AP CALCULUS SUMMER REVIEW BC

Welcome to AP Calculus! These questions are intended as review (with one or two exceptions) and are due to me by email on or before **AUGUST 31.**

If a particular topic is unknown or you have forgotten details email me immediately so that I may explain and/or give hints.

ASK ALL YOUR QUESTIONS BEFORE AUGUST 31. DO NOT hesitate, for any reason, to ask about any of these important ideas via email.

SHOW YOUR WORK IN AN ORGANIZED FASHION WITH PROBLEMS IN NUMERICAL ORDER. You may use a calculator only on problems marked with an $*$.

If you have a problem partially done and cannot finish then scan it and email it to me and I will supply hints.

If you find you are spending too much time on these or can't get started on a particular type, PLEASE ASK for assistance! I AM HERE TO HELP.

ANSWERS YOU GIVE ON THE QUESTION SHEET ARE IGNORED.

I Simplify 1-4. Show the work that leads to your answer.

1. $\dfrac{x-4}{x^2-3x-4}$
2. $\dfrac{x^3-8}{x-2}$
3. $\dfrac{5-x}{x^2-25}$
4. $\dfrac{x^2-4x-32}{x^2-16}$

5. Expand $x^{\frac{3}{2}}(x+x^{\frac{5}{2}}-x^2)$

6. Expand and then evaluate the sum $\displaystyle\sum_{n=0}^{5}\dfrac{(n-1)^2}{2}$

7. Factor and answer with only one negative exponent: $\quad 7x^{-5}+4x^{-3}+2x^3$

8. Factor and answer with only one negative exponent: $\quad 2x^{-\frac{3}{4}}+8x^{-\frac{7}{4}}-6x^{\frac{1}{4}}$

II

1. Is $\left\{(x,y): y=\sqrt{4-x^2}\right\}$ a function and if so, what is its domain?

2. Given $F(x)=\sqrt{x+9}$ find and simplify $\dfrac{F(x+h)-F(x)}{h}, h\neq 0$

3. Sketch the graph and determine its **DOMAIN**:

   a: $\dfrac{(x+3)(x+2)}{(x-4)(x+1)}$
   b: $f(x)=\dfrac{x^2-25}{x+5}$
   c: $f(x)=\dfrac{(x^2-4)(x-3)}{x^2-x-6}$

Include the DOMAIN and RANGE also for the rest:

d: $f(x) = \begin{cases} 9 - x^2 & x \neq -3 \\ 10 & x = -3 \end{cases}$ e: $f(x) = \lfloor x \rfloor$ ($\lfloor x \rfloor$ means the greatest integer less than or equal to x)

f: $f(x) = 3\lfloor 2x \rfloor$ g: $f(x) = \begin{cases} x + 3, & x < -5 \\ \sqrt{25 - x^2}, & -5 \leq x \leq 5 \\ 3 - x, & x > 5 \end{cases}$ h. $f(x) = \lfloor x - 4 \rfloor$

4. The **unit step function** is defined to be $U(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$. Sketch the graph of

   a: U(x)   b: $xU(x)$ c: $(x + 1)U(x + 1) - xU(x)$

5. The **signum function** is defined to be : $\text{sgn}(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$. For each function below, sketch the graph.

   a: x sgn(x)   b: $x - 2\,\text{sgn}(x)$

6. Write this function as a piecewise function (like 3d or 3g) **without** using the absolute value symbol: $|x^2 - 1|$. Sketch a graph of this function.

III

1. If $f(x) = \dfrac{x + 1}{x - 1}$ and $g(x) = \dfrac{1}{x}$, write a **formula** for and give the **domain** of:

   a: $f \cdot g$ b: $f \circ g$ c: $f \circ f$

2. If $f(x) = \sqrt{x - 2}$ and $g(x) = x^2 - 2$, write a **formula** for and give the **domain** of:

   a: $f \circ g$ b: $g \circ f$ c: $f \circ f$

3. Express $h(x) = \sqrt{x^2 - 4}$ as the composition of two functions, f and g $(f \circ g)(x)$, in **THREE** different ways.

4. Is $f(x) = \sqrt[3]{x}$ even, odd or neither? Prove analytically (algebraically) using the definition (**NOT** graphically/geometrically). Look up the definition if necessary.

5. Is $f(x) = \dfrac{x^2 - 5}{2x^3 + x}$ even, odd or neither? Prove analytically (**NOT** graphically).

6. Write the function $f(x) = |x - 2| - |x + 2|$ without using absolute value bars AND state whether f(x) is even, odd or neither with a defense for your answer.

FUNCTIONS

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read "*f* of *g* of *x*" Means to plug the inside function (in this case $g(x)$) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find $f(g(x))$.

$$f(g(x)) = f(x - 4)$$
$$= 2(x - 4)^2 + 1$$
$$= 2(x^2 - 8x + 16) + 1$$
$$= 2x^2 - 16x + 32 + 1$$
$$f(g(x)) = 2x^2 - 16x + 33$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

1. $f(2) =$ _____

2. $g(-3) =$ _____

3. $f(t + 1) =$ _____

4. $f[g(-2)] =$ _____

5. $g[f(m + 2)] =$ _____

6. $[f(x)]^2 - 2g(x) =$ _____

Let $f(x) = \sin(2x)$ **Find each exactly.**

7. $f\left(\dfrac{\pi}{4}\right) =$ _____
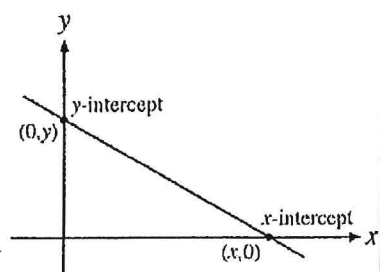
8. $f\left(\dfrac{2\pi}{3}\right) =$ _____

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$. **Find each.**

9. $h[f(-2)] =$ _____

10. $f[g(x - 1)] =$ _____

11. $g[h(x^3)] =$ _____

INTERCEPTS OF A GRAPH

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: Given the function $y = x^2 - 2x - 3$, find all intercepts.

$x-$int. $(Let\ y = 0)$
$0 = x^2 - 2x - 3$
$0 = (x-3)(x+1)$
$x = -1\ or\ x = 3$
$x-$intercepts $(-1,0)\ and\ (3,0)$

$y-$int. $(Let\ x = 0)$
$y = 0^2 - 2(0) - 3$
$y = -3$
$y-$intercept $(0,-3)$

Find the x and y intercepts for each.

12. $y = 2x - 5$

13. $y = x^2 + x - 2$

14. $y = x\sqrt{16 - x^2}$

15. $y^2 = x^3 - 4x$

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POINTS OF INTERSECTION

Use substitution or elimination method to solve the system of equations.
Remember: You are finding a POINT OF INTERSECTION so your answer is an ordered pair.

At the point of intersection (x, y) coordinates are the same for each line.
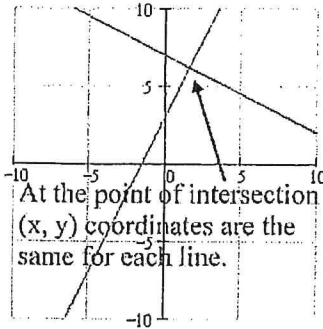
CALCULATOR TIP

Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2nd Trace) and hit INTERSECT.

Example: Find all points of intersection of $\begin{array}{l} x^2 - y = 3 \\ x - y = 1 \end{array}$

ELIMINATION METHOD

Subtract to eliminate y

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

Plug in $x = 2$ and $x = -1$ to find y

Points of Intersection: $(2,1)$ and $(-1,-2)$

SUBSTITUTION METHOD

Solve one equation for one variable.

$y = x^2 - 3$

$y = x - 1$

Therefore by substitution $x^2 - 3 = x - 1$

$x^2 - x - 2 = 0$

From here it is the same as the other example

Find the point(s) of intersection of the graphs for the given equations.

16. $\begin{array}{l} x + y = 8 \\ 4x - y = 7 \end{array}$

17. $\begin{array}{l} x^2 + y = 6 \\ x + y = 4 \end{array}$
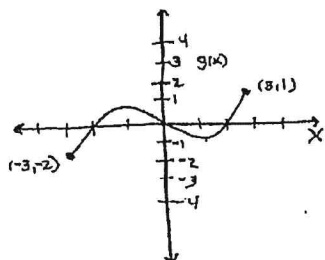
18. $\begin{array}{l} x = 3 - y^2 \\ y = x - 1 \end{array}$

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DOMAIN AND RANGE

Domain — All x values for which a function is defined (input values)
Range — Possible y or Output values

EXAMPLE 1

a) Find Domain & Range of $g(x)$.

The domain is the set of inputs (out) of the function.
Input values run along the horizontal axis.
The furthest left input value associated with a pt. on the graph is -3. The furthest right input values associated with a pt. on the graph is 3.
So Domain is [-3,3], that is all reals from -3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis.
The lowest output value of the function is -2. The highest is 1. So the range is [-2,1], all reals from -2 to 1.

EXAMPLE 2

Find the domain and range of $f(x) = \sqrt{4 - x^2}$
Write answers in interval notation.

DOMAIN
For $f(x)$ to be defined $4 - x^2 \geq 0$.
This is true when $-2 \leq x \leq 2$
Domain: $[-2, 2]$

RANGE
The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.
Range: $[0, \infty)$

Find the domain and range of each function. Write your answer in INTERVAL notation.

19. $f(x) = x^2 - 5$

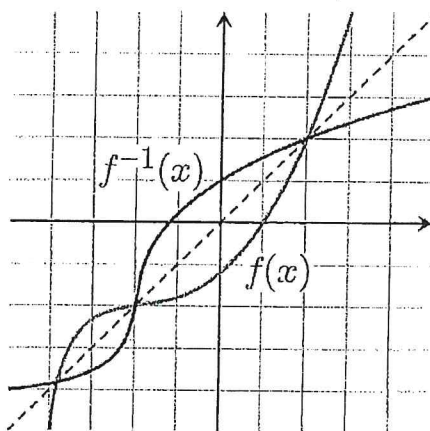20. $f(x) = -\sqrt{x + 3}$

21. $f(x) = 3 \sin x$

22. $f(x) = \dfrac{2}{x - 1}$

INVERSES

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value. Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x) = \sqrt[3]{x+1}$ Rewrite f(x) as y

$y = \sqrt[3]{x+1}$ Switch x and y

$x = \sqrt[3]{y+1}$ Solve for your new y

$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$ Cube both sides

$x^3 = y+1$ Simplify

$y = x^3 - 1$ Solve for y

$f^{-1}(x) = x^3 - 1$ Rewrite in inverse notation

Find the inverse for each function.

23. $f(x) = 2x + 1$

24. $f(x) = \dfrac{x^2}{3}$

25. $g(x) = \dfrac{5}{x-2}$

26. $y = \sqrt{4-x} + 1$

27. If the graph of $f(x)$ has the point (2, 7) then what is one point that will be on the graph of $f^{-1}(x)$?

28. Explain how the graphs of $f(x)$ and $f^{-1}(x)$ compare.

EQUATION OF A LINE

Slope intercept form: $y = mx + b$

Vertical line: $x = c$ (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$
* LEARN! We will use this formula frequently!

Horizontal line: $y = c$ (slope is 0)

Example: Write a linear equation that has a slope of ½ and passes through the point (2, -6)

Slope intercept form

$y = \dfrac{1}{2}x + b$ Plug in ½ for m

$-6 = \dfrac{1}{2}(2) + b$ Plug in the given ordered

$b = -7$ Solve for b

$y = \dfrac{1}{2}x - 7$

Point-slope form

$y + 6 = \dfrac{1}{2}(x - 2)$ Plug in all variables

$y = \dfrac{1}{2}x - 7$ Solve for y

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29. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

30. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

31. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

32. Use point-slope form to find a line passing through the point (2, 8) and parallel to the line $y = \dfrac{5}{6}x - 1$.

33. Use point-slope form to find a line perpendicular to $y = -2x + 9$ passing through the point (4, 7).

34. Find the equation of a line passing through the points (-3, 6) and (1, 2).

35. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3)

UNIT CIRCLE
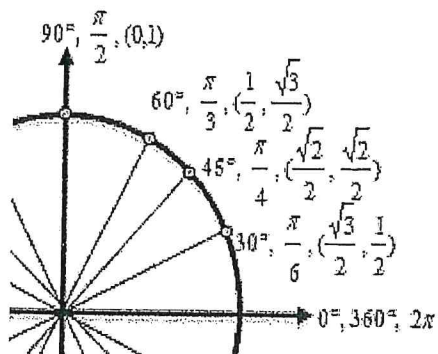
You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as sin/cos or the slope of the line.

Examples:

$$\sin\frac{\pi}{2}=1 \qquad \cos\frac{\pi}{2}=0 \qquad \tan\frac{\pi}{2}=und$$

*You must have these memorized OR know how to calculate their values without the use of a calculator.

36.   a.) $\sin\pi$ b.) $\cos\dfrac{3\pi}{2}$ c.) $\sin\left(-\dfrac{\pi}{2}\right)$ d.) $\sin\left(\dfrac{5\pi}{4}\right)$

    e.) $\cos\dfrac{\pi}{4}$ f.) $\cos(-\pi)$ g) $\cos\dfrac{\pi}{3}$ h) $\sin\dfrac{5\pi}{6}$

    i) $\cos\dfrac{2\pi}{3}$ j) $\tan\dfrac{\pi}{4}$ k) $\tan\pi$ l) $\tan\dfrac{\pi}{3}$

    m) $\cos\dfrac{4\pi}{3}$ n) $\sin\dfrac{11\pi}{6}$ o) $\tan\dfrac{7\pi}{4}$ p) $\sin\left(-\dfrac{\pi}{6}\right)$

TRIGONOMETRIC EQUATIONS

Solve each of the equations for $0 \leq x < 2\pi$.

37. $\sin x = -\dfrac{1}{2}$

38. $2\cos x = \sqrt{3}$

39. $4\sin^2 x = 3$

     **Recall $\sin^2 x = (\sin x)^2$

     **Recall if $x^2 = 25$ then $x = \pm 5$

40. $2\cos^2 x - 1 - \cos x = 0$ *Factor

TRANSFORMATION OF FUNCTIONS

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x - c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x + c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

41. Given $f(x) = x^2$ and $g(x) = (x - 3)^2 + 1$. How the does the graph of g(x) differ from f(x)?

42. Write an equation for the function that has the shape of $f(x) = x^3$ but moved six units to the left and reflected over the x-axis.

43. If the ordered pair (2, 4) is on the graph of $f(x)$, find one ordered pair that will be on the following functions:

     a) $f(x) - 3$ b) $f(x - 3)$ c) $2f(x)$ d) $f(x - 2) + 1$ e) $-f(x)$

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VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote <u>given the numerator does not equal 0 also</u> (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y = \dfrac{1}{x-2}$

Since when $x = 2$ the function is in the form 1/0 then the vertical line $x = 2$ is a vertical asymptote of the function.
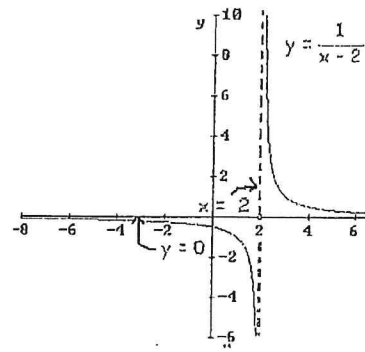
44. $f(x) = \dfrac{1}{x^2}$

45. $f(x) = \dfrac{x^2}{x^2 - 4}$

46. $f(x) = \dfrac{2+x}{x^2(1-x)}$

47. $f(x) = \dfrac{4-x}{x^2 - 16}$

48. $f(x) = \dfrac{x-1}{x^2 + x - 2}$

49. $f(x) = \dfrac{5x + 20}{x^2 - 16}$

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HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y = \dfrac{1}{x-1}$ (As x becomes very large or very negative the value of this function will

approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y = \dfrac{2x^2 + x - 1}{3x^2 + 4}$ (As x becomes very large or very negative the value of this function will

approach 2/3). Thus there is a horizontal asymptote at $y = \dfrac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \dfrac{2x^2 + x - 1}{3x - 3}$ (As x becomes very large the value of the function will continue to increase

and as x becomes very negative the value of the function will also become more negative).

Determine all Horizontal Asymptotes.

50. $f(x) = \dfrac{x^2 - 2x + 1}{x^3 + x - 7}$

51. $f(x) = \dfrac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

52. $f(x) = \dfrac{4x^2}{3x^2 - 7}$

53. $f(x) = \dfrac{(2x - 5)^2}{x^2 - x}$

54. $f(x) = \dfrac{-3x + 1}{\sqrt{x^2 + x}}$ * Remember $\sqrt{x^2} = \pm x$

This is very important in the use of limits.

EXPONENTIAL FUNCTIONS

Example: Solve for x

$$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$$

$$\left(2^2\right)^{x+1} = \left(2^{-1}\right)^{3x-2} \qquad \text{Get a common base}$$

$$2^{2x+2} = 2^{-3x+2} \qquad \text{Simplify}$$

$$2x + 2 = -3x + 2 \qquad \text{Set exponents equal}$$

$$x = 0 \qquad \text{Solve for x}$$

Solve for x:

55. $3^{3x+5} = 9^{2x+1}$

56. $\left(\frac{1}{9}\right)^x = 27^{2x+4}$

57. $\left(\frac{1}{6}\right)^x = 216$

LOGARITHMS

The statement $y = b^x$ can be written as $x = \log_b y$. They mean the same thing.
REMEMBER: A LOGARITHM IS AN EXPONENT

Recall $\ln x = \log_e x$

The value of e is 2.718281828... or $\lim\limits_{x \to \infty}\left(1 + \frac{1}{x}\right)^x$

Evaluate the following logarithms

58. $\log_7 7$

59. $\log_3 27$

Example: Evaluate the following logarithms

$\log_2 8 = ?$

In exponential for this is $2^? = 8$
Therefore $? = 3$
Thus $\log_2 8 = 3$

60. $\log_2 \dfrac{1}{32}$

61. $\log_{25} 5$

62. $\log_9 1$

63. $\log_4 8$

64. $\ln \sqrt{e}$

65. $\ln \dfrac{1}{e}$

PROPERTIES OF LOGARITHMS

$$\log_b xy = \log_b x + \log_b y \qquad \log_b \frac{x}{y} = \log_b x - \log_b y \qquad \log_b x^y = y \log_b x \qquad b^{\log_b x} = x$$

Examples:

Expand $\log_4 16x$	Condense $\ln y - 2\ln R$	Expand $\log_2 7x^5$
$\log_4 16 + \log_4 x$	$\ln y - \ln R^2$	$\log_2 7 + \log_2 x^5$
$2 + \log_4 x$	$\ln \frac{y}{R^2}$	$\log_2 7 + 5\log_2 x$

Use the properties of logarithms to evaluate the following

66. $\log_2 2^5$

67. $\ln e^3$

68. $\log_2 8^3$

69. $\log_3 \sqrt[5]{9}$

70. $2^{\log_2 10}$

71. $e^{\ln 8}$

72. $9\ln e^2$

73. $\log_9 9^3$

74. $\log_{10} 25 + \log_{10} 4$

75. $\log_2 40 - \log_2 5$

76. $\log_2 \left(\sqrt{2}\right)^5$

EVEN AND ODD FUNCTIONS

Recall:

Even functions are functions that are symmetric over the y-axis.

    To determine algebraically we find out if $f(x) = f(-x)$

    (*Think about it what happens to the coordinate $(x, f(x))$ when reflected across the y-axis*)

Odd functions are functions that are symmetric about the origin.

    To determind algebraically we find out if $f(-x) = -f(x)$

    (*Think about it what happens to the coordinate $(x, f(x))$ when reflected over the origin*)

State whether the following graphs are even, odd or neither, show ALL work.

77. _____

78. _____
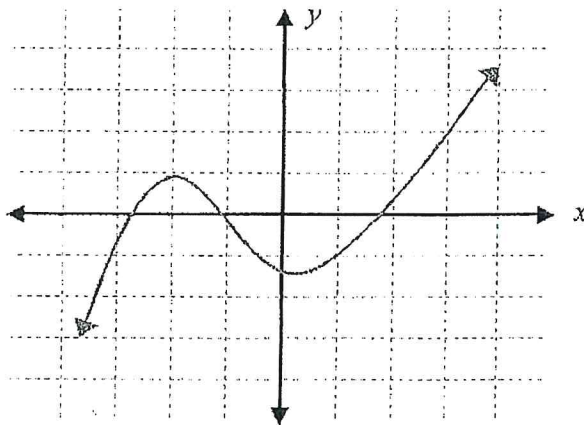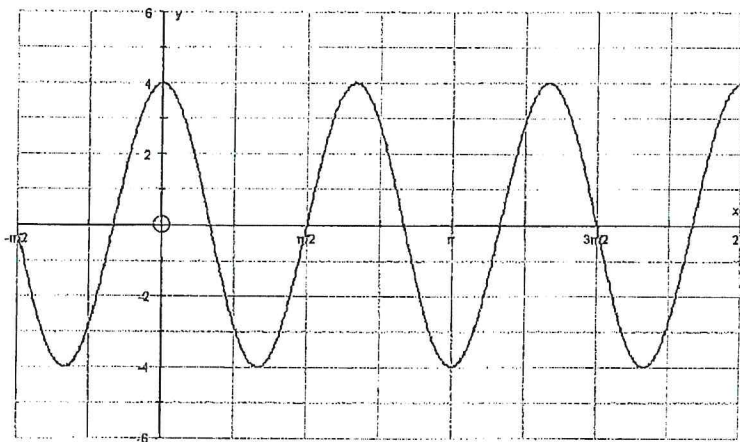
79. _____

$$f(x) = 2x^4 - 5x^2$$

80. _____

$$g(x) = x^5 - 3x^3 + x$$

81. _____

$$h(x) = 2x^2 - 5x + 3$$

82. _____

$$j(x) = 2\cos x$$

83. _____

$$k(x) = \sin x + 4$$

84. _____

$$l(x) = \cos x - 3$$

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Fill in The Unit Circle

Positive:
Negative:

Positive:
Negative:

$(\underline{\quad}, \underline{\quad})$

$(\underline{\quad}, \underline{\quad})$

$(\underline{\quad}, \underline{\quad})$

$(\underline{\quad}, \underline{\quad})$

$(\underline{\quad}, \underline{\quad})$

$(\underline{\quad}, \underline{\quad})$

$(\underline{\quad}, \underline{\quad})$

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$(\underline{\quad}, \underline{\quad})$

Positive:
Negative:

Positive:
Negative:

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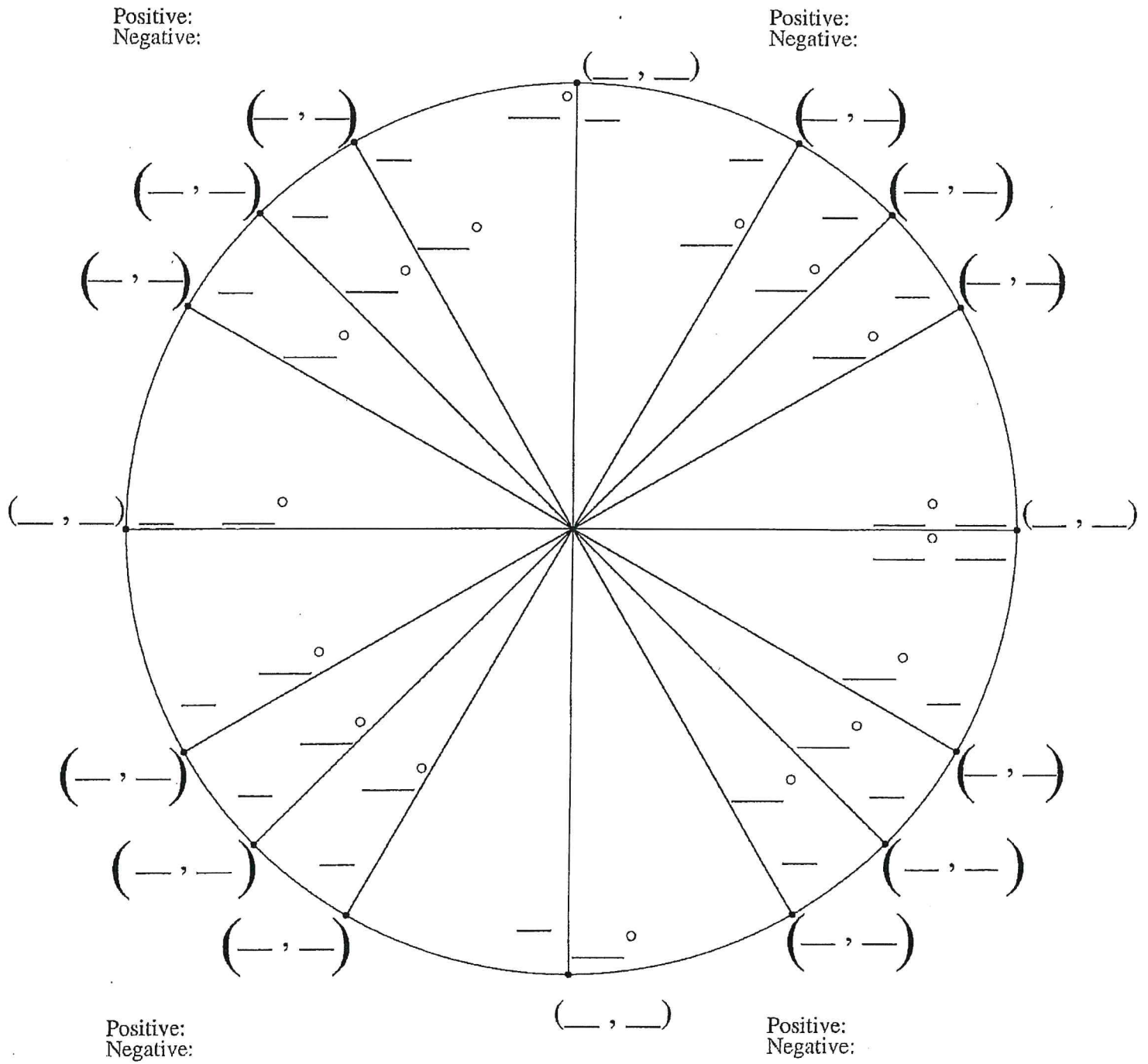
Parametric curves are relations $(x(t), y(t))$ for which both x and y are defined as functions of a **third** variable, t. As in $x = f(t)$ and $y = g(t)$. Essentially, we now will have the ability to not only tell <u>where</u> an object is given a point (x, y), but also <u>when</u> the object is at that point (x, y).

Given:

$$y = -\frac{x^2}{72} + x$$

With the Parameter:

$x = 24\sqrt{2}t$

$y = -16t^2 + 24\sqrt{2}t.$
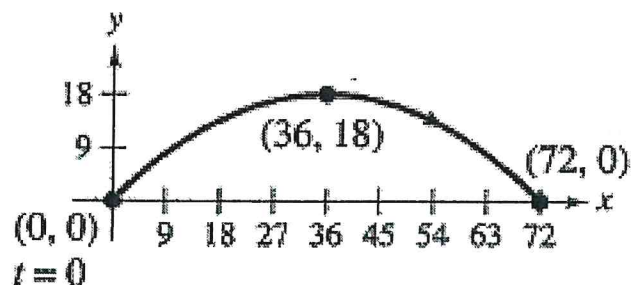
Sketch by hand. Indicate direction. Find the domain and range....

Example 1: Sketch the curve given by the parametric equations: $\begin{cases} x = t^2 - 4 \\ y = \dfrac{t}{2} \end{cases}$, for $-2 \le t \le 3$

t						
x						
y						
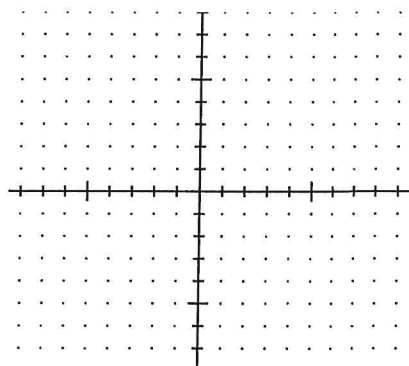
Example 2: A particle moving in the coordinate plane in such a way that $x(t) = 3t - 6$ and $y(t) = t - 4$ for $0 \leq t \leq 5$.

t						
x						
y						
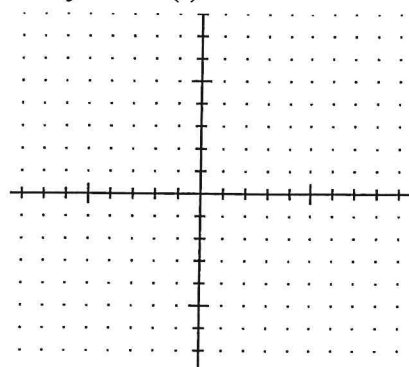
Example 3: Sketch the curve and the orientation given by $x(t) = 2 - t$ and $y(t) = t^2$ for $-2 \leq t \leq 3$

t						
x						
y						
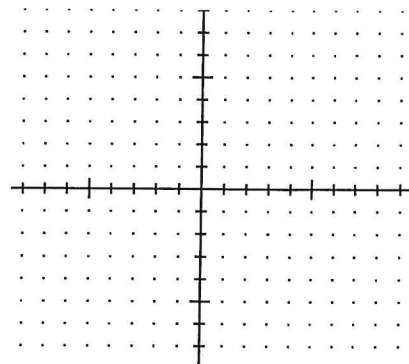
Example 4: Sketch the curve and the orientation given by $x(t) = 2\cos\theta$ and $y(t) = 6\sin\theta$ for $\theta = 0, \dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}, \pi, \dfrac{5\pi}{4}, \dfrac{3\pi}{2}, 2\pi$.

T								
x								
y								
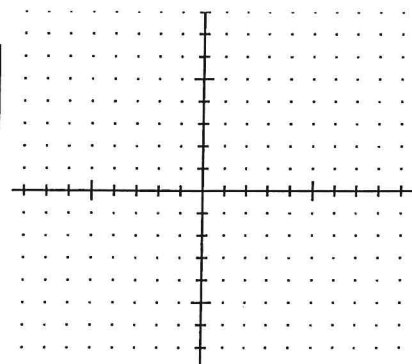
Use a graphing calucator....
Example 5 : Use a graphing calucator to graph the curves represented by the parametric equations. Tell which curves are functions.

a. $\begin{cases} x = t^2 \\ y = t^3 \end{cases}$

b. $\begin{cases} x = t \\ y = t^3 \end{cases}$

c. $\begin{cases} x = t^2 \\ y = t \end{cases}$

d. $\begin{cases} x = t^2 \\ y = 2 + t \end{cases}$, from $-3 \leq t \leq 3$

e. $\begin{cases} x = 4\sin\theta \\ y = 3\cos\theta \end{cases}$

Eliminate the parameter....

Example 6: Find the Cartesian (rectangular) equations of the curve. Identify the curve represented by the equations.

a. $\begin{cases} x = \dfrac{1}{\sqrt{t+1}} \\ y = \dfrac{t}{t+1} \end{cases}$

b. $\begin{cases} x = 3\cos\theta \\ y = 4\sin\theta \end{cases}$, for $0 \le \theta \le 2\pi$

c. $x = t - 2$ and $y = \dfrac{1}{t-1}$

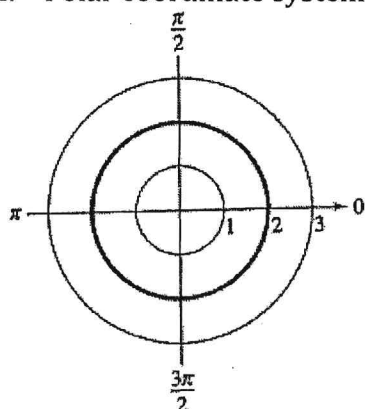d. $x = t^2 - 1$ and $y = t + 2$

e. $x = t^2 + 2$ and $y = t^2 - 1$

f. $x = 3 + 2\cos\theta$ and $y = 1 + \sin\theta$

I. Polar coordinate system
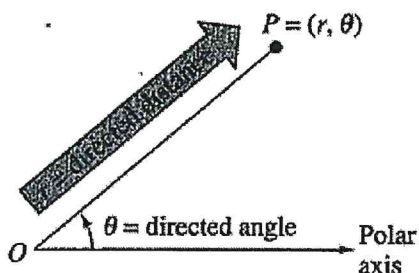
Plotting Points in the Polar Coordinate System
Example 1: Plot A and B on the first graph, then C and D on the second graph.

$$A = \left(1, \frac{\pi}{4}\right) \qquad B = \left(5, \frac{7\pi}{6}\right) \qquad\qquad\qquad C = \left(3, -\frac{\pi}{2}\right) \qquad D = \left(-4, \frac{2\pi}{3}\right)$$
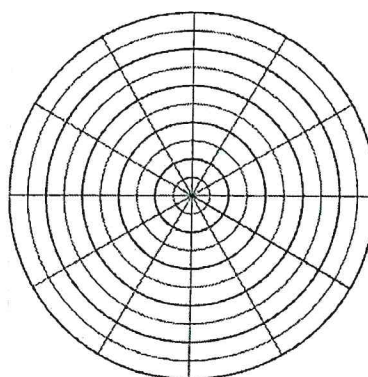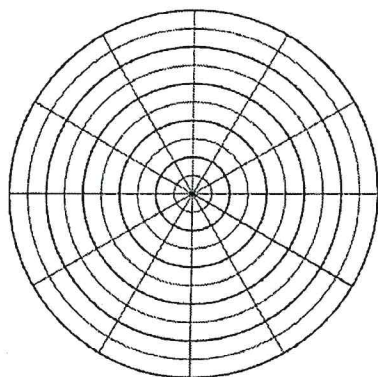
Unlike the Cartesian coordinate system, in which each point on the plane is expressed by a unique coordinate pair, each point in the coordinate plane can be represented by an infinite number of polar coordinate pairs.

Example 2.

A. Find and graph two polar coordinate pairs that represents the same point on the plane as $\left(2, \frac{\pi}{6}\right)$.

B. Find and graph polar coordinate pairs that represent the same point on the plane as $\left(-3, -\frac{2\pi}{3}\right)$ on the interval $[0, 2\pi]$.
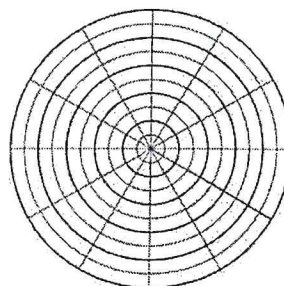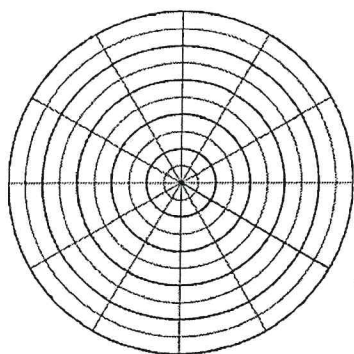
Coordinate Conversion

Recall: $\qquad x = r\cos\theta \qquad y = r\sin\theta \qquad r = \sqrt{x^2 + y^2} \qquad \tan\theta = \dfrac{y}{x}$
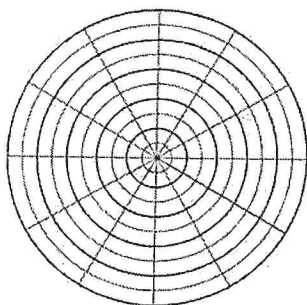
Example 3: Convert each point to rectangular coordinates. Graph the polar coordinates.

a. $(2, \pi)$ 　　 b. $\left(\sqrt{3}, \dfrac{\pi}{6}\right)$ 　　 c. $(-3, \pi)$ 　　 d. $\left(7, \dfrac{4\pi}{3}\right)$ 　　 e. $\left(-2, \dfrac{7\pi}{4}\right)$
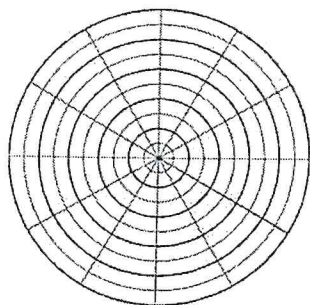
Example 4: Rectangular-to-Polar Conversion. Graph the polar coordinates.

a. $(-1, 1)$ 　　 b. $(0, 2)$ 　　 c. $(4, -4)$ 　　 d. $(-1, \sqrt{3})$
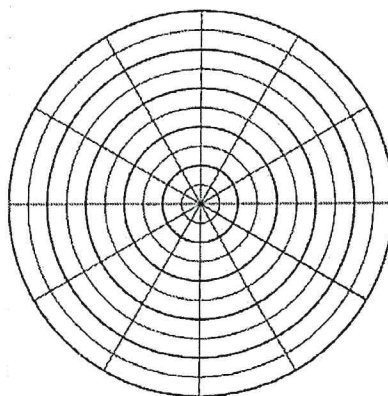

Equation Conversion

Example 5:
Describe the graph of each polar equation and find the corresponding rectangular equation.

a. $r = 2$ 　　 b. $\sin\theta = \dfrac{\pi}{3}$ 　　 c. $r = \sec\theta$
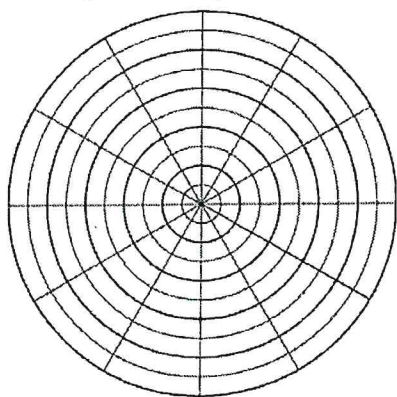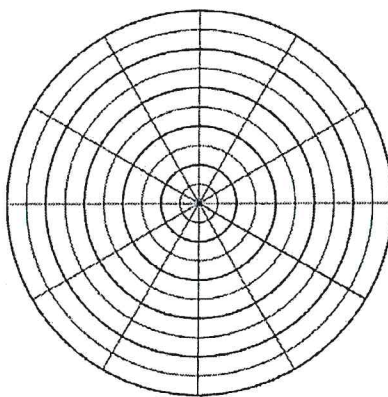
Example 6: $r = a\cos\theta$
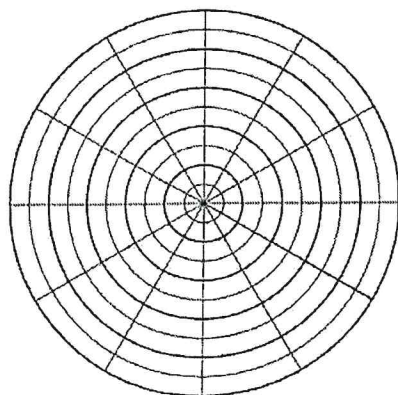
Example 7: $r = a\sin\theta$

Example 6: Graph $r = 3\cos\theta$

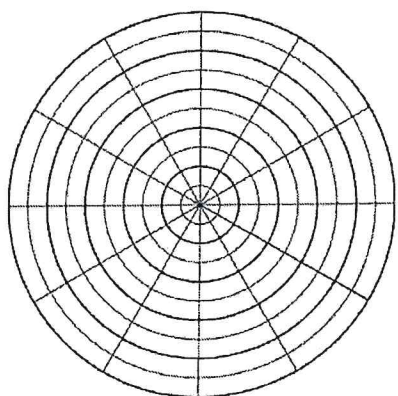**Example 7:** Graph $r = 5$

Example 6: Graph $\theta = -\dfrac{\pi}{6}$

Example 7: Graph $r = 4\cos 2\theta$