FELLOWSHIP
CHRISTIAN SCHOOL
COLOSSIANS 1: 9-12

EST. 1986

Entering

Dual Enrollment
College Algebra

Summer Math Packet

Summer 2020

Students,

As we have completed a unique semester we have decided summer packets are more important than ever, given we had to do half of the spring semester remotely.

Therefore, this packet is to be completed by the first day of school and will be graded for completion this year. We will not have an assessment over the topics in this packet as we have in years past. However, we want you to use this packet as a way to get prepared for the next course.

It is a mistake to do this entire packet at the beginning of the summer. We want these techniques to be relatively fresh in your mind in the fall. If you work a couple of problems a day, the whole packet will be completed in no time. Please show all steps when working through the packet.

As a math department, we hope you take this seriously, as we sincerely wish for you to be successful throughout this next year. Your preparation over the summer will be rewarded in unexpected ways during the year.

Here are some helpful websites to use, if needed:

- www.khanacademy.org
- www.patrickjmt.com
- www.youtube.com to find specific math related topics with accompanying videos

Sincerely,

Fellowship Math Department

<u>Radicals:</u>

<u>Radicals:</u>

To simplify means that 1) no radicand has a perfect square factor <u>and</u>

2) there is no radical in the denominator (rationalize).

Recall – the **Product Property** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

<u>Examples</u>: Simplify $\sqrt{24} = \sqrt{4} \cdot \sqrt{6}$ find a perfect square factor

$= 2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ split apart, then multiply both the numerator and the denominator by $\sqrt{2}$

$= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$ multiply straight across and simplify

If the denominator contains 2 terms –

multiply the numerator and the denominator by the ***conjugate*** of the denominator

The ***conjugate*** of $3 + \sqrt{2}$ is $3 - \sqrt{2}$ (the sign changes between the terms).

Simplify each of the following.

1. $\sqrt{32}$

2. $\sqrt{(2x)^8}$

3. $\sqrt[3]{-64}$

4. $\sqrt{49m^2n^8}$

5. $\sqrt{\frac{11}{9}}$

6. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

Rationalize.

7. $\frac{1}{\sqrt{2}}$

8. $\frac{3}{2 - \sqrt{5}}$

Complex Numbers:

Form of complex number - $a+bi$

Where a is the "real" is part and bi is the "imaginary" part

Always make these substitutions $\sqrt{-1}=i$ and $i^2=-1$

- To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5}=\sqrt{-1}\cdot\sqrt{5}$ Pull out $\sqrt{-1}$
$= i\sqrt{5}$ Make substitution

Example: $(i\sqrt{5})^2 = i\sqrt{5}\cdot i\sqrt{5}$ List twice
$= i^2\sqrt{25}$ Simplify
$= (-1)(5) = -5$ Substitute

- Treat i like any other variable when +, -, ×, or ÷ (but always simplify $i^2=-1$)

Example: $2i(3+i)=2(3i)+2i(i)$ Distribute
$= 6i+2i^2$ Simplify
$= 6i+2(-1)$ Make substitution
$= -2+6i$ Simplify and rewrite in complex form

- Since $i=\sqrt{-1}$, no answer can have an 'i' in the denominator **RATIONALIZE!!**

Simplify.

9. $\sqrt{-49}$

10. $6\sqrt{-12}$

11. $-6(2-8i)+3(5+7i)$

12. $(3-4i)^2$

13. $(6-4i)(6+4i)$

Rationalize.

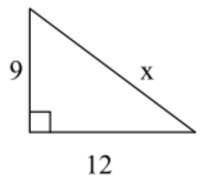
14. $\frac{1+6i}{5i}$

Geometry:

Pythagorean Theorem (right triangles): $a^2 + b^2 = c^2$

Find the value of x.

15.



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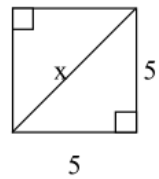

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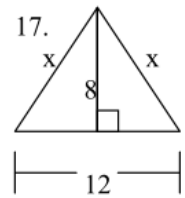
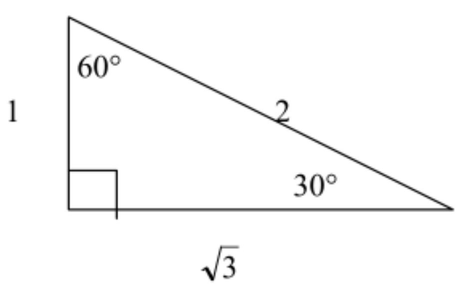
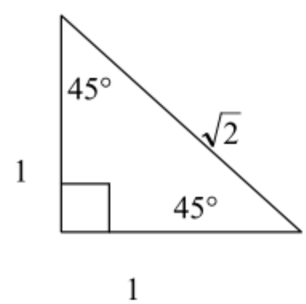
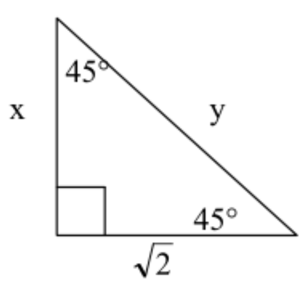


18. A square has perimeter 12 cm. Find the length of the diagonal.

* In $30° - 60° - 90°$ triangles, sides are in proportion $1, \sqrt{3}, 2$.	*In $45° - 45° - 90°$ triangles, sides are in proportion $1, 1, \sqrt{2}$.
	

Solve for x and y.

19.



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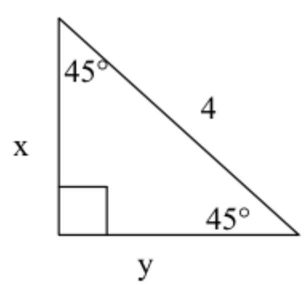

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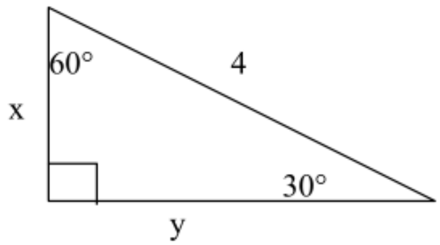

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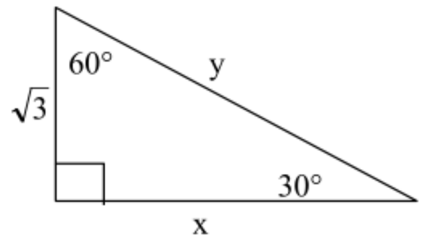


<u>Equations of Lines:</u>

Slope intercept form: $y = mx + b$	**Vertical line:** x = c (slope is undefined)
Point-slope form: $y - y_1 = m(x - x_1)$	**Horizontal line:** y = c (slope is 0)
Standard Form: $Ax + By = C$	**Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1}$

23. State the slope and y-intercept of the linear equation: $5x - 4y = 8$.

24. Find the x-intercept and y-intercept of the equation: $2x - y = 5$

25. Write the equation in standard form: $y = 7x - 5$

Write the equation of the line in slope-intercept form with the following conditions:

26. slope = -5 and passes through the point (-3, -8)

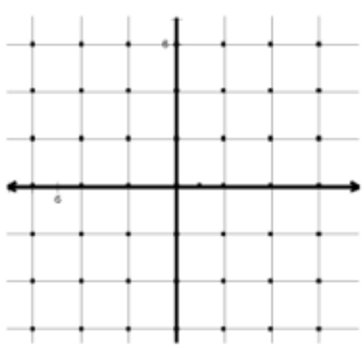
27. passes through the points (4, 3) and (7, -2)

28. x-intercept = 3 and y-intercept = 2

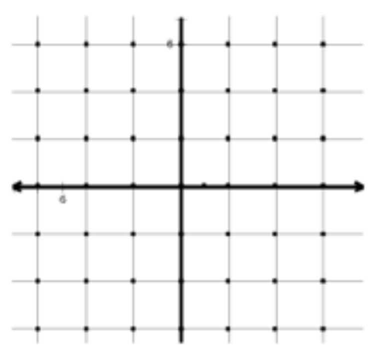
Graphing:

Graph each function, inequality, and / or system.

29. $3x - 4y = 12$

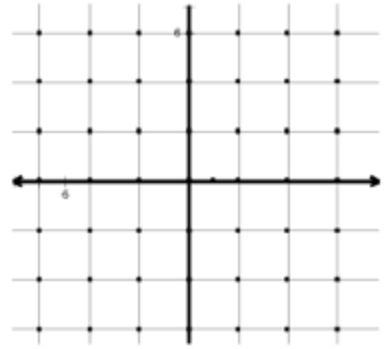


30. $\begin{cases} 2x + y = 4 \\ x - y = 2 \end{cases}$

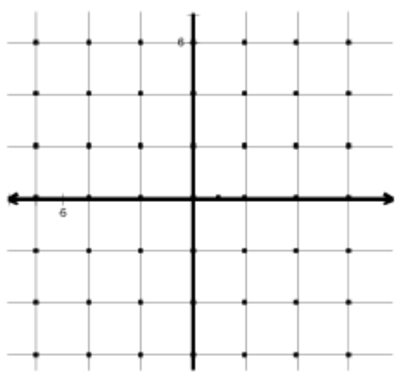


31. $y < -4x - 2$

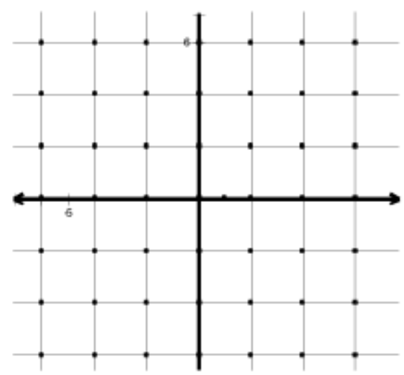
32. $y + 2 = |x + 1|$



33. $y > |x| - 1$



34. $y + 4 = (x - 1)^2$



Vertex: ___________________

x-intercept(s): ___________________

y-intercept(s): ___________________

Systems of Equations:

$3x + y = 6$

$2x - 2y = 4$

Substitution:	**Elimination**:
Solve 1 equation for 1 variable.	Find opposite coefficients for 1 variable.
Rearrange.	Multiply equation(s) by constant(s).
Plug into 2nd equation.	Add equations together (lose 1 variable).
Solve for the other variable.	Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

$y = 6 - 3x$	solve 1st equation for y	$6x + 2y = 12$	multiply 1st equation by 2
$2x - 2(6 - 3x) = 4$	plug into 2nd equation	$\underline{2x - 2y = 4}$	coefficients of y are opposite
$2x - 12 + 6x = 4$	distribute	$8x = 16$	add
$8x = 16$	simplify	$x = 2$	simplify
$x = 2$			

Plug $x = 2$ back into original

$$3(2) + y = 6$$
$$6 + y = 6$$
$$y = 0$$

Solve each system of equations. Use any method.

35. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

36. $\begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$

37. $\begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$

Exponents:

TWO RULES OF ONE

1. $a^1 = a$

Any number raised to the power of one equals itself.

2. $1^a = 1$

***One to any power is one*.**

ZERO RULE

3. $a^0 = 1$

Any nonzero number raised to the power of zero is one.

PRODUCT RULE

4. $a^m \cdot a^n = a^{m+n}$

When multiplying two powers that have the same base, add the exponents.

QUOTIENT RULE

5. $\dfrac{a^m}{a^n} = a^{m-n}$

When dividing two powers with the same base, subtract the exponents.

POWER RULE

6. $(a^m)^n = a^{m \cdot n}$

When a power is raised to another power, multiply the exponents.

NEGATIVE EXPONENTS

7. $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$

Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

38. $5a^0$

39. $\dfrac{3c}{c^{-1}}$

40. $\dfrac{2ef^{-1}}{e^{-1}}$

41. $\dfrac{(n^3 p^{-1})^2}{(np)^{-2}}$

Simplify.

42. $3m^2 \bullet 2m$

43. $(a^3)^2$

44. $(-b^3 c^4)^5$

45. $4m(3a^2 m)$

Polynomials:

To add / subtract polynomials, combine like terms.

EX: $8x-3y+6-(6y+4x-9)$ *Distribute the negative through the parantheses.*

$=8x-3y+6-6y-4x+9$ *Combine terms with similar variables.*

$=8x-4x-3y-6y+6+9$

$=4x-9y+15$

Simplify.

46. $3x^3+9+7x^2-x^3$

47. $7m-6-(2m+5)$

To multiplying two binomials, use FOIL.

EX: $(3x-2)(x+4)$ *Multiply the first, outer, inner, then last terms.*

$=3x^2+12x-2x-8$ *Combine like terms.*

$=3x^2+10x-8$

Multiply.

48. $(3a + 1)(a - 2)$

49. $(s + 3)(s - 3)$

50. $(c - 5)^2$

51. $(5x + 7y)(5x - 7y)$

Factoring.

Follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

a.) If you have one (other than 1) factor it out front.
b.) If you don't have one, move on to STEP 2.

STEP 2: How many terms does the polynomial have?

2 Terms

a.) Is it difference of two squares? $a^2 - b^2 = (a+b)(a-b)$

EX: $x^2 - 25 = (x+5)(x-5)$

b.) Is it sum or difference of two cubes? $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

EX: $m^3 + 64 = (m+4)(m^2 - 4m + 16)$
$p^3 - 125 = (p-5)(p^2 + 5p + 25)$

3 Terms

$x^2 + bx + c = (x+\quad)(x+\quad)$ **Ex:** $x^2 + 7x + 12 = (x+3)(x+4)$

$x^2 - bx + c = (x-\quad)(x-\quad)$ $x^2 - 5x + 4 = (x-1)(x-4)$

$x^2 + bx - c = (x-\quad)(x+\quad)$ $x^2 + 6x - 16 = (x-2)(x+8)$

$x^2 - bx - c = (x-\quad)(x+\quad)$ $x^2 - 2x - 24 = (x-6)(x+4)$

4 Terms -- Factor by Grouping

a.) Pair up first two terms and last two terms
b.) Factor out GCF of each pair of numbers.
c.) Factor out front the parentheses that the terms have in common.
d.) Put leftover terms in parentheses.

Ex: $x^3 + 3x^2 + 9x + 27 = (x^3 + 3x^2) + (9x + 27)$
$= x^2(x+3) + 9(x+3)$
$= (x+3)(x^2+9)$

Factor completely.

52. $z^2 + 4z - 12$

53. $6 - 5x - x^2$

54. $2k^2 + 2k - 60$

55. $-10b^4 - 15b^2$

56. $9c^2 + 30c + 25$

57. $9n^2 - 4$

58. $27z^3 - 8$

59. $2mn - 2mt + 2sn - 2st$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use quadratic formula.

EX:

$x^2 - 4x = 21$	*Set equal to zero FIRST.*
$x^2 - 4x - 21 = 0$	*Now factor.*
$(x+3)(x-7) = 0$	*Set each factor equal to zero.*
$x + 3 = 0 \quad x - 7 = 0$	*Solve each for x.*
$x = -3 \qquad x = 7$	

Solve each equation.

60. $x^2 - 4x - 12 = 0$

61. $x^2 + 25 = 10x$

62. $x^2 - 14x + 40 = 0$

Rational Algebraic Expressions:

<u>Multiplying and Dividing</u>.
Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.

EX:

$$\frac{x^2+10x+21}{5-4x-x^2}\bullet\frac{x^2+2x-15}{x^3+4x^2-21x}$$ *Factor everything completely.*

$$=\frac{(x+7)(x+3)}{(5+x)(1-x)}\bullet\frac{(x+5)(x-3)}{x(x-3)(x+7)}$$ *Cancel out common factors* in *the top and bottom.*

$$=\frac{(x+3)}{x(1-x)}$$ *Simplify.*

Simplify.

63. $\frac{5z^3+z^2-z}{3z}$

64. $\frac{m^2-25}{m^2+5m}$

65. $\frac{10r^5}{21s^2}\bullet\frac{3s}{5r^3}$

66. $\frac{a^2-5a+6}{a+4}\bullet\frac{3a+12}{a-2}$

67. $\frac{6d-9}{5d+1}\div\frac{6-13d+6d^2}{15d^2-7d-2}$

68. Function Names

Match the following equations to their description.

_____1.	$f(x)=\frac{2}{3}\left\|4x+5\right\|-3$	A.	Linear Function
_____2.	$f(x)=\frac{2}{3}\sqrt[3]{4x+5}-3$	B.	Quadratic Function
_____3.	$f(x)=\frac{2}{3}\cdot\frac{1}{4x+5}-3$	C.	Absolute Value Function
_____4.	$f(x)=\frac{2}{3}(4x+5)^4-3(4x+5)^2-2$	D.	Cubic Function
_____5.	$f(x)=\frac{2}{3}(4x+5)^3-3$	E.	Cube Root Function
_____6.	$f(x)=\frac{2}{3}(4x+5)-3$	F.	Square Root Function
_____7.	$f(x)=\frac{2}{3}(4x+5)^2-3$	G.	Rational Function
_____8.	$f(x)=\frac{2}{3}\sqrt{4x+5}-3$	H.	Polynomial Function