Brockton High School
Content: Math II
Week of: June 8 – June 14, 2020
Greetings Math II Students! We hope you are safe and well with your families!
This assignment is for the week, use your time wisely. **You do not have to complete this in one sitting**. Here is the lesson plan for this week:

Goals for this week: Module 5 on Mathia
<u>Learning Objectives:</u> Students will be able to: 1. Perform operations on polynomials (Standards A.SSE.1a, A.APR.1)
<u>Literacy Objectives:</u> Students will be able to: 1. to compute, interpret and explain numbers. 2. to read, break down, and solve a word problem. 3. to explain and/or interpret relationships of space and time. *(https://www.bpsma.org/schools/brockton-high-school/about-us/mission-literacy-charts)*

Carnegie Learning (use with Carnegie Resources provided below) Carnegie Learning (Log-in through Clever)	
<u>Instructional Video Links: Please watch these videos to help guide you.</u> 1. Introduction to Polynomials 2. Adding Polynomials 3. Subtracting Polynomials 4. Multiplying Binomials using FOIL 5. Multiplying Binomials using factor table #1 6. Multiplying Binomials using factor table #2	**<u>Printable Resources</u>**: Skills Practice: ● Module 4 topic 1: This time, with polynomials ● Multiplying binomial worksheet
Practice Activities: <u>On-Line:</u> All students now have access to an on-line program called Mathia! ● Mathia- If you are already in Mathia, please continue to work in the program. ● If you are new to Mathia: Please see the log-in information below.	**<u>Key Terms</u>**: ● **Monomial:** an expression that consists of a single term that is either a constant, a variable, or a product of constant and a variable ● **Binomial:** a polynomial with exactly two terms ● **Polynomial:** the sum or difference of more than one monomial ● **Complex Number:** A number that can be written in the form a + bi where a and b are real numbers and i is the imaginary unit ● **Term:** The parts of an expression that are added together (a monomial)

- **Like Terms:** Terms that have identical variables and exponents.
- **Variable:** a letter used to represent one of more numbers
- **Exponent:** the number of times the base is used as a factor of repeated multiplication
- **Order of Operations:** the set of rules for evaluating an expression (PEMDAS)
- **Distributive Property:** for any real numbers a, b, and c is true that

$$a(b + c) = ab + ac.$$

Extension Activities:

Stretch

Consider the binomials $(x + 3)$, $(2x + 1)$, and $(x - 4)$.

1. Without multiplying, make a conjecture about the degree of the product of these binomials. Explain how you determined your answer.
2. Without multiplying, make a conjecture about the number of terms in the product of these binomials. Explain your reasoning.
3. Two students determine the product of the 3 binomials using two different methods. Student 1 uses a multiplication table, and Student 2 uses the distributive Property. Their work is shown below. Determine which student multiplied correctly and identify the mistake the other student made. Explain how you determined your answer.

Student 1

$\cdot$	x	3	$2x$	1
x	x^2	$3x$	$2x^2$	x
-4	$-4x$	-12	$-8x$	-4

The product is $3x^2 - 8x - 16$.

Student 2

$$(x + 3)(2x + 1)(x - 4) = (2x^2 + 7x + 3)(x - 4)$$
$$= 2x^3 - x^2 - 25x - 12$$

The product is $2x^3 - x^2 - 25x - 12$.

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Log-in Information

1. http://studentintranet.bpsma.org/
2. Click on Clever
3. Under Math, click on Carnegie Learning (CL)
4. Username: 6-digit BHS school ID # @bpsma.org
   Password: Date of birth bps 1920
   Example: Student (Michael) with ID #:123456
   Date of birth: January 1st, 2000
   Username: 123456@bpsma.org
   Password: 01012000bps1920

Additional Support

Email:

- Please email your math teacher with specific questions.

Office Hours:

- Click here for teacher Office hours!

Carnegie Learning Resource

1

This Time, With Polynomials

Adding, Subtracting, and Multiplying Polynomials

Warm Up

Rewrite each expression by combining like terms.

1. $-3x + 4y - 9x - 5y$

2. $2xy^2 + 5x^2y - 7xy + xy^2$

3. $6 - m^2 + 5m^2$

4. $-8 - (-4k) + 7 + 1 - 4k$

Learning Goals

- Name polynomials by number of terms or degree.
- Understand that operations can be performed on functions as well as numbers.
- Add, subtract, and multiply polynomials.
- Explain why polynomials are closed under addition, subtraction and multiplication.
- Recognize and use special products when multiplying binomials.

Key Terms

- polynomial
- monomial
- binomial
- trinomial
- degree of a polynomial
- closed, closure
- difference of two squares
- perfect square trinomial

You know that a linear expression is one type of polynomial expression. What are other polynomial expressions, and how do you add, subtract, and multiply them?

GETTING STARTED

Sorting It Out

You are familiar with many types of mathematical expressions. Cut out the 12 expressions located at the end of this lesson. Analyze and sort them into groups based upon common characteristics.

1. **Summarize the groups you formed by listing the expressions that you grouped together and your description for each group. Use mathematical terms in your descriptions.**

2. **Compare your groups of expressions to your classmates' groups. Describe any similarities and differences.**

3. **Jimmy and Andrew agree that $4x - 6x^2$ and $25 - 18m^2$ belong in the same group. They each are adding the expressions shown to the group. Who is correct? Explain your reasoning.**

Jimmy	Andrew
$5 - 7h$	$y^2 - 4y + 10$
$78j^3 - 3j$	$-3 + 7n + n^2$
$-13s + 6$	

4. **What characteristics do all twelve expressions share?**

ACTIVITY **1.1**

Categorizing Polynomials

Previously, you worked with linear expressions in the form $ax + b$ and quadratic expressions in the form $ax^2 + bx + c$. Each is also part of a larger group of expressions known as *polynomials*.

A **polynomial** is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients. A polynomial in one variable is the sum of terms of the form ax^k, where a is any real number and k is a non-negative integer. In general, a polynomial is of the form $a_1x^k + a_2x^{k-1} + \ldots + a_nx^0$. Within a polynomial, each product is a term, and the number being multiplied by a power is a coefficient.

Worked Example

The polynomial $m^3 + 8m^2 - 10m + 5$ has four terms. Each term is written in the form ax^k.

- The first term is m^3.
- The power is m^3, and its coefficient is 1.
- In this term, the variable is m and the exponent is 3.

1. Write each term from the worked example and identify the coefficient, power, and exponent. The first term has already been completed for you.

	1st	2nd	3rd	4th
Term	m^3			
Coefficient	1			
Variable	m			
Power	m^3			
Exponent	3			

2. Identify the terms and coefficients in each polynomial.

a. $-2x^2 + 100x$

b. $4m^3 - 2m^2 - 5$

c. $y^5 - y + 3$

Polynomials are named according to the number of terms they have. Polynomials with only one term are **monomials**. Polynomials with exactly two terms are **binomials**. Polynomials with exactly three terms are **trinomials**.

The degree of a term in a polynomial is the exponent of the term. The greatest exponent in a polynomial determines the **degree of the polynomial**. In the polynomial $4x + 3$, the greatest exponent is 1, so the degree of the polynomial is 1.

3. Khalil says that $3x^{-2} + 4x - 1$ is a trinomial with a degree of 1 because 1 is the greatest exponent. Jazmin disagrees and says that this is not a polynomial at all because the power on the first term is not a whole number. Who is correct? Explain your reasoning.

4. Determine whether each expression is a polynomial. Explain your reasoning.

$5^x + 4^{x-1} + 3^{x-2}$ $\qquad$ $x^2 + \sqrt{x}$ $\qquad$ $x^4y + x^3y^2 + x^2y$

A polynomial is written in general form when the terms are in descending order, starting with the term with the largest degree and ending with the term with the smallest degree.

5. Revisit the cards you sorted in the Getting Started.

   a. Identify any polynomial not written in general form and rewrite it in general form on the card.

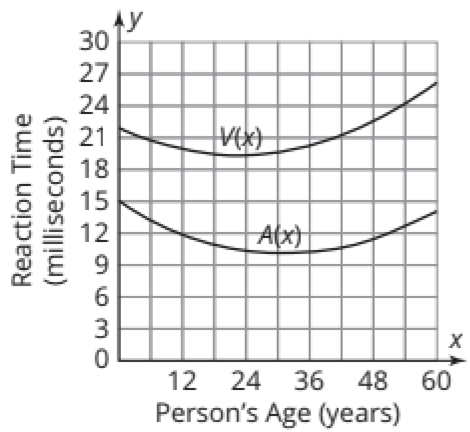
   b. Identify the degree of each polynomial and write the degree on the card.

   c. Glue each card in the appropriate column based on the number of terms in each polynomial. Write your own polynomial to complete any empty boxes.

Monomial	Binomial	Trinomial

ACTIVITY 1.2 Interpreting the Graphs of Polynomial Functions

The graphs of functions $V(x)$ and $A(x)$ are shown. The function $V(x)$ models people's reaction times to visual stimuli in milliseconds, based upon the age of a person in years. The function $A(x)$ models people's reaction times to audio stimuli in milliseconds based on the age of a person in years.



1. Interpret the graphs of the functions.

a. Describe the functions $V(x)$ and $A(x)$.

b. Write a summary to describe people's reaction times to visual stimuli and audio stimuli.

c. Do you think a person would react faster to a car horn or a flashing light? Explain your reasoning.

2. Estimate the age that a person has the quickest reaction time to each stimuli. Explain how you determined each answer.

a. visual stimuli

b. audio stimuli

Many times, auto insurance companies use test results similar to the ones shown to create insurance policies for different drivers.

3. How do you think the information provided in the graphic representation may be used by an auto insurance company?

4. Consider a new function $h(x)$, where $h(x) = V(x) - A(x)$. What does $h(x)$ mean in terms of the problem situation?

5. Write a report about drivers' reaction times to visual and audio stimuli. Discuss actions that may improve drivers' reaction times and distractions that may worsen drivers' reaction times. Discuss the importance of flashing lights and sirens on emergency vehicles.

Ask yourself:

How can you incorporate information about auto insurance rates and a driver's age in your report?

ACTIVITY
1.3

Adding and Subtracting Polynomial Functions

You are playing a new virtual reality game called "Species." You are an environmental scientist who is responsible for tracking two species of endangered parrots, the orange-bellied parrot and the yellow-headed parrot. Suppose the orange-bellied parrots' population can be modeled by the function $B(x)$, where x represents the number of years since the current year. Suppose that the population of the yellow-headed parrot can be modeled by the function $H(x)$.

$$B(x) = -18x + 120$$

$$H(x) = 4x^2 - 5x + 25$$

The two polynomial functions are shown on the coordinate plane.

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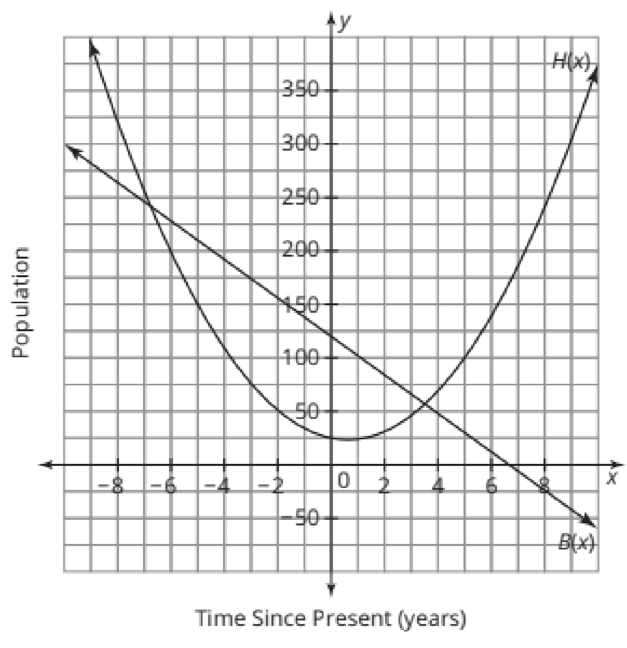


Ask yourself:

One place to start the sketch of $T(x)$ would be to consider the y-intercept for each function. What would the new y-intercept be for $T(x)$?

Your new task in this game is to determine the total number of these endangered parrots each year over a six-year span. You can calculate the total population of parrots using the two graphed functions.

1. Use the graphs of $B(x)$ and $H(x)$ to determine the function, $T(x)$, to represent the total population of parrots.

a. Write $T(x)$ in terms of $B(x)$ and $H(x)$.

b. Predict the shape of the graph of $T(x)$.

c. Sketch a graph of $T(x)$ on the coordinate plane shown. First choose any 5 x-values and add their corresponding y-values to create a new point on the graph of $T(x)$. Then connect the points with a smooth curve. Record the values in the table.

x	$B(x)$	$H(x)$	$T(x)$

d. Did the graph of $T(x)$ match your prediction in part (b)? Identify the function family to which $T(x)$ belongs.

You can write a function, $T(x)$, in terms of x to calculate the total number of parrots at any time.

parrots at any time.

Worked Example

$T(x) = B(x) + H(x)$	Write $T(x)$ in terms of two known functions.
$T(x) = (-18x + 120) + (4x^2 - 5x + 25)$	Substitute the functions in terms of x.
$T(x) = 4x^2 + (-18x + (-5x)) + (120 + 25)$	Use the Commutative Property to reorder and the Associative Property to group like terms.
$T(x) = 4x^2 - 23x + 145$	Combine like terms.

2. Choose any two x-values in your table. Use the new polynomial function, $T(x)$, to confirm that your solution in the table for those times is correct. Show your work.

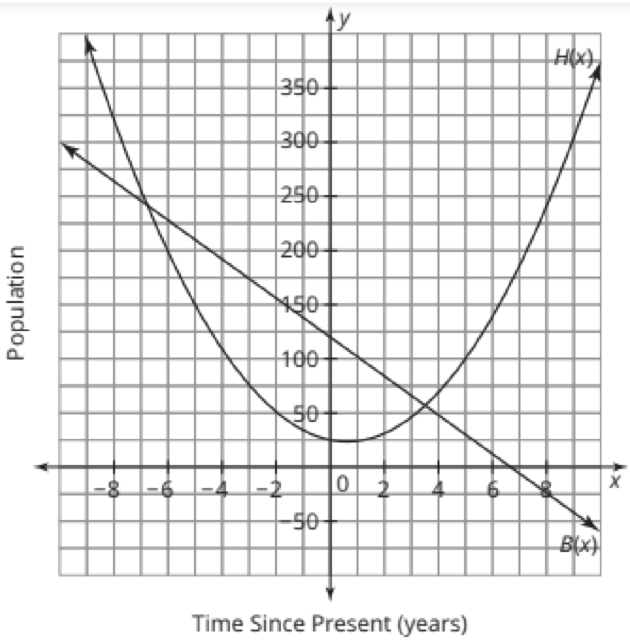
Remember:

The table feature on a graphing calculator is an efficient tool to determine y-values.

3. Use technology to confirm that your graph and the remaining solutions in the table are correct. Explain any discrepancies and how you corrected them.

4. Zoe says that using $T(x)$ will not work for any time after 6 years from now because by that point the orange-bellied parrot will be extinct. Is Zoe's statement correct? Why or why not?

Throughout the game "Species," you must always keep track of the difference between the population of each type of species. If the difference gets to be too great, you lose the game. The graphs of $B(x) = -18x + 120$ and $H(x) = 4x^2 - 5x + 25$ are shown.



5. Use the graphs of $B(x)$ and $H(x)$ to determine the function, $D(x)$, to represent the difference between the populations of each type of species.

a. Write $D(x)$ in terms of $B(x)$ and $H(x)$.

b. Predict the shape of the graph of $D(x)$.

c. Sketch a graph of $D(x)$ on the coordinate plane shown. First choose any 5 x-values and subtract their corresponding y-values to create a new point on the graph of $D(x)$. Then connect the points with a smooth curve. Record the values in the table.

x	$B(x)$	$H(x)$	$D(x)$

d. Did the graph of $D(x)$ match your prediction in part (b)? Identify the function family to which $D(x)$ belongs.

6. **Write a function, *D*(x), in terms of x to calculate the difference between the population of the orange-bellied parrots and the yellow-headed parrots. Write *D*(x) as a polynomial in general form.**

Think about:

Refer to the Worked Example for adding polynomials as a guide.

7. **Choose any two x-values in your table. Use your new polynomial function to confirm that your solution in the table for those times is correct. Show your work.**

8. **Use technology to confirm that your graph and the remaining solutions in the table are correct. Explain any discrepancies and how you corrected them.**

9. Eric uses his function $D(x) = -4x^2 - 13x + 95$ to determine that the difference between the number of orange-bellied parrots and the number of yellow-headed parrots 7 years from now will be -192. Is Eric correct or incorrect? If he is correct, explain to him what his answer means in terms of the problem situation. If he is incorrect, explain where he made his error and how to correct it.

10. The next round of the Species game included the red-winged parrot, whose population can be modeled by the function $W(x) = -9x + 80$ and the rainbow lorikeet parrot, whose population can be modeled by the function $L(x) = 2x^2 - 4x + 10$. In both cases, x represents the number of years since the current year.

    a. Write a function, $S(x)$, in terms of x to calculate the total number of red-winged parrots and rainbow lorikeet parrots at any time.

    b. Write a function, $M(x)$, in terms of x to calculate the difference in the number of red-winged parrots and rainbow lorikeet parrots at any time.

    c. Calculate $S(4)$ and $M(4)$. Interpret the meaning of your results.

    d. In four years, how many red-winged parrots will there be? How many rainbow lorikeet parrots will there be?

ACTIVITY 1.4

Combining Functions and Addressing Closure

In this activity, you will practice adding and subtracting polynomials.

1. Analyze each student's work. Determine the error and make the necessary corrections.

Marco

$3x^2 + 5x^2 = 8x^4$

Kamiah

$2x - (4x + 5)$

$2x - 4x + 5$

$-2x + 5$

Alexis

$(4x^2 - 2x - 5) + (3x^2 + 7)$

$(4x^2 + 3x^2) - (2x) - (5 + 7)$

$7x^2 - 2x - 12$

Consider each polynomial function.

$A(x) = x^3 + 5x^2 - 9$ $\quad B(x) = -3x^2 - x + 1$ $\quad C(x) = 2x^2 + 7x$ $\quad D(x) = -2x^2 - 8x$

2. Determine each function. Write your answers in general form.

a. $J(x) = A(x) + C(x)$

b. $K(x) = D(x) - B(x)$

c. $L(x) = C(x) + D(x)$

d. $M(x) = B(x) - A(x)$

e. $N(x) = A(x) - C(x) - D(x)$

In Question 1, Julie uses the Distributive Property to multiply a monomial and a binomial. She wants to use the Distributive Property to multiply any polynomials.

Worked Example

Consider the polynomials $x + 5$ and $x - 2$. You can use the Distributive Property to multiply these polynomials.

Distribute x to each term of $(x - 2)$, and then distribute 5 to each term of $(x - 2)$.

$$(x + 5)(x - 2) = (x)(x - 2) + (5)(x - 2)$$
$$= x^2 - 2x + 5x - 10$$
$$= x^2 + 3x - 10$$

Think about:

How can you use technology to check your answers?

4. Use the Distributive Property to determine each product. Write the polynomial in general form.

a. $(5x - 1)(2x + 1)$

b. $(x - 7)(x + 7)$

c. $(x + 2)(x - 9)$

d. $(2x^2 + 1)(3x^2 + x - 1)$

5. Explain the mistake in Cheyanne's thinking. Then determine the correct product.

Cheyanne

$$(x + 4)^2 = x^2 + 16.$$

I can just square each term to determine the product.

6. Based on the definition of closure, are polynomials closed under the operation of multiplication? Justify your answer.

ACTIVITY **1.6**

Special Products When Multiplying Binomials

In this activity you will investigate the product of two linear factors when one is the sum of two terms and the other is the difference of the same two terms, and when the two linear factors are the same.

1. Determine each product.

a. $(x - 4)(x + 4) =$ ____________________

$(x + 4)(x + 4) =$ ____________________

$(x - 4)(x - 4) =$ ____________________

b. $(x - 3)(x + 3) =$ ____________________

$(x + 3)(x + 3) =$ ____________________

$(x - 3)(x - 3) =$ ____________________

c. $(3x - 1)(3x + 1) =$ ____________________

$(3x + 1)(3x + 1) =$ ____________________

$(3x - 1)(3x - 1) =$ ____________________

d. $(2x - 1)(2x + 1) =$ ____________________

$(2x + 1)(2x + 1) =$ ____________________

$(2x - 1)(2x - 1) =$ ____________________

2. What patterns do you notice between the factors and the products?

3. Multiply each pair of binomials.

$(ax - b)(ax + b) =$ ____________________

$(ax + b)(ax + b) =$ ____________________

$(ax - b)(ax - b) =$ ____________________

In Questions 1 and 3, you should have observed a few special products. The first type of special product is called the *difference of two squares*. The **difference of two squares** is an expression in the form $a^2 - b^2$ that has factors $(a - b)(a + b)$.

4. Label the expressions in Questions 1 and 3 that are examples of the difference of two squares.

The second type of special product is called a *perfect square trinomial*. A **perfect square trinomial** is an expression in the form $a^2 + 2ab + b^2$ or the form $a^2 - 2ab + b^2$. A perfect square trinomial can be written as the square of a binomial.

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

5. Label the expressions in Questions 1 and 3 that are examples of perfect square trinomials.

6. Use special products to determine each product.

a. $(x - 8)(x - 8)$

b. $(x + 8)(x - 8)$

c. $(x + 8)^2$

d. $(3x + 2)^2$

e. $(3x - 2)(3x - 2)$

f. $(3x - 2)(3x + 2)$

Assignment

Write

Match each definition with its corresponding term.

1. polynomial
2. term
3. coefficient
4. monomial
5. binomial
6. trinomial
7. degree of a term
8. degree of a polynomial

a. a polynomial with only 1 term
b. the degree of the term with the greatest exponent
c. a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients
d. a polynomial with exactly 3 terms
e. any number being multiplied by a power within a polynomial expression
f. each product in a polynomial expression
g. a polynomial with exactly 2 terms
h. the exponent of a term in a polynomial

Remember

- The difference of two squares is an expression in the form $a^2 - b^2$ that has factors $(a + b)(a - b)$.
- A perfect square trinomial is an expression in the form $a^2 + 2ab + b^2$ or in the form $a^2 - 2ab + b^2$ that has the factors $(a + b)^2$ and $(a - b)^2$, respectively.

Practice

1. Ramona and James each build a rocket launcher. They launch a model rocket using Ramona's launcher and on its way back down it lands on the roof of a building that is 320 feet tall. The height of the rocket can be represented by the equation $H_1(x) = -16x^2 + 200x$, where x represents the time in seconds and $H_1(x)$ represents the height. Ramona and James take the stairs to the roof of the building and re-launch the rocket using James's rocket launcher. The rocket lands back on the ground. The height of the rocket after this launch can be represented by the equation $H_2(x) = -16x^2 + 192x + 320$.
   a. Compare and contrast the polynomial functions.
   b. Use technology to sketch a graph of the functions.
   c. Does it make sense in terms of the problem situation to graph the functions outside of Quadrant I? Explain your reasoning.
   d. Explain why the graphs of these functions do not intersect.
   e. Ramona believes that she can add the two functions to determine the total height of the rocket at any given time. Write a function $S(x)$ that represents the sum of $H_1(x)$ and $H_2(x)$. Show your work.

f. Is Ramona correct? Explain your reasoning.

g. Subtract $H_1(x)$ from $H_2(x)$ and write a new function, $D(x)$, that represents the difference. Then, explain what this function means in terms of the problem situation.

2. Determine whether each expression is a polynomial. If so, identify the terms, coefficients, and degree of the polynomial. If not, explain your reasoning.

a. $-2b^4 + 4b - 1$

b. $6 - g^{-2}$

c. $8h^4$

d. $9w - w^3 + 5w^2$

e. $x^{\frac{1}{2}} + 2$

f. $\frac{4}{5}y + \frac{2}{3}y^2$

3. Given $A(x) = x^3 - 5x + 4$, $B(x) = 2x^2 + 5x - 6$, and $C(x) = -x^2 + 3$, determine each function. Write your answer in general form.

a. $D(x) = B(x) + C(x)$

b. $E(x) = A(x) + B(x)$

c. $F(x) = A(x) - C(x)$

d. $G(x) = C(x) - B(x)$

e. $H(x) = A(x) + B(x) - C(x)$

f. $J(x) = B(x) - A(x) + C(x)$

4. Determine each product.

a. $(x - 7)(x - 7)$

b. $(x + 10)(x - 10)$

c. $(x + 6)^2$

d. $(2x + 5)^2$

e. $(2x - 5)(2x - 5)$

f. $(2x - 5)(2x + 5)$

Using a Factor Table to Multiply Binomials

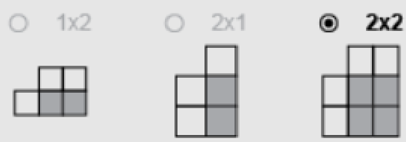
When a monomial is multiplied to a polynomial, we apply the distributive property. The distributive property is also applied when multiplying polynomials together. Each term of one polynomial is multiplied to every term of the other polynomial. A factor table is a useful visual for this concept.

Example:

Use a factor table to multiply $3x + 4$ and $x - 6$.

In this example, we are multiplying a binomial by a binomial. In other words, two terms by two terms. Therefore, we need a 2 x 2 factor table.

Select the dimensions of the Factor Table.



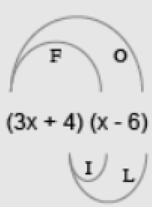
Enter each expression's terms as headings in the table. (Be aware of the signs of each term.)

	x	-6
$3x$		
4		

Multiply each term in a row by each term in a column to complete the table.

.

	x	-6
$3x$	$3x^2$	$-18x$
4	$4x$	-24



The product of $3x - 4$ and $x - 6$ is $3x^2 - 18x + 4x - 24$.

Combine like terms to write in simplest form: $3x^2 - 14x - 21$

Skills Practice

Name ______________________________ Date ______________

Use a factor table to multiply.

1. $3x + 4$ and $2x + 2$

2. $5m + 3$ and $4m + 6$

3. $6t + 5$ and $7t - 5$

4. $4x + 2$ and $4x - 2$

5. $10w - 1$ and $9w + 8$

6. $y + 12$ and $5y + 15$