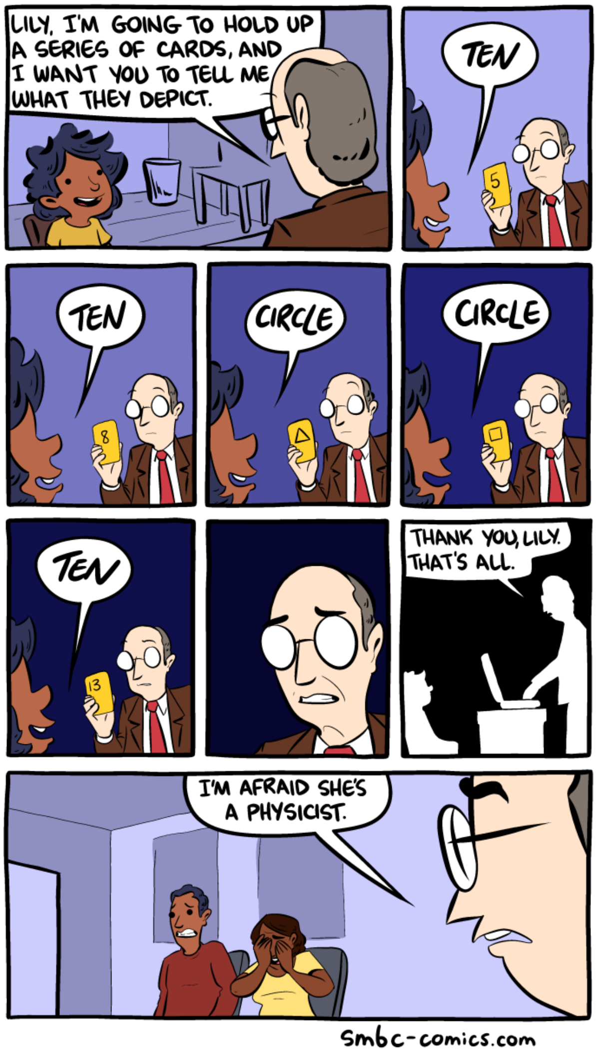

Figure 3: https://www.smbc-comics.com/comic/a-severe-disorder

3 Units, Scientific Notation, and Conversions

Science is largely comprised of observation and analysis of physical phenomena. That is, we intelligently measure the real world (hopefully) and think about what our measurements mean. Because experimentation and data collection are both so important to developing physics, we have to make sure everyone's on the same page about what we've measured.

3.1 Units

How do we label the quantities that we measure? What happens to units when we do math operations on data?

By convention, we label basic dimensional information with the following units (abbreviations in

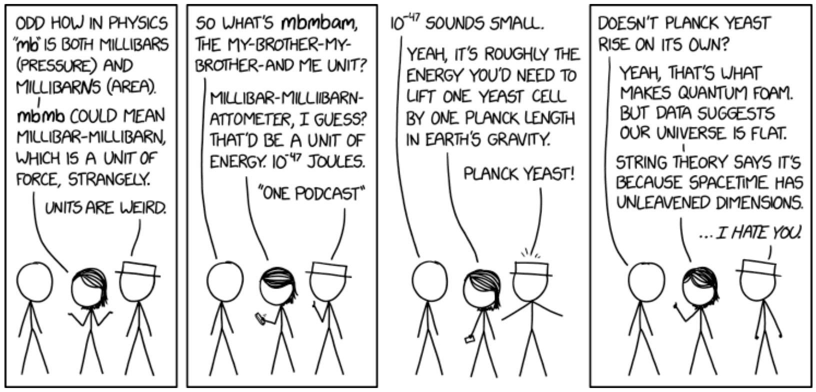

Figure 4: https://xkcd.com/2312/

parentheses), using prefixes to indicate order of magnitude:

Quantity	Unit	Symbol
Distance	meters	m
Time	seconds	s
Mass	kilograms *	kg
Electric current intensity	Amperes	A
Temperature	Kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

Table 1: SI Base Units

These 7 units of measure comprise the SI base units. Any other unit you encounter is called a derived unit because it can be derived from mathematical combinations of base units. It's pretty cool that any scientific unit you've ever seen can come from just these simple ideas.

As an example, consider the unit of force: a Newton. Newtons don't appear on the list above, so they're not a base unit. How do we combine the above values to get a Newton? Multiply a unit of mass by a unit of length, and divide by the square of a unit of time:

$$N := kg \times \frac{m}{s^2}$$

Here, I use the := operator to signify assignment, or "is defined to be." This is a computer science and mathematics convention.

In the above case, you can consider units to behave nicely according to the rules of algebra. If you perform mathematical operations on a set of measurements, the same operations act on the units of those measurements.

You'll also encounter unitless (dimensionless) values with some regularity. You'll have to know how to deal with these and what they indicate. Often, these represent ratios between values with

*this one comes with a prefix attached already. It would be odd to use millikilograms, so we just refer to this kind of unit as "grams" and leave it be.

similar units (dimensions), or are constants that simply scale measurements for one reason or another. Coefficients of friction or ratios between masses are good examples of dimensionless quantities in physics. From geometry, you might consider the example of scale factors.

3.2 Scientific Notation

Scientific notation exists for two main reasons.

1. Since we work so much with measurements from experiments, it's necessary to somehow indicate the precision of our measuring instruments. It'd be a pain to include some long narrative explanation for each value or measurement, so we take a shortcut of indicating that precision numerically through the use of significant figures (or significant digits; these two terms can be used interchangeably).

2. We'll on occasion be dealing with very large and/or very small values. Instead of using some unreadable decimal format like 0.00000000000000000000001 $kg\frac{m}{s}$ for a momentum reading, instead we'll use a notation that can give us a better understanding of how small the value is: 1×10^{-23} $kg\frac{m}{s}$. Not only is this format more compact, but it is also more effective in conveying information about the value.

Rules for significant digits (and how they are maintained or changed through mathematical operations) can be found in many places online, so I won't recount them here. You'll generally be expected to use these correctly for laboratory exercises (we'll also later introduce the idea of uncertainty), but I won't penalize you for problems unless you're more than two or three significant figures off.

Rules for scientific notation are simple, and can also be found online (Khan Academy has resources for both significant figures and scientific notation, for example.). You'll also use this when appropriate for laboratory assignments. It is important that you are able to read these correctly and use them with conversions and graphs. A problem below will test your abilities.

3.3 Conversions

We now find it necessary to make use of a table of unit prefixes.

Prefix	Multiple
yocto	10^{-24}
zepto	10^{-21}
atto	10^{-18}
femto	10^{-15}
pico	10^{-12}
nano	10^{-9}
micro	10^{-6}
milli	10^{-3}
centi	10^{-2}
deci	10^{-1}

Table 2: SI prefixes with negative exponents

While you'll occasionally have to use these in calculations, it is important to pay attention to the

Prefix	Multiple
deca	10^1
hecto	10^2
kilo	10^3
mega	10^6
giga	10^9
tera	10^{12}
peta	10^{15}
exa	10^{18}
zetta	10^{21}
yotta	10^{24}

Table 3: SI prefixes with positive exponents

ways in which scaling units also scales slopes on a graph. This will be particularly important in our analysis of experimental data.

3.4 Problems

3.4.1 Unit Conversions

There are two major unit systems used in science, where the MKS (meter-kilogram-second) system is typically seen in some areas of science, while the CGS (centimeter-gram-second) system, largely on the decline, is seen in others like astronomy. Suppose a hapless, ancient scientist records the force between two charged objects, measured as 8.37×10^7 dynes. A dyne is equivalent to one gram times one centimeter per second squared ($g \times \frac{cm}{s^2}$).

a) Convert this measurement to Newtons.

b) We wish to determine the work done by this force if it acts on the object across a distance of 4.0×10^2 centimeters. Please write your answer in scientific notation in the SI (MKS) system of units. Be careful to calculate significant digits appropriately.

c) What were our spherical cows in part b of this question?

3.4.2 A Millikilofemtobarn of Crows

A barn is a unit of area covering 10^{-28} m^2. Given that a crow has a mass of 1.3 kg, a length of 0.6 m, and a width of 3.0×10^1 cm, how many barns do three crows cover? Assuming every crow is uniformly dense, how much mass of crow fits in a barn?

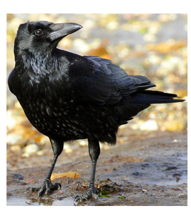

Figure 5: Corvus Corone. Image courtesy of Wikipedia. https://en.wikipedia.org/wiki/Crow

4 Trigonometry

I'll assume that you have reasonable knowledge from Precalculus this year. You'll need to know simple identities and the unit circle; you can definitely refresh using Khan Academy.

4.1 Problems

4.1.1 Ramp angles

In your free time this summer, you decide to build a skateboarding ramp to pull off some sweet tricks. You have a long sheet of plywood, of length L, and some cinder blocks with which to prop it up. You incline the ramp at angle θ.

a) What is the height of the ramp? How far horizontally does the ramp extend?

b) Assume now that the cinder blocks have dimensions $w = \frac{L}{8}, h = \frac{L}{8}, d = \frac{L}{4}$. Given one cinder block, how far from the base of the ramp should you place the block (and in what orientation) to reach the angle θ? At what point on the ramp does the cinder block hit the plywood?

c) Is this a stable configuration? If so, explain. If not, why not? Is another placement of the cinder block possible to reach a stable configuration?

d) Is there some angle $0 < \phi < 90$ that no placement of the block will properly allow? If so, please list limiting angles and other relevant information. If not, explain why not.

e) What are your spherical cows for this problem?

4.1.2 Skate Wars: A New Ramp

You've improved in your skills and have decided to invest in a prefabricated ramp of length L and base angle θ to avoid some of the issues (and injuries) you ran into when using your cinder block wonder. You'd like to test this higher quality ramp for both acceleration, friction, and acceptable load. As a prerequisite to this information, you have to figure out some reference angles. The colored lines on the ramp represent dowels used to aid in your calculations. The blue line below is perpendicular to the ramp surface, the purple line is parallel to the floor, and the green line is perpendicular to the floor.

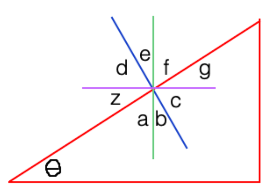

Figure 6: Problem 4.1.2. A ramp with dowels.

a) Find the measure of each lettered angle.

b) Is this ramp more likely to be an effective skateboarding ramp than the one from the previous problem? Why, or why not?

5 Vectors

5.1 Basic Concepts

Vectors will be the bread and butter of much of the work we do this year. One day you'll all better understand vectors (and vector spaces) through the mathematics of linear algebra, but for now we'll consider only some basic properties.

First, vectors can be thought of differently depending on the area of application. In computer science, we consider vectors to be lists of numbers. In physics, we consider vectors to be quantities that are able to be represented as arrows with a size and direction.

Figure 7: https://www.smbc-comics.com/comic/scalars

5.2 Magnitude-Angle Notation

In general, the most natural interpretation of this idea is to think about how big your quantity is and in which direction it acts. Because direction is easy to indicate using an angle in a standard coordinate reference frame (and you've had practice with this using trigonometry), we can clearly see that a 5N push of a sled along the y-axis (if we place an axis on top of the real world) could be represented as a force vector of 5N @ 90° from the +x-axis.

5.3 Component Notation

Using trigonometry, we can take some vector, say 89 kg m/s of momentum at 140° from the +x-axis and break it down into horizontal and vertical components. That is, if we shine a light on the vector perpendicular to the x-axis, what is the length of the shadow that is cast onto the axis? This would simply be a leg of the triangle. We can determine that the x-component of the above vector would be negative as the angle is in excess of 90°.
To find the x-component of the vector, we take $89cos(140)$.
To find the y-component of the vector, we take $89sin(140)$.

In general, with a reference angle θ from the +x-axis, and some vector of magnitude (length) R, we find that:

$$x = Rcos(\theta)$$
$$y = Rsin(\theta)$$

This is useful when decomposing any vector quantity (forces, velocities, accelerations, momenta, etc) into pieces along the axes so that mathematical operations then become trivial.

In general, we can consider some force vector $\vec{F}$ to have components along all relevant coordinate axes. You would have, for example, F_x, F_y, F_z as components of $\vec{F}$ in 3-dimensional space. I will calculate F_x and F_y below using the notation that $||\vec{F}||$ is the magnitude of $\vec{F}$, which is a vector at angle θ from the +x-axis.

$$F_x = ||\vec{F}||cos(\theta)$$
$$F_y = ||\vec{F}||sin(\theta)$$

We can recompose these into the original vector by using a combination of trigonometry and the pythagorean theorem. First, we can find the magnitude of $\vec{F}$ like so:
1) $F_x = ||\vec{F}||cos(\theta)$
2) $F_x^2 = ||\vec{F}||^2 cos^2(\theta)$ by squaring line 1 on both sides
and
3) $F_y = ||\vec{F}||sin(\theta)$
4) $F_y^2 = ||\vec{F}||^2 sin^2(\theta)$ by squaring line 3 on both sides
adding lines 2 and 4 together, we get:
5) $F_x^2 + F_y^2 = ||\vec{F}||^2 cos^2(\theta) + ||\vec{F}||^2 sin^2(\theta)$
6) $F_x^2 + F_y^2 = ||\vec{F}||^2 (cos^2(\theta) + sin^2(\theta))$ by factoring out $||\vec{F}||^2$
7) $F_x^2 + F_y^2 = ||\vec{F}||^2$ by simplifying through the pythagorean identity.
8) $\sqrt{F_x^2 + F_y^2} = ||\vec{F}||$

By a longer/similar process requiring two angles in 3-dimensional space, we can see that

$$\sqrt{F_x^2 + F_y^2 + F_z^2} = ||\vec{F}||$$

This derivation is left as an exercise for the reader.

Likewise, we can find the angle of the resultant vector $\vec{F}$ simply by knowing the values of F_x and F_y. Since these are legs of a right triangle with $\vec{F}$ as the hypotenuse,
$tan(\theta) = \frac{F_y}{F_x}$
$tan^{-1}(\frac{F_y}{F_x}) = \theta$.

You will see this approach again in many places, including a discussion of polar coordinates in mathematics. We will revisit these topics at a later time.

5.4 Mathematical Operations on Vectors

Our first lab will delve into the applications of vector operations to actual real-world physics. In general, however, there are several important mathematical operations that we need to consider.

First, we can add two vectors (or subtract two vectors) mathematically by adding (or subtracting) their components and recomposing into a new vector. For example, if $\vec{C} = \vec{A} + \vec{B}$, $C_x = A_x + B_x$ and $C_y = A_y + B_y$. Since we have the components of C, we can find the magnitude and angle of C by the results of the previous section. The same process works for subtraction.

Two vectors can only be added if their units match (as an indication of underlying similarity of concept). Would it make sense to add a distance to a time? A force to a momentum? What could this even mean in a physical setting?

We also have the ability to multiply a vector by a scalar value. For example, say a robot needs have a 50N force at 45° from the +x-axis applied to move forward. If the force were increased by a factor of 5, each force component would be multiplied by 5. This would have the effect of increasing the magnitude of the force to 250N, but would keep the angle at 45° from the +x-axis (try it on your own to validate). In general, this multiplication by a scalar will multiply all components by that same scalar. Note that this preserves units.

5.5 Further Vector Topics

Looking forward, we will also cover the two flavors of vector multiplication: the scalar product and the cross product. There will be a discussion of unit vectors and associated notation, along with some use of matrices. Please look at these topics ahead of time to familiarize yourself with the concepts. For a more comprehensive look at vectors, matrices, and linear algebra, check out this youtube playlist by Grant Sanderson from his channel 3Blue1Brown.

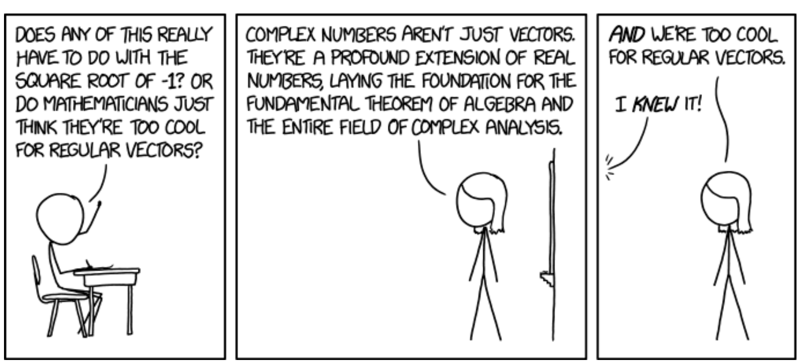

Figure 8: https://xkcd.com/2028/

5.6 Problems

5.6.1 Basketball Passing Drill

Three basketball players are passing a ball for the purpose of practicing a play. The first player imparts a momentum[†]of 80 kg m/s at an angle of 40° from the horizontal axis, measured as parallel to the home end of the court. The second player has to run and tips the ball toward the third player by adding 40 kg m/s of momentum at an angle of 120° to the horizontal axis.

a) In which direction and with how much momentum is the basketball now moving?

b) Suppose that, from the third player's position, the ball needs to leave her hands with a momentum of 100 kg m/s at an angle of 5° from the horizontal in order to make the shot. How much momentum should she add to the ball to score? Is this even possible?

5.6.2 Vectors in Conflict

If you have any two vectors that are noncollinear[‡], they will certainly define a plane in which they both rest. Suppose you have two such vectors. Determine a procedure to make a third vector such that:

a) The third vector never intersects with the plane above.

b) The third vector lies in the plane.

c) The third vector is perpendicular to the plane.

6 Basic Kinematics

6.1 Basic Ideas of Motion

Motion is something that is intuitive to us predators. It's often hard to describe things that we find intuitive, though it ends up being necessary in the study of physics.

First, we distinguish between things that are moving and things that are not moving. Consider a potato on a table. If you're shown two pictures of the potato, you will probably encounter one of the following situations:

1. The two pictures are identical, in that they are literally copies of each other. They were taken at the same time, from the same camera.

2. The two pictures were not taken at the same time, but nonetheless show the same image. The potato rests in the same location in the frame.

3. The two pictures show the background from the same perspective, but the potato is in a different location in the frame between the two pictures (or might be absent in one of the pictures).

[†]Momentum is a vector quantity.
[‡]do not lie on the same line

4. The two pictures show the background from different perspectives, and the potato might or might not be in the same location.

From these examples, we can discern some information about the nature of motion. First, any measurement of motion requires some investigation of time, location, and distance.

Second, it turns out that we have to consider reference frames when making sense of motion measurements and calculations. This means that it's important to keep track of where the camera is and how it is moving in our whole situation.

6.2 Position and Reference Frames

We'll define position to be the coordinates (the location) of an object in a given reference frame. For our purposes here, a reference frame is a set of axes that assigns a coordinate to every point in space and/or time. We typically use something called a right-handed coordinate system, which will come into play later.

Sometimes it's convenient to move our axes around. Any transformation you can do to a function (translation, rotation, reflection) can be done to a coordinate system as well. To simplify things, we will just consider translation (moving the whole system around, but not rotating or reflecting it). For this purpose, pay attention to the origin!

As an example, consider that a potato rests on a table. If we label the lower left corner of the table as the origin $(0, 0)$, then the potato might rest at $(4, 3)$.

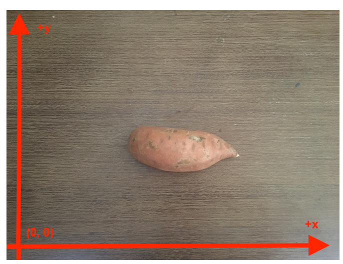

Figure 9: A sweet potato in a particular coordinate frame.

Now, let's move the axes around. Note that the origin now begins at the lower right corner of the table.

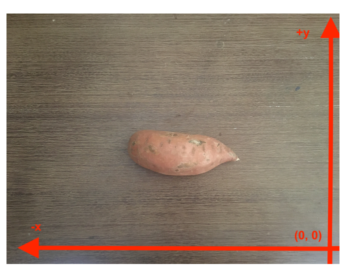

Figure 10: A sweet potato in a translated coordinate frame.

Thus, the new coordinates of the potato might be $(-4, 3)$, despite no movement on the part of the potato. You can think, in one sense, that we've translated the potato, but in reference to the background image, what we've actually done is translate the axes! Any transformation acting on a point (or set of points) can also be thought of as a transformation of a coordinate system. We'll find this useful later[§].

In general, we will consider position to be a vector quantity in multiple dimensions. Some object's position $\vec{A}$ would have components A_x, A_y, A_z in 3-D space. These would represent the object's location in reference to the origin.

6.3 Displacement and Distance

The definition of displacement is pretty obvious from the name. An object that has been displaced has been moved from its place. Displacement measures how much movement there's been, and in which direction. We can consider a displacement in many directions at once, or in one direction at a time. In three dimensions, component-wise:

$$\Delta x = x_1 - x_0$$
$$\Delta y = y_1 - y_0$$
$$\Delta z = z_1 - z_0$$

You may use my simple displacement demonstration here[¶] to get a better sense the difference between displacement and distance.

Clearly, we can do better by packing this information into one vector object, though it is definitely convenient to consider how components change individually. For now, assume that $\vec{b} = (x_1, y_1, z_1), \vec{a} = (x_0, y_0, z_0)$, and the displacement vector between those two locations, $\vec{s} = (x, y, z)$.

$$\vec{s} = \vec{b} - \vec{a}$$

Here, $\vec{s}$ is the displacement vector, while $\vec{a}$ and $\vec{b}$ represent the position of the object relative to the origin before and after the displacement.

Not only are you considering how much an object is moved when it is displaced, but also which way it moves.

Distance is the length of the journey a particle takes, and is therefore just a number. If I run around a track once, my distance traveled is 400m, but my displacement is 0. This path-length concept is useful, but will not be the major focus of our studies.

6.4 Velocity and Speed

Speed is defined to be distance covered per unit time. We won't spend an enormous amount of time considering speed or distance in this class, but it's important to understand that speed is a scalar value since both distance and time are scalar values. Speed has no direction, only magnitude. It can also be significantly different in magnitude from velocity due to the difference in definition between

[§] As an aside, there is a particular notation for evaluating positions in different reference frames, but is beyond the scope of summer work; we'll visit these ideas in the new school year.

[¶] https://www.openprocessing.org/sketch/751670; use the space, t, and a keys to show information, and the arrow keys to move. Further options are available in the comments of the code itself. Press r to reset.

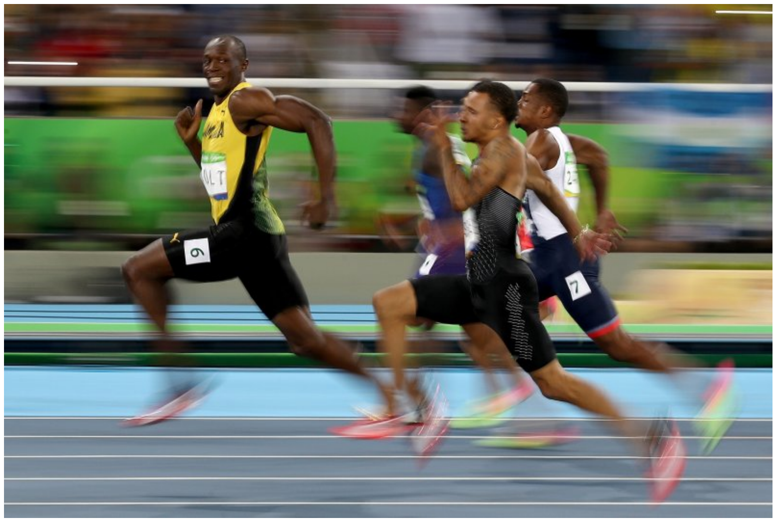

Figure 11: Usain Bolt, 2016 Olympics. Photo by Cameron Spencer, Getty Images. Taken from https://time.com/4452585/usain-bolt-2016-rio-olympics/.

displacement and distance.

Velocity is defined to be displacement per unit time.To be more precise, we should consider that we have two different flavors of velocity: average velocity and instantaneous velocity. Average velocity is the one you're used to. While the equation below considers only the x-component, the principle remains the same in multiple dimensions:

$$\overline{v}_x = \frac{\Delta x}{\Delta t}$$

Clearly, we see here that average velocity is displacement over time. We'll see later that instantaneous velocity is the time-derivative of position, but this is a topic for October. It is important to note that velocity takes into account direction, so velocity changes if direction changes but speed stays the same. This has implications for our definition of acceleration.

6.5 Acceleration

Acceleration is defined to be a change in velocity. An object accelerates when it turns, speeds up, or slows down. In general,

$$\vec{a} = \frac{\vec{v}}{t}$$

Where same caveats apply in relation to instantaneous and average values. Acceleration is the time-derivative of velocity.

If there is acceleration of an object, then its velocity is changing. If there is no acceleration, the velocity (both magnitude and direction) remains the same.

To further investigate these ideas, you should play around with my vector motion demonstration, linked below‖.

‖https://www.openprocessing.org/sketch/754789; use the space, t, d, and a keys to show information, and the arrow keys to move. Further options are available in the comments of the code itself. Press r to reset.

6.6 Problems

6.6.1 Mario Kart, But For Real

Sheik is accelerating down a long, straight hill on Rainbow Road of length L, starting from rest at the top after previously being knocked off the course. She makes it to the bottom in time t, right before she has to make a sharp turn.

a) Using only the definitions of displacement, average velocity, and acceleration, show that v_f, her final velocity, is $2\overline{v}$.

b) Show that her total displacement in that time, L, is equal to $\frac{at}{2}$.

c) What assumptions were made to derive these two equations?

d) If Policeman Toad is at the bottom and uses the derivations above to conclude that Sheik was going 1.5 times the speed limit when she hit the curve, does Sheik have a strong defense by discussing the assumptions in part c above? Why, or why not? What if he found she would be going 2 times the speed limit?

e) If the road were now curved (like going down to the bottom of a bowl from the top) instead of straight, would Sheik be able to make it to the bottom sooner? Justify your answer.

6.6.2 Cat and Mouse

A cat and a mouse agree to a game on a running track. The mouse will run along the track (which is composed of two straight sections and two curved sections), and the cat must run in a straight line to intercept it. Assume the 400m track is composed of two 31.8m radius semi-circles and two 100m straight sections. These are combined together into an "oval-ish" shape.

a) Assume the mouse starts off at a speed of 4.2 m/s. How fast must the cat move to catch the mouse at the 150 meter mark, assuming the starting line is the 0m mark?

b) Now, assume the mouse is accelerating at a rate of 1 m/s^2. How fast must the cat move to catch the mouse by the 200m mark?

c) Assume the cat has uniform acceleration in part b. What is the cat's final speed by the 200m mark?

7 Forces

This will be our first unit of the year. It's necessary to have a basic background in ideas of motion (kinematics) before moving forward here. We'll consider two types of systems this year: static systems, wherein there is no relative motion within the system; and dynamic systems, in which there is relative motion between components of the system.

Forces are interactions between objects. These interactions are either pushing interactions or pulling interactions. We'll only consider the origin of forces in pairs of objects. Imagine a single object in the universe. Can forces exist in such a circumstance (consider the object to be a single point)?

7.1 Newton's Laws of Motion

I'll refer to Newton's Laws of Motion by the abbreviations N1, N2, and N3. Herein are simplified versions that are more useful for conceptual understanding, but you are welcome to research the original wording if you prefer. I list them here in order of importance**.

N3. Every force is the result of an interaction between two objects. The force produced by the interaction acts on each object in opposite directions, but is the same size.

**https://en.wikipedia.org/wiki/Newton%27s_laws_of_motion contains a look at a translation of Newton's original Latin text.

N2. The net force (sum of all forces) acting on an object is equal to the product of its mass and its acceleration. In general,

$$\vec{F}_{net} = \Sigma\vec{F} = m\vec{a}$$

Note that $\vec{F}$ and $\vec{a}$ are vectors, denoted with an arrow above the symbol.

N1. A special case of N2. Law of inertia: objects are lazy. The more stuff they're made out of, the lazier they are, and thus the harder they are to change the motion of. No force means no acceleration, and vice versa. An object doesn't change its velocity (no turning, speeding up, or slowing down) in the absence of a net force or, alternately, an object's velocity only changes if a nonzero net force is applied. Symbolically,

$$\vec{a} = \vec{0} \Leftrightarrow \vec{F}_{net} = \vec{0}$$

The $\Leftrightarrow$ symbol means "if and only if" and can also be called the biconditional symbol. Also note that I use $\vec{0}$ here, which is really only something you'll see in mathematics; here, it indicates that all components are zero.

In general, we can get an idea of equilibrium from N1, which is really only a special case of N2. Equilibrium happens when the net force on an object is zero. An object can be moving in a reference frame and in equilibrium. We call this dynamic equilibrium. An object can also be staying still in a reference frame; we call this state static equilibrium.

N2 (and, by extension, N1) requires that a reference frame must be inertial (non-accelerating). We can have our camera move at a constant rate and N1 still applies, but we have to take extra considerations if our camera is accelerating. We'll discuss the implications of this later on.

7.2 Some Particular Forces

There are four fundamental forces in the universe, based on our current understanding: the force of gravity, the weak nuclear force (responsible for radioactive decay of atoms), the strong nuclear force (responsible for maintaining the structure of the nucleus), and the electromagnetic force. Of these, our main focus will be the force of gravity in the purely Newtonian sense. However, many of the interactions we discuss are also dependent directly on the electromagnetic force, so we must take this into consideration as well. You'll need a basic sense of Coulomb's Law and chemical bonding to really get the connections.

Some common forces that you will encounter in this course are listed in the table below, along with relevant symbols.

Force name	Common Symbols	Mediating Fundamental Force
Normal force	F_N, N	electromagnetic force
Gravity	F_g, W	gravitational force
Tension	F_T, T	electromagnetic force
Spring force	F_s	electromagnetic force
Friction	f, F_f, F_s, F_k	electromagnetic force

Table 4: Particular forces

7.3 Free Body Diagrams

You should recall how to use free body diagrams from your previous physics course. In general, they are useful tools for evaluating physics problems in this class. You should label these with only actual forces, not components.

In a free body diagram, a dot represents the object, while each force acting on the object is drawn as an arrow beginning on and pointing away from that dot. Label all forces using symbols above. You can draw components using a dotted line.

7.4 Problems

7.4.1 Raising the Roof

You're at a party when, all of a sudden, the room begins to shake uncontrollably. It turns out, after checking around, that the music definitely doesn't hit that hard, nor is the dancing particularly hopping. As the pillars in the corners of the room start to visibly crumble, everyone rushes to support the falling ceiling. Despite your best efforts, you and your hundred closest friends are having trouble raising the roof.

a) The ceiling, while not stopping, seems to have at least slowed to a constant rate. You estimate that each person pushes with an average of 200N of force upward. How massive is the ceiling?

b) Assume that Raise the Roof starts playing, giving you a burst of energy and increasing the force output of the group by 50%. How fast does the ceiling now accelerate?

c) Now assume the fire department has attached a crane to the top of the ceiling. How much upward force must it apply before every party-goer can escape? What kind of force is this?

7.4.2 We All Rise Together

You and your friends decide to jump into a new amusement park after your close encounter with the ceiling. The ride, which looks like a long bench attached to two elastic cords, is aptly named "The Slingshot." You are assured by the park employee that it's perfectly safe.

a) The two cords are attached at angle θ. You, your friends, and the bench all collectively have mass M. The bench is held in place by a hook attached to a trigger mechanism. Draw a free body diagram of the forces acting on the bench when the mechanism and elastic cords are waiting to launch?

b) The trigger is activated and the hook detaches. Draw a new free body diagram. What are the magnitude and direction of the net force on the bench? What is the acceleration vector of the bench (with y'all still on it)?

c) What were the spherical cows present in parts a and b?