Brockton High School
Content: Math III
Week of: June 8 to June 14, 2020
Greetings Math III Students! We hope you are safe and well with your families!
This assignment is for the week, use your time wisely. You do not have to complete this in one sitting. Here is the lesson plan for this week:

Goals for this week

Learning Objectives:
Students will be able to:

- Rewrite radicals
- Add, subtract, multiply and divide with radicals
- Distinguish exponential growth functions from exponential decay functions.
- Calculate compound interest and the change in population using an exponential growth function.
- Evaluate a simple logarithm.
- Recognize a graphed log function as the inverse of an exponential function.
- Solve an exponential and logarithmic problem with base 2, 10 and e
- Write a finite geometric series using summation notation.
- Calculate the sum of a finite geometric series.

(Standards A.REI.3, A.SSE.1, A.CED, 1, N.RN.1, F.BF. 4/5, F.IF)

Literacy Objectives:
Students will be able to:

1. to create, interpret and explain a table, chart or graph
2. to compute, interpret and explain numbers
3. to interpret and present statistics that support an argument or hypothesis
4. to identify, explain and make a prediction based on a pattern

(https://www.bpsma.org/schools/brockton-high-school/about-us/mission-literacy-charts)

Instructional Video Links: Please watch at least one of these videos to help guide you.

- https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:rational-exponents-radicals/x2f8bb11595b61c86:simplifying-square-roots/v/simplifying-square-roots-1
- https://www.khanacademy.org/math/algebra-home/alg-exp-and-log/miscellaneous-radicals/v/adding-and-simplifying-radicals
- https://www.youtube.com/watch?v=PcUUM-s1g-I
- https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:exponential-growth-decay/x2f8bb11595b61c86:exponential-vs-linear-growth/v/exponential-growth-functions
- https://www.khanacademy.org/economics-finance-domain/core-finance/interest-tutorial/compound-interest-tutorial/v/introduction-to-compound-interest
- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:logs/x2ec2f6f830c9fb89:log-intro/v/logarithms
- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:exp/x2ec2f6f830c9fb89:exp-eq-prop/v/solving-exponential-equations-with-exponent-properties
- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-factor/x2ec2f6f830c9fb89:geo-series/v/deriving-formula-for-sum-of-finite-geometric-series

Your teacher also has a list of additional instructional videos. Feel free to also search for your own tutorial videos to guide you.

Key Terms

- **Root of a number-** an *n*th root of a number *b* is a solution of the equation x^n = b
- **Index –** a number used to indicate what root is to be determined; it is placed above and to the left of the radical sign.
- **Radicand –** the quantity under a radical sign in an expression.
- **Perfect square -**
- **Exponential function –** a function of the form *f(x)* = ab^x where *a* is any non-zero real number and *b* is any real number greater than zero and not equal to one.
- **Logarithm –** the logarithm of a positive number is the exponent to which the base must be raised to result in that number.
- **Base of a power –** the number or variable that is repeatedly multiplied.
- **Geometric sequence –** a sequence of numbers in which the ration between any two consecutive numbers is a constant, called the constant ratio.

Practice Activities:
On-Line:
All students now have access to an on-line program called **Mathia (Carnegie Learning)**

- If you are **already in Mathia**, please continue to work in the program. The suggested pacing is to complete Module 4 Unit 5 by the end of the week.
- If you are **new to Mathia**: Please see the log-in information below and begin. The suggested pacing is to complete Module 4 Unit 5 by the end of the week.

Practice Activities:
Printable resources: Optional resources if technology is not available

Math 3 Printable Resources: All the Pieces of the puzzle pages M3-171 through M3-183

Extension Activities: Complete the STRECTH problem on page M#-184 and send to your teacher.

Log-in Information

1. Log-in to Clever(if you need to sign in follow number 2 then 3; if you automatically log in go to 3)
2. If you need to sign in to Clever: Username: 6-digit BHS school ID # @bpsma.org
   Password: Date of birth bps 1920
   Example: Student with ID #:123456
   Date of birth: January 1st, 2000
   Username: 123456@bpsma.org
   Password: 01012000bps1920
3. Under Math (in the left- hand column) click on Carnegie Learning (MATHia)
4. Pick up where you left off in Mathia

Additional Support

- Please email your math teacher with specific questions.
- Here is a list of math teachers' office hours:
- https://brocktonpublicschools-my.sharepoint.com/:x:/g/personal/danielcorbett_bpsma_org/EWk_ij9UwjpPtRAHBUkEpS4B3vue-_IG8VYz0AwG9ovJjQ?e=4%3arkcL1r&at=9&CT=1588698277992&OR=OWA-NT&CID=8d0078f2-9a27-460f-e632-64578875ee60

1

All the Pieces of the Puzzle

Logarithmic Expressions

Warm Up

Convert each logarithmic equation to an exponential equation. Then solve for the unknown.

1. $\log_{10} 0.1 = x$

2. $\log_2 16 = x$

3. $\log_{10} 10^9 = x$

4. $\log_2 \frac{1}{4} = x$

Learning Goals

- Convert exponential equations into logarithmic equations.
- Convert logarithmic equations into exponential equations.
- Solve exponential and simple logarithmic equations.
- Estimate the values of logarithms on a number line.
- Evaluate logarithmic expressions.

Key Term

- logarithmic expression

You know that logarithmic functions are inverses of exponential functions. How can you use inverses to solve exponential and logarithmic equations?

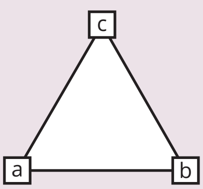
Two-Way Street

Recall that a logarithmic function is the inverse of an exponential function.

1. Write the equivalent form of the given exponential or logarithmic equation.

Remember:

Use the Triangle of Power.



$a^c = b \Leftrightarrow \log_a b = c$

Exponential Form $y = b^x$	$\Leftrightarrow$	Logarithmic Form $x = \log_b y$
$12^2 = 144$	$\Leftrightarrow$	
	$\Leftrightarrow$	$\log_{16} 4 = \frac{1}{2}$
$10^5 = 100{,}000$	$\Leftrightarrow$	
	$\Leftrightarrow$	$\ln 20.086 \approx 3$
$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$	$\Leftrightarrow$	
	$\Leftrightarrow$	$\log_9 27 = \frac{3}{2}$
	$\Leftrightarrow$	$\log_2 x = 8$
$6^x = 36$	$\Leftrightarrow$	
$n^5 = 243$	$\Leftrightarrow$	$\log_n 243 = 5$

ACTIVITY 1.1

Solving Logarithmic Equations

When you evaluate a logarithmic expression (logarithm), you are determining the value of the exponent in the corresponding exponential expression.

$$\text{base}^{\text{exponent}} = \text{argument} \Leftrightarrow \log_{\text{base}} (\text{argument}) = \text{exponent}$$

The variables of the logarithmic equation have the same restrictions as the corresponding variables of the exponential equation. The base, b, must be greater than 0 but not equal to 1; the argument must be greater than 0; and the value of the exponent has no restrictions.

It is important to become familiar with how the base, argument, and exponent fit into a *logarithmic equation*. A **logarithmic equation** is an equation that contains a logarithm.

To write a logarithmic equation, sometimes it is helpful to consider the exponential form first and then convert it to logarithmic form.

1. Arrange the given terms to create a true logarithmic equation.

a. $49, 2, 7$

b. $-3, 6, \frac{1}{216}$

c. $4, 4, 1$

d. $256, 4, 4$

Let's consider the relationship between the base, argument, and exponent. You can use that relationship to solve for any unknown in a logarithmic equation.

It is important to note that you can convert a logarithmic equation to an exponential equation regardless of which term is unknown.

Worked Example

To solve for any unknown in a simple logarithmic equation, begin by converting it to an exponential equation.

Argument Is Unknown	**Exponent Is Unknown**	**Base Is Unknown**
$\log_4 y = 3$	$\log_4 64 = x$	$\log_b 64 = 3$
$4^3 = y$	$4^x = 64$	$b^3 = 64$
$64 = y$	$4^x = 4^3$	$b^3 = 4^3$
	$x = 3$	$b = 4$

2. Justify the last step of each case in the worked example.

a. If $4^3 = y$, why does $y = 64$?

b. If $4^x = 4^3$, why does $x = 3$?

c. If $b^3 = 4^3$, why does $b = 4$?

3. **Solve for the unknown in each logarithmic equation.**

a. $\log_8 64 = n$

b. $\log_n \frac{1}{16} = -2$

c. $\log_{\frac{1}{2}} 64 = n$

d. $\log n = -3$

e. $\log_n \sqrt[3]{49} = \frac{2}{3}$

f. $\log_9 27 = n$

4. **Write three logarithmic expressions that are equivalent to each given expression. Explain your strategy.**

a. $\log_5 625$

b. $\log_7 \frac{1}{7}$

c. $\log_{64} 8$

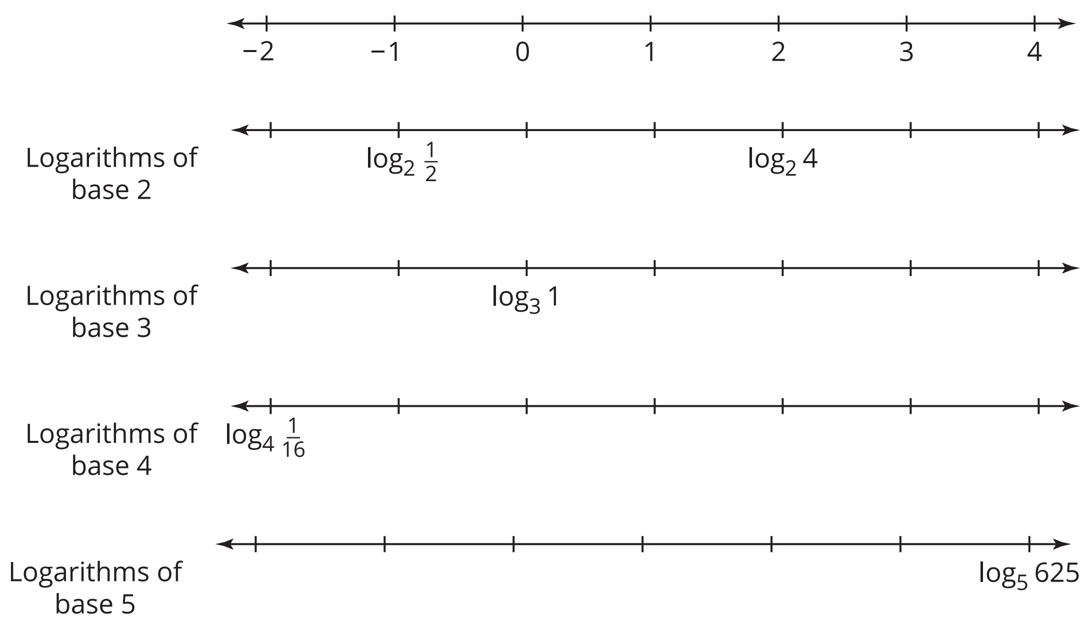
d. $\log_2 -2$

ACTIVITY 1.2

Estimating with Logarithms

A logarithm is an exponent, so it can be any real number, even an irrational number.

1. Label each number line using logarithmic expressions with the indicated base.



Describe the restrictions on the variables when appropriate.

2. Compare the logarithms on the number lines.

a. Analyze all the logarithms that are equivalent to 0. Write a general statement using the base *b* to represent this relationship.

b. Analyze all the logarithms that are equivalent to 1. Write a general statement using the base *b* to represent this relationship.

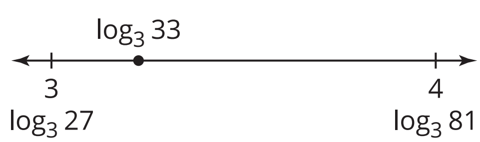
c. Rewrite the general statements from parts (a) and (b) in exponential form. Use exponent rules to verify that each statement is true.

You can estimate the value of a logarithm that is not an integer using a number line as a guide.

Worked Example

Estimate the value of $\log_3 33$.

To estimate $\log_3 33$ to the tenths place, identify the closest integer logarithm whose argument is less than 33 and the closest integer logarithm whose argument is greater than 33 on a number line that represents base 3.



Closest integer logarithm whose argument is less than 33:	Logarithm you are estimating:	Closest integer logarithm whose argument is greater than 33:
$\log_3 27$	$\log_3 33$	$\log_3 81$

You know that $\log_3 27 = 3$ and $\log_3 81 = 4$. This means the estimate of $\log_3 33$ is between 3 and 4.

$$\log_3 27 < \log_3 33 < \log_3 81$$
$$3 < x < 4$$

Next, estimate the decimal digit.

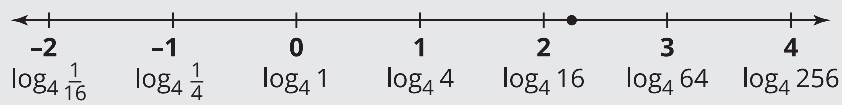
Because 33 is closer to 27 than to 81, the value of $\log_3 33$ is closer to 3 than to 4.

In this case, 3.2 is a good estimate for $\log_3 33$.

3. **Sutton and Silas were each asked to estimate $\log_4 28$.**

Sutton

I estimated $\log_4 28$ using the number line.



$$\log_4 16 < \log_4 28 < \log_4 64$$
$$2 < x < 3$$
$$\log_4 28 \approx 2.3$$

Silas

I estimated $\log_4 28$ by converting the log into exponential form and estimating based on powers of 4.

$$\log_4 28 = x$$
$$4^x = 28$$

I know that $4^2 = 16$ and $4^3 = 64$ so the estimate of $\log_4 28$ must be between 2 and 3.

$$\log_4 28 \approx 2.4$$

Silas did not use the number line, but his estimate was about the same as Sutton's. Will Silas's method always work?

4. **Estimate each logarithm to the tenths place and explain your reasoning.**

a. $\log_2 10$

b. $\log_5 4$

c. $\log_4 300$

d. $\log 2500$

5. Mark and Scotty were each asked to determine which base was used to estimate the value of $\log_b 58 = 2.9$.

Mark

The log of 58 falls between 2 and 3 when the base is 4.

$\log_4 16 < \log_b 58 < \log_4 64$

$2 < 2.9 < 3$

So, $b \approx 4$.

Scotty

The log of 58 falls between 2 and 3 when the base is 5.

$\log_5 25 < \log_b 58 < \log_5 125$

$2 < 2.9 < 3$

So, $b \approx 5$.

Who is correct? Explain your reasoning.

6. Use the number lines from Question 1 to determine the appropriate base of each logarithm.

a. $\log_b 108 = 2.9$

b. $\log_b 0.4 = -1.3$

c. $\log_b 74 = 3.1$

ACTIVITY

1.3

Estimating with Natural Logarithms

You have estimated with logarithms that have rational number bases. Let's consider how you can estimate the values of natural logarithms—those with a base of *e*.

1. **For a fixed base greater than 1, as the value of the argument gets larger, what happens to the value of the logarithm? Provide an example to illustrate your statement.**

2. **Plot $\log_2 18$, $\log_3 18$, $\log_4 18$, and $\log_5 18$ on the appropriate number lines in Question 1 in the previous activity. Then use the number lines to estimate the numeric value of each logarithm to the tenths place. Verify your answers in exponential form.**

3. **For a fixed argument, when the value of the base is greater than 1 and increasing, what happens to the value of the logarithm?**

4. **How could you use the number lines to predict the value of ln 18?**

5. **Make a prediction for the value of ln 18.**

NOTES

TALK the TALK

Always, Sometimes, Never

Complete each sentence with *always, sometimes, or never* to make it true. Explain your reasoning.

1. **The value of a logarithm is ______________ equal to the exponent of the corresponding exponential equation.**

2. **The argument of a logarithmic expression is ______________ a negative number.**

3. **The value of a logarithm is ______________ equal to a negative number.**

4. **The base of a logarithm is ______________ a negative number.**

5. **A logarithm is ______________ a value that is not an integer.**

6. **For a base greater than 1, if $b > c$ then the value of $\log_a b$ is ______________ greater than $\log_a c$.**

7. **If $a > b$, then the value of $\log_a 1$ is ______________ less than $\log_b 1$.**

8. **The base of a logarithm is ______________ equal to 1.**

Assignment

Write

Describe how to estimate the value of a logarithm.

Remember

The value of a logarithmic expression is equal to the value of the exponent in the corresponding exponential expression.

For a fixed base greater than 1, as the value of the argument increases, the value of the logarithm increases as well.

For a fixed argument, when the value of the base is greater than 1 and increasing, the value of the logarithm is decreasing.

Practice

1. Solve for the unknown in each logarithmic equation.

   a. $\log 1000 = n$

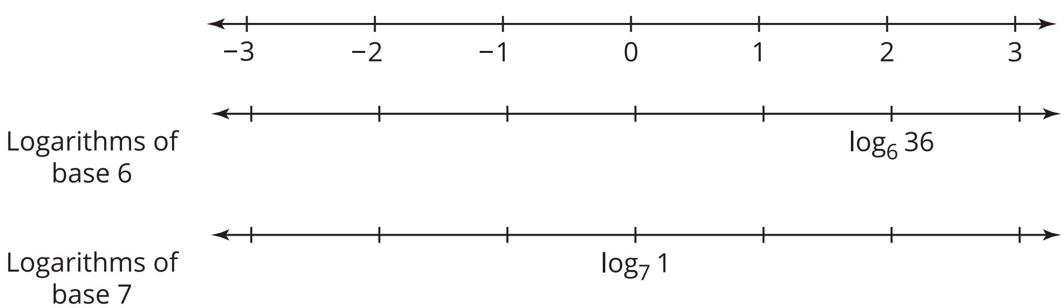
   b. $\log_n \frac{1}{27} = -3$

   c. $\log_{\frac{1}{3}} 81 = n$

   d. $\log_8 16 = n$

2. Consider base 6 and base 7 logarithms.

   a. Label each number line using logarithmic expressions with the indicated base to match the given number line.



   b. Estimate $\log_6 40$.

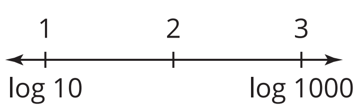
   c. Estimate $\log_7 40$.

   d. Estimate $\log_6 200$.

   e. Estimate $\log_7 100$.

3. Estimate $\log_2 15$ to the nearest tenth. Explain your reasoning.

4. Two students are trying to finish labeling the number line with the base 10 logarithmic expression that equals 2. Dylan says the missing logarithm should be log 505 because 505 is halfway between 10 and 1000, just like 2 is halfway between 1 and 3. Jakob disagrees. He says the missing logarithm should be log 100.



   a. Which student is correct? Explain your reasoning.

   b. For a logarithm with a base greater than 1, how does the argument change for every increase of 1 in the value of the logarithm?

   c. Estimate log 55. Explain your reasoning.

Stretch

Solve the equation $4 = \log_2(x) + \log_2(x - 6)$.

Review

1. Given: $f(x) = 1.5^x$ and $g(x) = -\frac{1}{2}f(4x)$.
   a. Describe the transformation of $f(x)$ that produces $g(x)$.
   b. Write $g(x)$ as an exponential function.
2. Consider $s(x) = 3^{x-1}$, which is a transformation of the function $f(x) = 3^x$.
   a. Describe the transformation(s) of $f(x)$ to produce $s(x)$.
   b. Write the equations of the inverse functions $f^{-1}(x)$ and $s^{-1}(x)$.
   c. Describe the transformation(s) on the graph of $f^{-1}(x)$ to produce $s^{-1}(x)$.
3. Given $p(x) = 2^x$ and $t(x) = 3p(x + 1) + 7$.
   a. Describe the transformation of $p(x)$ that produces $t(x)$.
   b. Write $t(x)$ as an exponential function.
4. Consider the function $h(x)$, which is formed by translating the function $g(x) = \log_3 x$ right 2 units and down 1 unit.
   a. Write $h(x)$ in terms of $g(x)$.
   b. Complete the table by determining the corresponding point on $h(x)$ for each reference point on $g(x)$.

Reference Point on $g(x)$	**Corresponding Point on $h(x)$**
$(\frac{1}{3}, -1)$	
(1, 0)	
(3, 1)	
(9, 2)	

   c. Write $h(x)$ as a logarithmic function.
   d. List the domain, range, and any asymptotes of the logarithmic function $h(x)$.
5. Determine the quadratic equation that goes through the points (−1, 10), (2, 4), and (3, −6).