Week of: June 8 to June14, 2020

Grade: 8 **Content:** Math

Learning Objective Blurb:

Greetings 8th graders! We hope you are safe and well with your families!
This week, students learn to use the substitution method to solve systems of linear equations. They use substitution to solve systems of linear equations including systems with no solution or with infinite solutions. They define variables, write systems of equations, solve systems, and interpret the meaning of the solution in terms of the problem context.
We've included some video links to help you if you get stuck!   This work will not be graded, just do your best and have fun!

Carnegie Learning: Use with Carnegie Resources provided below:
Video 1: Systems of Equations using substitution Part 1
https://vimeo.com/402915532
Video 2: Systems of Equations using substitution Part 2
https://vimeo.com/403300643
Printable Resources:
Skills Practice: Module 3, Topic 2, Lesson 3: ***The Country Fair***
Family Guide below

Practice Activities:

On-Line
All students now have access to an on-line program called Mathia!
Mathia- If you are already in Mathia, please continue to work in the program. This is covered in Module 3 Unit 5.
If you are new to Mathia: Please see the log-in information attached.

Video Links:

Khan Academy: Refresh your memory with any or all of the following:
Solving Systems of Equations using Substitution
https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-systems-topic/cc-8th-systems-with-substitution/v/the-substitution-method

Module 3: Modeling with Linear Equations

TOPIC 2: SYSTEMS OF LINEAR EQUATIONS

In this topic, students analyze and solve pairs of simultaneous linear equations. Throughout the topic, students write systems of equations to represent problem situations. To build fluency with solving systems of linear equations using inspection, graphing, and substitution, students write and solve additional systems of linear equations, using the structure of the equations in the system to determine the most efficient solution strategy.

Where have we been?

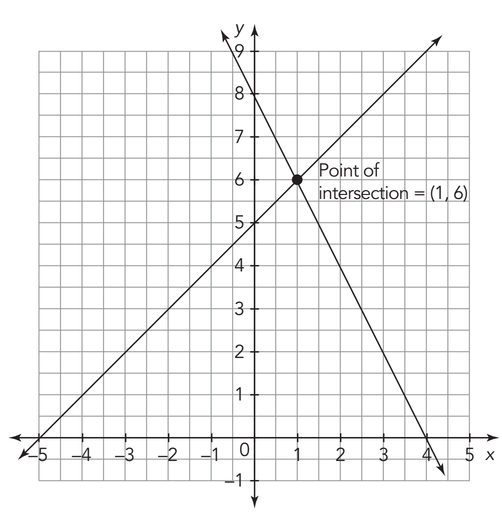
In this topic, students utilize a great deal of what they have learned in this course and previous courses about linear relationships, tables, graphs, and equations, and proportionality to solve problems and investigate solutions to multiple linear equations.

Where are we going?

Students' experiences in this topic provide the foundation for a more rigorous and abstract study of systems of equations in high school. In high school, students will solve systems that include equations that are not linear, and they will use algebraic and graphical techniques to solve systems of inequalities.

Modeling a Solution to a System with a Point of Intersection

If a system of two linear equations has one solution, that solution can be modeled as the point of intersection of the graphs of the two equations. For this system, when $x = 1$ and $y = 6$, both equations are true.



$$\begin{cases} y = x + 5 \\ y = -2x + 8 \end{cases}$$

Myth: Memory is like an audio or video recording.

Let's play a game. Memorize the following list of words: strawberry, grape, watermelon, banana, orange, peach, cherry, blueberry, raspberry. Got it? Good. Some believe that the brain stores memories in pristine form. Memories last for a long time and do not change—like a recording. Without looking back at the original list, was apple on it?

If you answered "yes," then go back and look at the list. You'll see that apple does not appear, even though it seems like it should. In other words, memory is an active, reconstructive process that takes additional information, like the category of words (e.g., fruit), and makes assumptions about the stored information.

This simple demonstration suggests memory is not like a recording. Instead, it is influenced by prior knowledge and decays over time. Therefore, students need to see and engage with the same information multiple times to minimize forgetting (and distortions).

#mathmythbusted

Talking Points

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning more about systems of equations and solving systems.

Questions to Ask

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

Key Terms

break-even point

When one graphed line represents the cost of an item and the other line represents the income from selling the item, the point of intersection is called the break-even point.

solution of a linear system

The solution of a linear system is an ordered pair (x, y) that is a solution to both equations in the system.

substitution method

The substitution method is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

The County Fair

3

Using Substitution to Solve Linear Systems

WARM UP

Analyze each system of equations. What can you conclude about the value of y in each?

1. $\begin{cases} x = 12 \\ y = x + 22 \end{cases}$
2. $\begin{cases} x = 0 \\ y = x - 45 \end{cases}$
3. $\begin{cases} x = y \\ y = 2x - 10 \end{cases}$
4. $\begin{cases} x = y + 3 \\ y = 2x - 10 \end{cases}$

LEARNING GOALS

- Write a system of equations to represent a problem context.
- Solve a system of equations algebraically using substitution.
- Interpret a solution to a system of linear equations in terms of the problem situation.
- Solve real-world and mathematical problems with two linear equations in two variables.

KEY TERMS

- standard form of a linear equation
- substitution method

Suppose you graph a system of equations, but the point of intersection is not clear from the graph? How can you determine the solution to the system?

Getting Started

Goats, Chickens, and Pigs

At the county fair, farmers bring some of their animals to trade with other farmers. To make all trades fair, a master of trade oversees all trades. Assume all chickens are of equal value, all goats are of equal value, and all pigs are of equal value.

- In the first trade of the day, 4 goats were traded for 5 chickens.
- In the second trade, 1 pig was traded for 2 chickens and 1 goat.
- In the third trade, Farmer Lyndi put up 3 chickens and 1 pig against Farmer Simpson's 4 goats.

1. Is this a fair trade? If not, whose animals are worth more? How could this be made into a fair trade?

ACTIVITY
3.1 Introduction to Substitution

In this lesson, you will explore systems of equations that may or may not be accurately solved using graphs. As you have seen, reasoning can also be used to solve systems. In the next activities, you will learn about solving systems algebraically.

The **standard form of a linear equation** is $Ax + By = C$, where A, B, and C are constants and A and B are not both zero.

Janet was helping her mother make potato salad for the county fair and was asked to go to the market to buy fresh potatoes and onions. Sweet onions cost $1.25 per pound, and potatoes cost $1.05 per pound. Her mother told her to use the $30 she gave her to buy these two items.

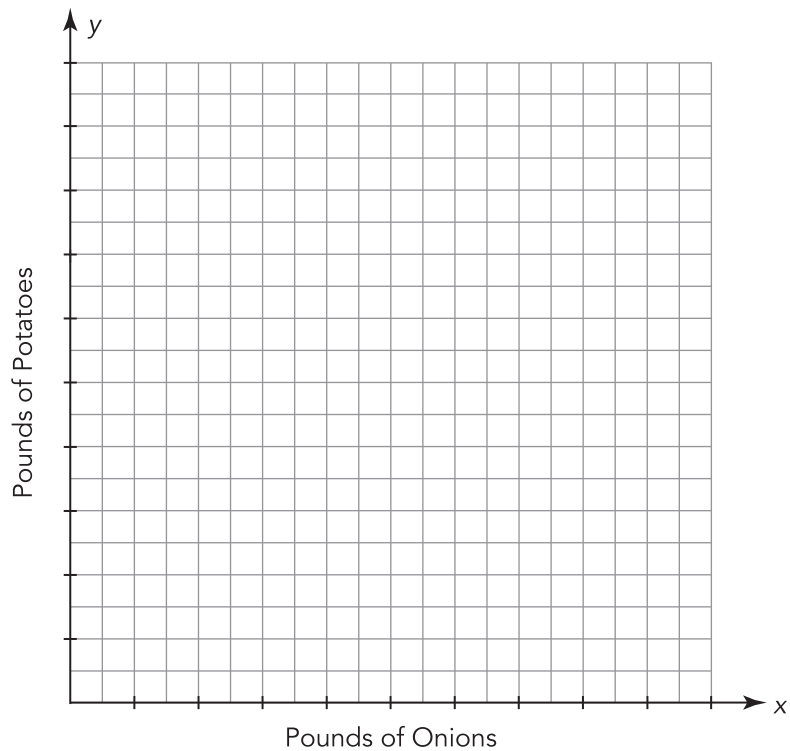
1. **Write an equation in standard form that relates the number of pounds of potatoes and the number of pounds of onions that Janet can buy for $30. Use x to represent the number of pounds of onions, and y to represent the number of pounds of potatoes that Janet can buy.**

2. **Janet's mother told her that the number of pounds of potatoes should be 8 times greater than the number of pounds of onions in the salad. Write an equation in x and y that represents this situation.**

3. **Will 1 pound of onions and 8 pounds of potatoes satisfy both equations? Explain your reasoning.**

4. **Create graphs of both equations. Choose your bounds and intervals for each quantity.**

Variable Quantity	Lower Bound	Upper Bound	Interval



5. **Can you determine the exact solution of this linear system from your graph? Explain your reasoning.**

6. **Estimate the point of intersection from your graph.**

In many systems, it is difficult to determine the solution from the graph. There is an algebraic method that can be used called the *substitution method*. The **substitution method** is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope of the line and b is the y-intercept of the line.

WORKED EXAMPLE

Let's consider the system you wrote.

$$\begin{cases} 1.25x + 1.05y = 30 \\ y = 8x \end{cases}$$

Because $y = 8x$ is in slope-intercept form, use this as the first equation.

Step 1: To use the substitution method, begin by choosing one equation and isolating one variable. This will be considered the first equation.

Step 2: Now, substitute the expression equal to the isolated variable into the second equation.

Substitute $8x$ for y in the equation $1.25x + 1.05y = 30$.

Write the new equation.

$$\begin{aligned} 1.25x + 1.05y &= 30 \\ 1.25x + 1.05(8x) &= 30 \end{aligned}$$

You have just created a new equation with only one unknown.

Step 3: Solve the new equation.

$$\begin{aligned} 1.25x + 8.40x &= 30 \\ 9.65x &= 30 \\ x &\approx 3.1 \end{aligned}$$

Therefore, Janet should buy approximately 3.1 pounds of onions.

Now, substitute the value for x into $y = 8x$ to determine the value of y.

$$y = 8(3.1) = 24.8$$

Therefore, Janet should buy approximately 24.8 pounds of potatoes.

Step 4: Check your solution by substituting the values for both variables into the original system to show that they make both equations true.

NOTES

7. Check that the solution is correct. Show your work.

8. What is the solution to the system? What does it represent in terms of the problem situation?

9. Compare your solution using the substitution method to the solution on your graph. What do you notice?

ACTIVITY
3.2

Substitution with Special Systems

Samson and Adrian are helping to set up the booths at the fair. They are each paid $7 per hour to carry the wood that is needed to build the various booths. Samson arrives at 7:00 A.M. and begins working immediately. Adrian arrives 90 minutes later and starts working.

1. **Write an equation that gives the amount of money that Samson will earn, *y*, in terms of the number of hours he works, *x*.**

2. **How much money will Samson earn after 90 minutes of work?**

3. **Write an equation that gives the amount of money Adrian will earn, *y*, in terms of the number of hours since Samson started working, *x*.**

4. **How much money will each student earn by noon?**

5. Will Adrian ever earn as much money as Samson? Explain your reasoning.

6. Write a system of linear equations for this problem situation.

7. Analyze the system of linear equations. What do you know about the solution of the system by observing the equations? Explain your reasoning.

Let's see what happens when we solve the system algebraically.

8. Since both equations are written in slope-intercept form as expressions for *y* in terms of *x*, substitute the expression from the first equation into the second equation.

a. Write the new equation.

b. Solve the equation for *x*.

c. Does your result for *x* make sense? Explain your reasoning.

9. **What is the result when you algebraically solve a linear system that contains parallel lines?**

On Monday night, the fair is running a special for the the local schools: if tickets are purchased from the school, you can buy student tickets for \$4 and adult tickets for \$4. You buy 5 tickets and spend \$20.

10. **Write an equation that relates the number of student tickets, *x*, and the number of adult tickets, *y*, to the total amount spent.**

11. **Write an equation that relates the number of student tickets, *x*, and the number of adult tickets, *y*, to the total number of tickets purchased.**

12. **Write both equations in slope-intercept form.**

13. **Analyze the system of linear equations. What do you know about the solution of the system by looking at the equations?**

NOTES

Let's see what happens when you solve the system algebraically.

14. Since both equations are now written in slope-intercept form as expressions for *y* in terms of *x*, substitute the expression from the first equation into the second equation.

a. Write the new equation and solve the equation for *x*.

b. Does your result for *x* make sense? Explain your reasoning.

15. How many student tickets and adult tickets did you purchase?

16. What is the result when you algebraically solve a linear system that contains two lines that are actually the same line?

ACTIVITY 3.3

Solving Systems by Substitution

Write and solve a system of equations to solve each problem.

1. The admission fee for the fair includes parking, amusement rides, and admission to all commercial, agricultural, and judging exhibits. The cost for general admission is $7, and the price for children under the age of 5 is $4. There were 449 people who attended the fair on Thursday. The admission fees collected amounted to $2768.

a. Write a system of equations in standard form for this situation. Use *x* to represent the number of people 5 and over, and use *y* to represent the number of children under 5 years of age.

b. Without solving the system of linear equations, interpret the solution.

c. Solve the system of equations using the substitution method. Then interpret the solution of the system in terms of the problem situation.

2. **The business manager for a band must make $236,000 from ticket sales to cover costs and make a reasonable profit. The auditorium where the band will play has 4000 seats, with 2800 seats on the main level and 1200 on the upper level. Attendees will pay $20 more for main-level seats.**

   a. **Write a system of equations with *x* representing the cost of the main-level seating and y representing the cost of the upper-level seating.**

   b. **Without solving the system of linear equations, interpret the solution.**

   c. **Solve the system of equations using the substitution method. Then interpret the solution of the system in terms of the problem situation.**

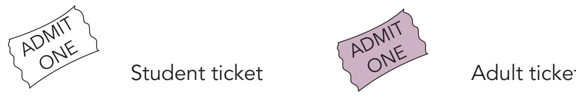
3. Ms. Ross told her class that tomorrow's math test will have 20 questions and be worth 100 points. The multiple-choice questions will be 3 points each, and the open-ended response questions will be 8 points each. Determine how many multiple-choice and open-ended response questions will be on the test.

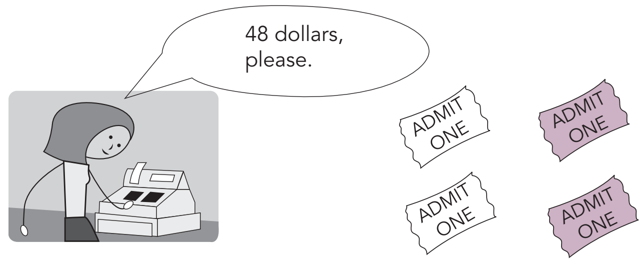
   a. Write a system of equations. Describe your variables.

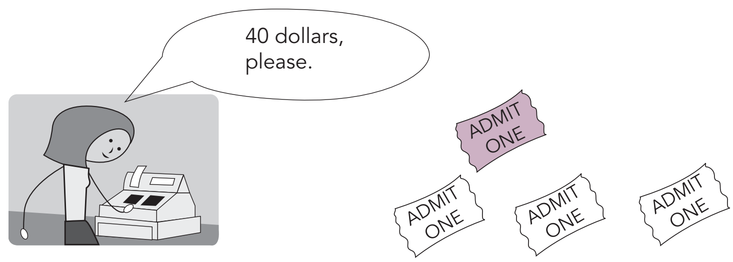
   b. Without solving the system of linear equations, interpret the solution.

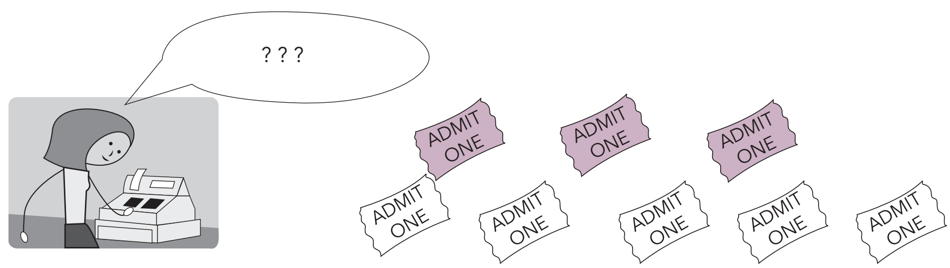
   c. Solve the system of equations using the substitution method. Then interpret the solution of the system in terms of the problem situation.

4. **Ashley is working as a cashier at the sports arena. What should she tell the next person in line?**









Write and solve a system of equations that represents the problem situation. Define the variables. Then determine the cost of each type of ticket. Finally, state the amount Ashley charges the third person.

5. **Alex is applying for positions at two different electronic stores in neighboring towns. The first job offer is a $200 weekly salary plus 5% commission on sales. The second job offer is a $75 weekly salary plus 10% commission.**

   a. **Write a system of equations that represents the problem situation. Define the variables. Then solve the system of linear equations and interpret the solution in terms of the problem situation.**

   b. **What is the difference in the weekly pay between stores if Alex sells $3000?**

   c. **What is the difference in the weekly pay if he sells $4225?**

   d. **Which job offer would you recommend Alex take? Explain your reasoning.**

Alex's sales targets for each job would be between $1500 and $3000 weekly. Each manager tells Alex the same thing: "Some weeks are better than others, depending on the time of year and the new releases of technology."

NOTES

TALK the TALK

The Substitution Train

1. Determine the solution to each linear system by using the substitution method. Check your answers algebraically.

a. $\begin{cases} 2x + 3y = 34 \\ y = 5x \end{cases}$

b. $\begin{cases} y = 4x + 2 \\ y = 3x - 2 \end{cases}$

c. $\begin{cases} 3x + 2y = 4 \\ 2x - y = 5 \end{cases}$

d. $\begin{cases} 3x + y = 8 \\ 6x + 2y = 10 \end{cases}$

Assignment

Write

Explain how to use the substitution method to solve systems of linear equations.

Remember

When a system has no solution, the equation resulting from the substitution step has no solution.

When a system has infinite solutions, the equation resulting from the substitution step has infinite solutions.

Practice

1. Serena is trying to become more environmentally conscious by making her own cleaning products. She researches different cleaners and decides to make furniture polish using olive oil and lemon juice. She wants to make enough to fill two 24-ounce bottles.
   a. Write an equation in standard form that relates the amount of olive oil and lemon juice to the total amount of mixture Serena wants to make. Use x to represent the amount of lemon juice and y to represent the amount of olive oil.
   b. The recommendation for the mixture is that the amount of olive oil be twice the amount of lemon juice. Write an equation in terms of x and y as defined in part (a) that represents this situation.
   c. Use substitution to solve the system of equations. Check your answer.
   d. What does the solution of the system represent in terms of the mixture?
   e. The best price Serena can find for lemon juice is $0.25 per ounce. The best price she can find for olive oil is $0.39 per ounce. She buys a total of 84 ounces of lemon juice and olive oil, and spends $29.40. Write equations in standard form for this situation. Use x to represent the amount of lemon juice she buys, and use y to represent the amount of olive oil she buys.
   f. Solve the system of equations you wrote using the substitution method. Check your answer. Describe the solution in terms of the problem situation.
2. In an effort to eat healthier, Bridget is tracking her food intake by using an application on her phone. She records what she eats, and then the application indicates how many calories she has consumed.
   One day, Bridget eats 10 medium strawberries and 8 vanilla wafer cookies as an after-school snack. The caloric intake from these items is 192 calories. The next day, she eats 20 medium strawberries and 1 vanilla wafer cookie as an after-school snack. The caloric intake from these items is 99 calories.
   a. Write a system of equations for this problem situation. Define your variables.
   b. Without solving the system of linear equations, interpret the solution.
   c. Solve the system of equations using the substitution method. Check your work.
   d. Interpret the solution of the system in terms of the problem situation.
   e. Bridget's friend Monica also has a calorie counting application on her phone. The two friends decide to compare the two programs. Bridget eats 1 banana and 5 pretzel rods, and her application tells her she consumed 657 calories. Monica eats 1 banana and 5 pretzel rods, and her application tells her she consumed 656 calories. The girls want to know how many calories are in each food. Write a system of equations for this problem. Define your variables.
   f. Solve the system of equations using the substitution method. Interpret your answer in terms of the problem.

3. Write a system of linear equations to represent each situation. Then solve the system using substitution. Interpret the solution of the system in terms of the problem situation.
   a. James has 13 coins. The coins are nickels and quarters. The coins have a total value of \$2.05. Let n represent the number of nickels, and let q represent the number of quarters.
   b. Ms. Snyder is giving a 28-question test that is made up of 2-point questions and 4-point questions. The entire test is worth 100 points. Let t represent the number of 2-point questions, and let f represent the number of 4-point questions.
   c. The basketball team scored 82 points from 2-point and 3-point baskets. They make 38 baskets altogether. Let a represent the number of 2-point baskets, and let b represent the number of 3-point baskets.

4. Use the substitution method to determine the solution of each system of linear equations. Check your solutions.
   a. $\begin{cases} 9x + y = 16 \\ y = 7x \end{cases}$
   b. $\begin{cases} 3x + \frac{1}{2}y = -3.5 \\ y = -6x + 11 \end{cases}$
   c. $\begin{cases} y = -5x \\ 21x - 7y = 28 \end{cases}$
   d. $\begin{cases} 2x + 4y = -32 \\ y = -\frac{1}{2}x - 8 \end{cases}$

Stretch

Create a system of linear equations with solution (2, 5). Solve the system using substitution to verify your system has the given solution.

Review

1. Graph each system of linear equations to determine the solution to the system.
   a. $y = 34 - \frac{5}{2}x$ and $y = \frac{2}{5}x + 5$
   b. $y = 21x + 144$ and $y = 3(7x + 48)$
2. The population growth (in thousands) for a small town near Bay City can be represented by the expression $x + \frac{4}{5}(x + 315)$, where x represents the number of years since 2005. The population growth (in thousands) for a neighboring town can be represented by the expression $2x - \frac{1}{5}(x - 630)$, where x represents the number of years since 2005. When will the populations of the two towns be the same?
3. Two neighboring towns are not having population growth. In fact, they both have been losing population since 1995. The population decline for one of the towns (in thousands) can be represented by the expression $-\frac{2}{5}(x - 500)$, where x represents the number of years since 1995. The population decline for the other town (in thousands) can be represented by the expression $-\frac{1}{2}x + \frac{1}{10}(x + 2000)$, where x represents the number of years since 1995. When will the populations of the two towns be the same?
4. Solve each equation.
   a. $8(2m + 7) = 10(m + 11)$
   b. $-3(y + 20) = -9y$