AP Calculus BC Summer Review Packet

This packet is a review of information you learned in your previous math courses and are needed to be successful in AP Calculus BC. Therefore, this packet is due at the end of **CALCULUS BC Boot Camp**. It is to be completed CORRECTLY, NEATLY, and on a SEPARATE sheet of paper.

Your Calculus teacher will collect your work at the end of **CALCULUS BC Boot Camp**. Failure to turn in your completed work at the end of **CALCULUS BC Boot Camp** may jeopardize your ability to remain in the course.
(The problems that warrant the use of a graphing calculator are indicated with a 🖩.)

Trigonometry

I. Without using a calculator, determine the exact value of each expression. YOU ARE EXPECTED KNOW THE UNIT CIRCLE! Practice at: www.kwarp.com/portfolio/trigspinner.html

1. $\sin \pi$ **2.** $\cos \frac{\pi}{4}$ **3.** $\tan \frac{5\pi}{6}$ **4.** $\sec \frac{4\pi}{3}$ **5.** $\sin^{-1}\left(\tan \frac{3\pi}{4}\right)$

II. Trigonometric Identities

6. Write the 3 Pythagorean Identities

7. Write the 3 double angle identities for cosine: $\cos 2x = ?$

8. Write the double-angle identity for sine: $\sin 2x = ?$

III. Simplify.

9. $\frac{\sin 2x}{\sin x}$ **10.** $\sin^2 x + \cot^2 x \sin^2 x$ **11.** $\csc x - \cos^2 x \csc x$ **12.** $\frac{\sin^2 x + \sin x - 6}{\sin x + 3}$

IV. Solve.

13. Solve for m: $m \cos x + m \sin x = \cos 2x$

14. Solve for x: $\cos x - \sin x = 0,\ 0 \le x \le \frac{\pi}{2}$

15. Solve for x: $\cos 2x = \sin x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

16. Solve for x: $\sec^2 x - \tan^2 x = \sin x,\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Solving & Simplifying Equations

I. Solve for *m*.

17. $2x + 8ym = 0$ **18.** $m = (y + xm)(1 + y)$

II. Solve for *x*, where *x* is a real number. Show the work that leads to your solution.

19. $x^2 - 7 = 2x + 8$ **20.** $\frac{4x^2 - 9}{x} = 0$ **21.** $(x + 3)^2 = 16$ **22.** $3x^2 + 10x = 8$

23. $(x + 3)^2(x - 1) + (x + 3)(x - 1)^2 = 0$

III. Simplify each expression.

24. $\frac{2x^2 - x - 3}{x + 1}$ **25.** $\frac{x^2 - 5x + 4}{x^2 - 2x - 8}$ **26.** $\frac{x^2 + 3x}{9 - x^2}$ **27.** $\frac{\frac{1}{x}}{\frac{1}{x^2 + 2x}}$ **28.** $\frac{x+1}{8x} \cdot \frac{4}{x^2 + 3x + 2}$

29. $\frac{1}{x} - \frac{1}{x+2}$ **30.** $\frac{2x}{x-1} + \frac{4}{x^2 - 4x + 3}$ **31.** $\frac{\frac{1}{x+2} - \frac{1}{2}}{x}$ **32.** $27^{2/3}$ **33.** $-4^{3/2}$

34. $\frac{8x^2y^{-1}}{2x^{-5}y}$ **35.** $(4x^{-5}yz^2)^0$ **36.** $(8x^{-3}y^3z^6)^{1/3}$ **37.** $(x^3y^{-1})(x^4y^2)$ **38.** $\ln 1$

39. $\ln 0$ **40.** $\ln e^2$ **41.** $e^{\ln 5}$ **42.** $e^{\ln 4+\ln 7}$ **43.** $e^{2\ln 3}$

44. $e^{\ln 4-\ln 3}$ **45.** $\log_5 25$ **46.** $\log_5\left(\frac{1}{5}\right)$ **47.** $\log_5\left(\sqrt[3]{5}\right)$

IV. Solve for *x*, where *x* is a real number. Show the work that leads to your solution.

48. $e^{2x} = 5$ **49.** $e^{3x} = -2$ **50.** $\ln 3x = -2$ **51.** $\log_2(x+3) + \log_2 x = 2$

Miscellaneous

I. Follow the directions for each problem. Show all computations that lead to your answer.

52. Given $f(x) = e^{-x}$, find $f(\ln 3)$.

53. Given $f(x) = \frac{1}{x} - \frac{1}{x+2}$, find $f(3)$.

54. Given $f(x) = 2|x-1| + 5$, find $f(0) - f(3)$.

55. Given $f(x) = x^2 + 2x - 3$, find $f(x+4) - f(4)$.

56. Find $f(x+h)$ for $f(x) = 2x^2 + 3x - 1$.

57. Find $\frac{f(x+h)-f(x)}{h}$ for $f(x) = x^2 + 3x - 2$.

58. Given $h(x) = f(x) - g(x)$, $f(x) = x + \frac{1}{x+1}$, and $g(x) = \frac{1}{x} + 2x$, find $h(2)$.

59. Let $f(x) = \begin{cases} 2x+7, & x < 2 \\ -x^2, & x \geq 2 \end{cases}$. Find $f(-3)$ and $f(5)$.

60. Let $f(x) = \begin{cases} x^{3/2}, & 0 \leq x < 5 \\ \ln 2x, & x \geq 5 \end{cases}$. Find $f(4)$ and $f(e^3)$.

61. Given $f(x) = 3x - 6$ and $g(x) = x^2 - 4$, find $f(x)g(x)$, $\frac{f(x)}{g(x)}$, $(g \circ f)(x)$, and $(f \circ g)(x)$.

62. Given $f(x) = 3x - 6$, find $f^{-1}(x)$.

63. Find the x-intercept(s): $4x^2 - 3y^2 + 2xy - 12 = 0$

64. Find the x-intercept(s) for $f(x) = -e^x + 2$.

65. Using a graphing calculator, find all point(s) of intersection of the graphs of $y = x^3 - 2x^2 + x - 1$ and $y = -x^2 + 3x - 2$. Round your answer to the nearest thousandths.

66. If the point $\left(4, -\frac{1}{3}\right)$ lies on the graph of the equation $2x + cy = 11$, find the value of c.

67. ▦ A drainpipe can hold 50 cubic feet of water before overflowing. There is 30 cubic feet of water in the pipe at time $t = 0$, where t is measured in hours. The amount of water that flows into the pipe is represented by the function $R(t) = 20 \sin\left(\frac{t}{25}\right)$ and the amount of water that drains out of the pipe is represented by the function $D(t) = -.01t^4 + 0.15t^3 + 0.48t^2$. Find the time, to the nearest thousandths, when the drainpipe overflows.

II. Using the point-slope form: $y - y_1 = m(x - x_1)$, write an equation for the line:

68. with slope -2, containing the point $(4,1)$

69. containing the points $(2, -3)$ and $(-5,6)$

70. parallel to $5x - 2y = 7$ and passing through $(3,2)$

71. perpendicular to the line $5x - 2y = 7$ and passing through $(3,2)$

Graphs & Limits

I. Use the given graph

72. Write the equation of the tangent line in point-slope form from the graph in Figure 1.

73. The graph in Figure 2 consists of line segments and a semi-circle. Find the area between the graph and the x-axis on the interval $-3 \le x \le 1$.

74. The graph in Figure 2 consists of line segments and a semi-circle. Find the area between the graph and the x-axis on the interval $-4 \le x \le 2$.

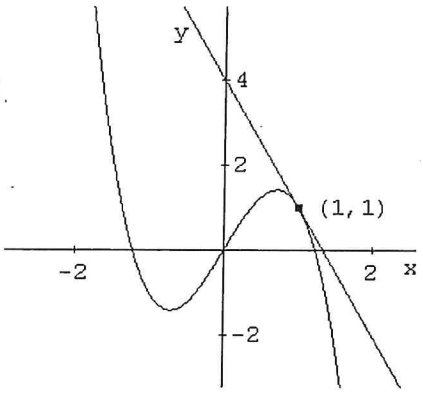


Figure 1

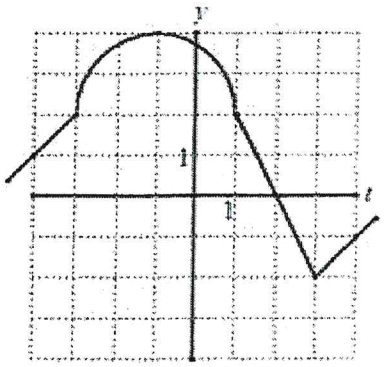


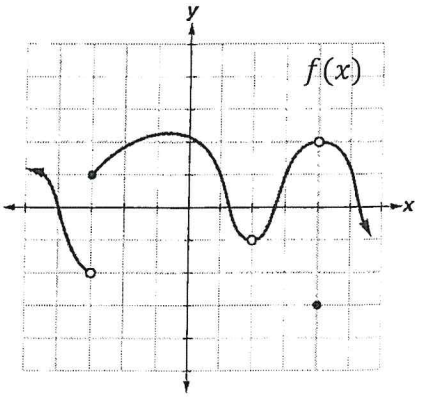
Figure 2

II. Identify the domain and range of the following equations.
(Keep in mind you should know your parent functions without a calculator.)

75. $y = x^3 - 2x^2 - 3x$ **76.** $y = x^2 + 3x - 10$ **77.** $y = e^x + 3$ **78.** $y = \ln(x - 1)$

79. $y = \sqrt{x + 2} - 1$ **80.** $y = |x - 2| + 3$ **81.** $y = \frac{1}{x}$ **82.** $f(x) = \begin{cases} -x + 3 & x < 0 \\ x^2 - 1 & x \ge 0 \end{cases}$

III. Use the drawing to the right to find the each limit/value.



83. $\lim_{x \to -3^-} f(x)$ **84.** $\lim_{x \to -3^+} f(x)$ **85.** $\lim_{x \to -3} f(x)$ **86.** $f(-3)$

87. $\lim_{x \to 2^-} f(x)$ **88.** $\lim_{x \to 2^+} f(x)$ **89.** $\lim_{x \to 2} f(x)$ **90.** $f(2)$

91. $\lim_{x \to 4^-} f(x)$ **92.** $\lim_{x \to 4^+} f(x)$ **93.** $\lim_{x \to 4} f(x)$ **94.** $f(4)$

IV. Find the limits algebraically.

95. $f(x) = \begin{cases} -x + 3 & x < 0 \\ x^2 - 1 & x \ge 0 \end{cases}$. Find $\lim_{x \to 0^-} f(x)$, $\lim_{x \to 0^+} f(x)$, and $\lim_{x \to 0} f(x)$ algebraically.

96. Find $f(x) = 3x + 2$, $g(x) = \frac{x^2 - 3x + 2}{x - 1}$. Find $\lim_{x \to 1} f(x)$ and $\lim_{x \to 1} g(x)$ algebraically.

97. ▦ Find $\lim_{x \to -5} \frac{x + 5}{\sqrt{x + 9} - 2}$ algebraically and numerically.

XV

1. Draw a representation of a unit circle in the center of a **full sheet of GRAPH paper (a link is available on the same site from which you downloaded this)** using a compass or something round. Draw a coordinate system with the origin at the center of the circle using a straight edge. Draw, with a protractor, all angles that are multiples of $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ and $\frac{\pi}{2}$ radians going once around the circle in standard position. For each angle θ, state the first two positive and the first two negative values of θ having that terminal side along with the $\sin\theta$, $\cos\theta$, $\tan\theta$ *and* $\sec\theta$. EXAMPLE: for the first positive angle you would write $\theta = \frac{\pi}{6}, \frac{13\pi}{6}, -\frac{11\pi}{6}, -\frac{23\pi}{6}, \sin\theta = \frac{1}{2}, \cos\theta = \ldots\ldots$ etc. This should be done carefully and ***from memory***, do NOT look these up! These need to be part of your general knowledge for this year! Look to see how well your values agree with the coordinates on the graph paper.

XVI

From Memory, without a calculator, state the exact values of the following:

1. a: $\sin\frac{17\pi}{3} =$ b: $\tan\frac{43\pi}{6} =$ c: $\cos\frac{97\pi}{4} =$ d: $\sin(-\frac{55\pi}{3}) =$

e: $\cos(-\frac{71\pi}{6}) =$ f: $\cot(-\frac{213\pi}{4}) =$ g: $\sec(-\frac{137\pi}{3}) =$ h: $\csc\frac{1000\pi}{6} =$

i: $\tan^{-1}(\cot(-\frac{13\pi}{4})) =$ j: $\sin(\cot^{-1}(-\sqrt{3})) =$ k: $\csc(\cos^{-1}(-\frac{\sqrt{3}}{2})) =$ l: $\sin^{-1}(\frac{\sqrt[3]{65}}{4}) =$

XVII

Suppose that the trigonometric functions were defined in exactly the way you have learned based on the unit circle **BUT** instead were based on a **UNIT SQUARE** instead of a unit circle. A unit square is ONE unit on each side, centered on the origin with sides parallel to the axes. With this **one** modification, what would the values of these trig functions now be? (NO Calculator)

1. $\sin\frac{\pi}{4} =$ 2. $\tan\frac{\pi}{4} =$ 3. $\cos\frac{3\pi}{4} =$ 4. $\csc(-\frac{19\pi}{4})$ 5. $\sin 30^o =$

6. $\cos(120^o) =$ 7. $\tan(-210^o) =$ 8. $\csc(1260^o) =$ 9. $\cos(43.72198^0) =$

10. $\sin(-948.6671^{0)}) =$ 11. $\sin^2 315^o + \cos^2 315 =$

Check your answers carefully and question anything for which you may need a reminder by emailing me.