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Parental Beliefs on the Efficacy of Productive Struggle and Their Relation to Homework-Helping Behavior

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Productive struggle—expending effort to make sense of something beyond one's current level of understanding—aids in learning mathematics concepts and procedures. In this study, we surveyed 197 parents with children in the 1st to the 5th grade on their beliefs about productive struggle. Beliefs were assessed via questionnaire and rating of a recorded lesson involving productive struggle. Parents also reported how often they helped with math homework and their child's ability in math. The results show that parents had diverse beliefs about the efficacy of productive struggle, with fathers favoring it more than mothers. A significant relation was found between parents' beliefs about productive struggle and reports of their child's ability in math. The findings of this study suggest that for productive struggle to be effective, parents must intentionally facilitate experiences through student-centered approaches. Programs for parents should emphasize specific evidence-based behaviors rather than broad generalizations about increased involvement with homework. Schools and educators should also provide guidance for parents to explain the potential harmful effects of gender stereotypes and parents' own math anxiety and to teach methods for limiting homework interaction while students grapple with difficult problems.

Keywords: Implicit attitudes; Mathematical understanding; Mathematics homework; Parent involvement; Productive struggle.

People commonly believe that activities that create disfluent processing of educational material (i.e., struggle) lead to inefficient learning. However, research has shown that struggle can be productive and actually promote deep learning (Kornell & Son, 2009). Findings from research in mathematics education and cognitive psychology and educational policies such as the Common Core State Standards in Mathematics (CCSSM) have prioritized sense-making activities as a central focus for improving understanding (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). These activities often require students to grapple with intrinsically challenging problems and tasks. Thus, it has become more important than ever to understand how individuals—in particular, parents—view productive struggle and how these views might relate to the way in which parents interact with their children around math. The current study

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investigated two modes of measuring parents' beliefs about productive struggle and analyzed the relationship between these indices and parents' math homework help.

Defining Productive Struggle

For the purposes of this study, we define *productive struggle* as the act of expending effort to make sense of something that is beyond one's current level of understanding (Hiebert & Grouws, 2007). Struggle is "productive" and does not cause frustration when students have sufficient knowledge and tools to tackle novel learning opportunities (Ermeling, Hiebert, & Gallimore, 2015). Social development theory describes this appropriately situated intellectual stretch as the *zone of proximal development*—the difference between what a learner can do without help and what he or she can do with help (Vygotsky, 1978). Distinguishing between productive and unproductive struggles in mathematics involves at least four critical factors: (a) Are students struggling with the mathematics or with the instructions or procedures for the task? (b) Do the students have sufficient prior knowledge to tackle the problem? (c) Are students able to make progress and meaningfully attempt a solution? (d) Are students struggling with the central mathematical concepts or with peripheral aspects of the problem that might be distracting them from the key ideas and learning goals? (Hiebert & Grouws, 2007; Kapur & Bielaczyc, 2012; Roll, Baker, Aleven, & Koedinger, 2014; Warshauer, 2015).

Productive Struggle in School

Research on the benefits of intentional struggle-filled learning has a long history, with thinkers such as John Dewey (Hiebert & Grouws, 2007), Vygotsky (1978), and Piaget (1952) theorizing about the benefits of exposing children to challenging learning experiences that allow them to construct their own understanding. However, only recently have studies begun to provide empirical evidence for the benefits underlying productive struggle during learning. As noted in our definition, productive struggle is based on the notion that destabilizing a child's knowledge of the world and exposing him or her to scaffolded activities that are just beyond his or her individual level of understanding drive learning. Productive struggle has become a prevalent topic in education circles recently because the CCSSM are heavily influenced by the notion of struggling and persisting with math problems. Initial evidence of a relationship between productive struggle and achievement in mathematics came from the Third International Mathematics and Science Study (TIMSS) video study. The goal of TIMSS was to assess how math and science were being taught in eighth-grade math classes in countries around the world and, in doing so, help unravel which teaching practices facilitated student achievement (Hiebert & Stigler, 2000; Hiebert et al., 2005). In higher performing regions like Japan and Hong Kong, for instance, teachers provided students with more opportunities to engage in struggle-inducing activities such as creating their own methods for solving problems based on previous lessons, writing their own proofs, and applying concepts to new types of problems (Hiebert et al., 2005). In contrast, students in the United States spent more time practicing well-learned procedural activities that had just been demonstrated by the teacher.

Practice on procedural skills can present opportunities for productive struggle if the skill must be adapted to fit novel problems or if students are required to justify

why a procedure works. Research in the area of deliberate practice also suggests that practice of procedural skills can present productive challenges for students when these activities are highly structured, contain a specific goal in mind, and require students to expend a great degree of effort with an intense focus on problems that are within their zone of proximal development (Ericsson, Krampe, & Tesch-Römer, 1993). However, although procedural practice can be a form of productive struggle, U.S. math instruction often converts important problems designed for deeper thinking, comparison, and analysis into routine exercises (Hiebert et al., 2005).

Productive struggle has also been illustrated in K–12 case studies that document teachers' efforts to move from traditional methods of lecture and practice of well-learned procedures to more open-ended instructional approaches that facilitate student inquiry around challenging problems (Ermeling, 2010; Reinhart, 2000; Smith, 2009). For instance, in the course of developing an inquiry-based professional development program for science teachers, Ermeling (2010) found that promoting productive struggle helped teachers to better assess what misconceptions students held about important concepts in physics, biology, and chemistry. In instances when the participating teachers failed to adhere to the struggle-promoting lesson plans, students responded to the lesson as if they understood the material but still failed to grasp the concept that was being taught. Productive struggle requires a substantial shift in how educators (both teachers and parents alike) approach challenging problems and tasks but can lead to students' deeper understanding and application of underlying concepts.

The claim that productive struggle can enhance deeper learning and understanding is also well supported by research from cognitive psychology investigating the mechanism underlying the benefits of struggle. For instance, many findings illustrate that increasing the difficulty of student learning activities results in the building of relevant associations (Carpenter, 2009), increased elaboration (Rowland, 2014) and semantic precision (Karpicke & Zaromb, 2010), and a heightened knowledge of how one knows what one knows and does not know (Benjamin & Bird, 2006; Toppino & Cohen, 2010), as well as potentiating subsequent learning (Richland, Kornell, & Kao, 2009). Indeed, decades of research on what is termed *desirable difficulties* suggests that activities that promote effortful study and retrieval can be efficacious (Bjork & Bjork, 2011; Clark & Bjork, 2014). Instructors can implement desirable learning principles by spacing out study episodes (Cepeda, Pashler, Vul, Wixted, & Rohrer, 2006), using a spiral curriculum (Bunce, VandenPlas, & Soulis, 2011), requiring students to self-generate information (Crutcher & Healy, 1989), interweaving topics during homework (Rohrer & Taylor, 2007), and providing students with frequent opportunities for retrieval practice (Khanna, Brack, & Finken, 2013; Lawrence, 2013). What all these practices have in common is that they force students to expend a great degree of effort in recalling content and hence create productive struggle experiences. Classroom cognitive science studies also find that engaging in what Kapur (2008, 2010, 2014) calls *productive failure* leads to better retention of material, better conceptual understanding, and greater variety in solution approaches to complex math problems. Kapur also postulates that engaging in productive failure, much like productive struggle, facilitates exploration of the problem space through repeated attempts at analyzing the problem (Kapur, 2008). Ultimately, situations

that require a student to struggle may activate prior knowledge and promote greater differentiation during the problem-solving phase, which in turn alleviates the adverse effects of higher cognitive load that is often reported by students in these studies, thus allowing them to outperform those students who do not experience struggle (Kapur, 2014).

Parental Perceptions of Productive Struggle

Although there is substantial evidence that productive struggle is beneficial to students within the classroom, little is known about how productive struggle might facilitate learning in other environments, such as the home. Historically, formal education in the United States was initially contained within the home, but as education transitioned to a public institution, parents gradually withdrew from the educational process and placed the onus of student achievement on schools and teachers (Jeynes, 2013). Parental involvement came to the attention of research circles around the 1970s when researchers began to notice the correlation between declining SAT scores and increases in divorces (Jeynes, 2013; McLanahan & Sandefur, 1994; "On Further Examination," 1977). This observation heightened focus on the role that parents play in shaping learning opportunities for their children. Additionally, the importance of parental involvement has become more relevant as the federal government has increased legislation of parental involvement in educational reform policies like Goals 2000 and No Child Left Behind (Epstein, 2005; Kohl, Lengua, & McMahon, 2000), which have encouraged school districts to engage parents in their children's education. Given the reported benefits of parental involvement, parents are commonly asked to be involved in their children's education by teachers and schools (Cooper, Lindsay, & Nye, 2000; Kohl et al., 2000; Patrikakou & Weissberg, 2000).

Even though there are various ways that a parent might get involved in their child's math education, many parents may interpret the call for parental involvement as primarily about helping children with math homework. Unfortunately, research on parental involvement around math homework has produced an inconclusive picture (Cooper et al., 2000; Hoover-Dempsey et al., 2001; Patall, Cooper, & Robinson, 2008), with some evidence suggesting that it could even be detrimental to children's education (Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015; Robinson & Harris, 2014). The role that parents play in fostering productive struggle and a positive attitude toward productive struggle during math homework help is important because parents serve as role models for how students view disfluent learning experiences (i.e., experiences where students process information slowly) around math. For instance, research on homework-helping interactions between parents and children shows that children often adopt their parents' attitudes and beliefs about the efficacy of struggle (Epstein, 2005; Haimovitz & Dweck, 2016; Kohl et al., 2000). In one recent study (Haimovitz & Dweck, 2016), researchers examined how parents' beliefs about the efficacy of failure relate to children's own beliefs about whether ability is something that is fixed or something that can grow with effort. They found that parents who viewed failure as something that could enhance (as opposed to debilitate) learning predicted what children subsequently believe about their capacity to grow in their own abilities.

Other studies in the area of homework help indicate that parents can transfer their own negative beliefs to their child during homework help, with a direct

influence on how children learn (Bhanot & Jovanovic, 2005; Gonida & Cortina, 2014; Pomerantz, Wang, & Ng, 2005). Pomerantz, Wang, and Ng (2005) investigated mothers' affect during homework interactions. Using a daily interview method for collecting data, they found that mothers reported a more negative affect in the form of annoyance and frustration on days that they assisted their children with homework. The authors concluded that the negative affect was not a result of mothers having to help their children but instead was a consequence of the mother perceiving the child as helpless. The negative affect reported by mothers was not predictive of their children's emotional and motivational functioning 6 months later as long as the mothers were able to maintain positive affect during the homework-helping interaction. Correspondingly, when mothers reported heightened levels of negative affect and were not able to maintain positive affect during the homework-helping interaction itself, children reported poor emotional functioning 6 months later. This study provides evidence that homework interactions can be a vehicle for transferring more than academic knowledge from a parent to a child. Most importantly, this study highlights the influence of perception: How a parent perceives the child not only affects how the parent chooses to approach the homework interaction but also influences the child's emotions and motivations for challenging educational activities.

Why do teachers and parents in the United States view struggle-filled learning as a mark of ineffective learning? Previous cross-cultural investigations suggest that people in the United States tend to hold a self-enhancing orientation that views failure and effort as threats to success (Heine et al., 2001). Researchers have also noted that Americans tend to ascribe higher value to experiences that are processed faster, smoother, and more fluently (Carrell & West, 2010; Jacoby & Brooks, 1984). For example, students in the United States rate classroom teaching as more effective when the instruction is less challenging and leads to higher immediate performance (Kornell & Hausman, 2016). It is not surprising, then, that students and teachers in the United States often assume that quick, snappy processing and instruction are the mark of effective learning, and these beliefs are passed down culturally within the home and the classroom (Haimovitz & Dweck, 2016; Stevenson & Stigler, 1994; Stigler & Hiebert, 2009).

When operating from this struggle-harms-learning premise, parents may make ineffective decisions regarding their children's education during homework helping. For instance, adults often fail to make good decisions about what leads to effective learning, and those decisions influence test performance (Kornell & Bjork, 2007). Similarly, parents might misunderstand the role of productive struggle in a child's learning and cause more harm than good when they seek to alleviate any signs of disfluent learning. Maloney, Ramirez, Gunderson, Levine, and Beilock (2015), for instance, demonstrated that among parents with math anxiety, a higher frequency of help with homework was associated with lower gains in their own children's math achievement across the school year (Maloney et al., 2015). Similarly, Bhanot and Jovanovic (2005) found that frequent parental attempts to help resulted in diminished confidence among their children.

The aforementioned studies help illustrate that the beliefs that parents bring to homework-helping interactions can significantly affect not only how children approach homework but also how they approach their struggle-filled academic

work in general. Of course, most parents enter these homework interactions with the intent of helping rather than hurting children's learning and disposition. However, the research outlined thus far suggests that parents who prevent students from experiencing productive struggle, perhaps by providing frequent homework help, may disrupt children's learning. Hence, there is an important impetus for understanding what parents believe regarding productive struggle.

Dual Beliefs Toward Productive Struggle

Many parents today face a different way of learning math compared to what they experienced within an educational system that required them to repeatedly solve well-learned math problems for the sake of improving speed and efficiency. Given that instructors (i.e., parents and teachers) often struggle to give up cultural scripts in how they teach math (Stigler & Hiebert, 1998), we surmised that parents may hold conflicting beliefs regarding the efficacy of struggle for learning: Although they self-report believing in the efficacy of productive struggle, their implicit distrust toward new methods that value productive struggle persists (Dovidio, Gaertner, Kawakami, & Hodson, 2002; Wilson, Lindsey, & Schooler, 2000).

One way we can learn more about parent's views of productive struggle in the home is by asking parents to self-report their beliefs toward components and behaviors that promote productive struggle (i.e., persistence, tolerating challenging problems, and working toward conceptual understanding). Asking parents what they think and feel about the underlying behaviors that make up productive struggle can give us insights into parents' deliberate, constructed, and explicit (or conscious) beliefs toward productive struggle. However, these beliefs may be heavily influenced by self-presentational concerns in which the parents attempt to match their beliefs to the experiment's expectations (Baumeister, 1982). Another possibility is that the explicit beliefs expressed by parents may not adequately capture their habitual, automatic, or implicit belief that often persists in memory (Wilson, 2011). The Dual Attitude Model proposed by Wilson, Lindsey, and Schooler (2000) suggests that when an individual changes his or her attitude, it is still possible for the original attitude to persist implicitly and be expressed indirectly. With new educational reforms dictating a focus on improving student reasoning, it is likely that even parents who in theory may subscribe to the new belief about productive struggle may still hold (at an implicit level) a deeply held negative belief about struggle-filled learning. If this is the case, then parents may simultaneously express both a positive and negative disposition toward activities and behaviors that promote productive struggle.

Therefore, we reasoned that although some parents may report believing in the efficacy of productive struggle activities, traditional conceptions around the harm of struggle will lead to a very different pattern of responses using an implicit or indirect measure of parents' beliefs around productive struggle. To get at parents' implicit beliefs, we asked parents to evaluate a video of a classroom lesson that engages students in the disfluent process of productive struggle. This implicit measure allowed us to assess what parents think about productive struggle when seeing it in practice. By using both explicit (i.e., self-report questionnaire) and implicit (evaluation of a classroom lesson video) attitudinal measures, we hoped

to capture parents' dual beliefs around the efficacy of productive struggle. Knowing more about parents' perceptions toward productive struggle can help us understand more about how parents can better facilitate learning math at home and how well parents have internalized math reforms focused on reasoning and struggle.

The Current Study

To that end, this study investigated parent perceptions of productive struggle using a questionnaire and a classroom lesson video where students were engaged in a sustained episode of productive struggle. We sampled 197 parents to assess the relation between their belief in the efficacy of productive struggle (using both explicit and implicit measures) and how often they helped their children with math homework. In addition, parents were asked to report their perceptions of their child's ability in math. Finally, we were also interested in examining whether parents' beliefs are moderated by the gender of both the parent and the child. We know that mathematics is a domain plagued by gendered ability beliefs (Beilock, Gunderson, Ramirez, & Levine, 2010) and that parents often treat their children differently according to their gender (Lytton & Romney, 1991). Even parents who view struggle as effective for learning may provide unsolicited help around homework for girls rather than boys (Bhanot & Jovanovic, 2005; Gunderson, Ramirez, Levine, & Beilock, 2012; Pomerantz et al., 2005). This study aimed to address the following questions:

1. What are parents' explicit and implicit beliefs about the efficacy of productive struggle?
2. Do parents' beliefs about the efficacy of productive struggle differ by parent or child gender?
3. Is there a relation between parents' beliefs in the efficacy of productive struggle and how often they help with math homework?
4. Do parents' perceptions of their child's ability in math relate to parents' beliefs about the efficacy of struggle?

Method

Participants

For our main sample, we recruited a set of 286 parents on Amazon Mechanical Turk (MTurk), an open online marketplace for getting work done by others. MTurk contains essential elements for conducting research, such as an integrated participant compensation system; a large participant pool; and a streamlined process of study design, participant recruitment, and data collection (Buhrmester, Kwang, & Gosling, 2011). Using this online platform, parents were recruited to answer questions regarding their beliefs about the efficacy of productive struggle, about interactions with their child during homework, and their impressions after watching an example classroom lesson video of productive struggle.

To qualify for participation in the study, parents had to answer two qualifying questions in the affirmative: "Are you a parent to a child in elementary school (grades 1st through 5th)?" and "Does your child's teacher assign homework?"

Parents who failed to answer either of the two questions affirmatively were skipped to the end of the survey and thanked for attempting to participate. Parents who qualified to participate were asked several additional questions throughout the main survey to assess participant fidelity. For instance, participants were asked at the beginning and end of the survey to indicate their child's date of birth. These questions were designed to verify that parents were not misrepresenting their parental status. We also embedded an attention check within the middle of the survey to ensure that participants were paying attention. Parents who had more than one child in Grades 1–5 were asked to focus on one child in particular. Those who completed the survey received $2 for their participation.

After omitting participants who did not qualify for our study, because they had no children, they were not consistent on the birthday questions, or they did not answer the attention questions correctly, our main study sample was reduced to a total of 197 individuals. The individuals who remained were parents who reported having children attending elementary school in the first through fifth grades within the United States. The mean age of the 197 parents was 34.2 (SD = 6.91), with 101 (51.3%) of them being females. The mean school grade level of the 197 children that the parents were asked to answer questions about was 2.72 (SD = 1.48), with 108 (54.8%) of them being males (see Table 1 for participant demographics).

Measures and Procedures

Participants who responded to the advertisement on MTurk and met all qualifications for participation were directed to the study's page at Survey Monkey, an online platform for administering surveys. There, the participants were presented with a survey consisting of the Belief in the Efficacy of Struggle Questionnaire (BESQ), the classroom lesson video and associated questions, and two questions about their homework-helping interactions with their children. The survey took approximately thirty minutes to complete. Each component of the survey is described in greater detail in the subsections below.

Belief in the Efficacy of Struggle Questionnaire (BESQ). Parent's explicit beliefs toward productive struggle were measured using a scale designed to tap into the operational definition of productive struggle, which is expending effort to make sense of something that is beyond one's current level of understanding (Hiebert & Grouws, 2007).

Parents were asked to express their agreement with learning situations that vary in the extent to which they might promote productive struggle as described in the literature (e.g., "Children will learn more if they are given math problems that take a long time to think through and solve"). Parents rated each item on a 5-point Likert scale ranging from 1 (Strongly Disagree) to 5 (Strongly Agree).

Survey construction began by generating a total of 49 items which the research team subsequently evaluated. We removed items that (a) were redundant, (b) were poorly worded, (c) were difficult to understand the intended meaning, or (d) did not fit our definitional framework. This initial screening reduced our sample of items to 18. We then pilot tested the remaining 18 items by recruiting a sample of 235 parents from MTurk. This sample of 235 participants was distinct from the

Table 1
Participant Demographics (N = 197)

Variable	Frequency	Percent
Ethnicity		
Native American	2	1.0
Asian	12	6.1
African American	21	10.7
Latino	16	8.1
White	132	67.0
Other	14	7.1
Income (US$)		
Less than 15,000	4	2.0
15,000 to 34,999	35	17.8
35,000 to 49,999	45	22.8
50,000 to 74,999	57	28.9
75,000 to 99,999	26	13.2
100,000 or more	30	15.2
Education		
Less than high school	1	0.5
High school or GED	20	10.2
At least 1 year of college	43	21.8
Associate or equivalent 2-year undergraduate degree	31	15.7
Bachelor's or equivalent 4-year undergraduate degree	69	35.0
Some graduate training (not completed)	7	3.6
Graduate degree	26	13.2

main study sample of 197 parents described above. Once again, we only included those participants who passed all the parent and attention screening questions described above. Each participant was tasked with completing the initial 18-item questionnaire using the same response scale described above. Participants were paid $0.75 for completing the 10-minute pilot survey. Based on this initial pilot, we removed items with average responses that were at the floor or ceiling, which reduced the total size of the questionnaire to 13 items. The remaining 13 items were grouped into the following three broad categories that are based on the understanding of behaviors that influence productive struggle as described in the literature: (1) Enable Extended Struggle versus Provide Immediate Assistance, (2) Encourage Conceptual Inquiry versus Teach/Review Known Procedures, and (3) Foster Tolerance for Mental Angst versus Act to Reduce Mental Angst. Internal consistency (using Cronbach's α) for 12 of the 13 items yielded a value of $\alpha = .857$, which was close to the internal consistency of all 13 items ($\alpha = .853$), and so Item 6 was omitted from the final scale (see the Appendix).

Classroom Lesson Video. As a component of the survey on Survey Monkey, we also measured parent's implicit beliefs toward the efficacy of productive struggle by asking them to watch a video clip that showcased second-grade students engaging in an episode of productive struggle in a Japanese classroom where productive struggle is a regular part of daily instruction. The lesson in the video consisted of a teacher posing the following challenging math problem in Japanese (the video had English subtitles) to a class of second-grade students: Solve the following equation by placing "+" and "–" signs in between the numbers on the left side, 1 2 3 4 5 6 7 8 9 = 100. Possible solutions might include 1 + 2 + 3 – 4 + 5 + 6 + 78 + 9 = 100 or 123 − 45 − 67 + 89 = 100. The teacher explicitly stated that for the first portion of the class, the students would have to try and solve the problem on their own. Students could ask questions and the teacher offered feedback without ever explicitly stating the answer to the problem.

From the original 10-minute clip, a total of 3 minutes and 35 seconds were selected and separated into three segments. The segments were designed to illustrate the progression of productive struggle during a math lesson from the beginning, middle, and end of the lesson. The three segments were shown sequentially with short descriptions in English before each segment explaining the context of the clip. Before each segment, participants were also asked to imagine that what they were watching was happening in an American classroom and to think about how they would feel if this lesson was taking place in their child's math classroom.

At the conclusion of the video, participants answered three questions. The first asked whether they thought the duration of the lesson was appropriate ("After introducing the problem, how long would you allow students to work before you stopped to explain the solutions?") using a 5-point scale (1: 1–5 minutes; 2: 5–10 minutes; 3: 15–20 minutes; 4: 20–25 minutes; and 5: 30 minutes or more). The second and third questions asked participants if they thought the lesson was appropriately difficult ("How appropriate is the level of difficulty in this lesson?") and if it was effective at engaging students to learn ("How effective is this approach for engaging students in learning?"). Participants were asked to answer these two questions using a 5-point scale with only two labels at the anchors (1: not at all appropriate/effective to 5: very appropriate/effective; $\alpha = .662$). The three items were averaged into a single score measuring parents' implicit belief toward productive struggle.

Homework-helping interaction questions. Parents were asked to answer two questions about how often they helped their child with math homework and their perception of their child's ability in math (adapted from Hyde, Else-Quest, Alibali, Knuth, & Romberg, 2006). The first item asked, "How often do you help your child with their math homework?" and had a 6-point response scale (1: never; 2: once a month; 3: 2–3 times per month; 4: once a week; 5: 2–3 times a week; and 6: every day). The second item "In terms of math, my child's current performance is…" had a 5-point response scale (1: poor; 2: fair; 3: good; 4: very good; and 5: excellent).

Results

We begin by describing parents' frequency of homework-helping behaviors. Across the entire sample, the distribution of responses was negatively skewed,

indicating that a large proportion of the parents in the sample helped their children often with math homework ($M = 4.85$, $SD = 1.08$). Approximately 70% of parents reported helping their children with math homework at least 2–3 times per week. To understand whether parental beliefs about the efficacy of productive struggle were related to homework-helping frequency, we analyzed data from the BESQ and responses to the Classroom Lesson Video. In the sections below, we outline our main findings by research question.

Question 1: What Are Parents' Explicit and Implicit Beliefs About the Efficacy of Productive Struggle?

Responses from the 12-item BESQ were computed into a mean score for every respondent (see Table 2). Mean responses on the BESQ ($M = 2.92$, $SD = 0.70$) were normally distributed (Kolmogorov-Smirnov test; $p > .05$). Parent's explicit beliefs in the efficacy of struggle were widely distributed, with 28% of parents believing that struggle is not efficacious for their children when doing math (i.e., responses at 2.5 and below on a 5-point scale). Responses to the three items about the Classroom Lesson Video were computed into a mean score ($M = 2.90$, $SD = 0.83$) for every parent. Parents who had a low mean score believed that the productive struggle lesson was ineffective at engaging students in learning, whereas parents who had a high mean score believed that the productive struggle lesson was effective at engaging students in learning. Mean responses to the Classroom Lesson Video were not normally distributed according to a Kolmogorov-Smirnov test ($p < .05$); however, the distribution did show variability in parents' implicit beliefs toward struggle (see Figure 1). Over 30% of the parents rated the classroom lesson as being ineffective, indicating a portion of parents who do not agree with productive struggle being used in the classroom setting to help children learn (i.e., responses at 2.5 and below on a 5-point scale).

Critically, we found that both measures of parents' beliefs toward productive struggle—the BESQ and the Classroom Lesson Video—were significantly correlated with each other, $r(195) = .231$, $p < .01$, though this correlation was not very strong. The weak correlation between the BESQ and the Classroom Lesson Video indicates that these two measures are measuring different things. These results also reveal that even within the modern Common Core landscape, parents exhibit a variety of views about productive struggle being used as a method to help enhance student learning.

Table 2
Descriptive Statistics

Measures	*N*	Mean	*SD*
Belief in the Efficacy of Struggle Questionnaire (BESQ)	197	2.92	0.70
Classroom lesson video	197	2.90	0.83
Math homework help frequency	197	4.85	1.08
Perceived ability in math	197	3.76	0.89

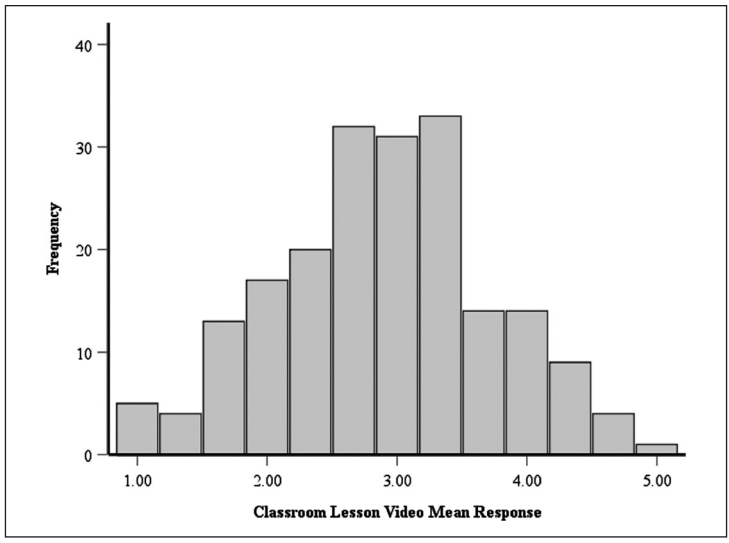

Figure 1. Distribution of parents' responses to the Classroom Lesson Video.

Table 3
Descriptive Statistics by Parent and Child Gender

		Mothers		Fathers	
Measures	Total (N = 197)	Daughters (N = 54)	Sons (N = 47)	Daughters (N = 35)	Sons (N = 61)
BESQ	2.92 (0.70)	2.67 (0.68)	2.99 (0.81)	2.92 (0.53)	3.09 (0.67)
Classroom lesson video	2.90 (0.83)	2.63 (0.91)	2.90 (0.69)	3.15 (0.86)	2.98 (0.81)
Math homework help frequency	4.85 (1.08)	4.89 (1.11)	5.15 (1.02)	4.80 (1.02)	4.61 (1.10)
Perceived ability in math	3.76 (0.89)	3.67 (0.97)	3.81 (0.95)	3.86 (0.77)	3.74 (0.85)

Question 2: Do Parents' Beliefs About the Efficacy of Struggle and Homework Helping Differ By Parent or Child Gender?

Beginning with parents' responses to the BESQ, we tested whether a two-way interaction existed between the parent gender and the gender of the target child (see Table 3 for means by parent and child gender). We found no evidence for a two-way interaction, $F(1, 193) = 0.57$, $p > .05$, $\eta_p^2 = .003$. We subsequently dropped the interaction term and ran a series of planned one-way models with the parent gender and gender of the target child as the predictor variable. We found an effect of the parent gender, $F(1, 195) = 4.53$, $p = .035$, $\eta_p^2 = .023$, with male parents ($M = 3.03$, $SD = 0.63$) reporting higher belief in the efficacy of struggle than female parents ($M = 2.82$, $SD = 0.76$). A separate one-way ANOVA also revealed a main effect of the gender of the target child, $F(1, 195) = 8.00$, $p < .01$, $\eta_p^2 = .039$, with parents of sons ($M = 3.05$, $SD = 0.73$) reporting higher explicit belief in the efficacy of struggle than did parents with daughters ($M = 2.77$, $SD = 0.63$).

These results indicate that, overall, the gender of the parent and child relates to their explicit expectations for the efficacy of struggle in math.

We next examined the responses to the classroom lesson video. Once again, we did not find evidence for a two-way interaction, $F(1, 193) = 3.29$, $p = .07$, $\eta_p^2 = .017$. We followed up by running a series of one-way ANOVAs with the parent gender and gender of the target child as predictors, respectively. We found evidence for a main effect of the parent gender, $F(1, 195) = 5.92$, $p = .016$, $\eta_p^2 = .029$, with fathers ($M = 3.05$, $SD = 0.83$) believing the classroom lesson to be more effective than mothers ($M = 2.76$, $SD = 0.82$). However, we did not find a main effect of child's gender, $F(1, 195) = 0.82$, $p = .37$, $\eta_p^2 = .004$.

Finally, we examined the frequency of help with math homework. There was no significant interaction of the parent and child gender, $F(1, 193) = 2.11$, $p = .14$, $\eta_p^2 = .011$. A subsequent one-way ANOVA revealed that the frequency of homework help differed as a function of parent's gender, $F(1, 195) = 4.75$, $p = .03$, $\eta_p^2 = .024$, with mothers ($M = 5.01$, $SD = 1.07$) reporting helping their children with math homework more often than fathers ($M = 4.68$, $SD = 1.07$). The frequency of how often parents helped with math homework as a function of the child's gender was not significant, $F(1, 193) = 0.05$, $p = .94$, $\eta_p^2 = .00$.

Given our findings that fathers exhibit a higher level of belief in the efficacy of struggle but report helping their children with homework less often raises an important question: Do parents who are more involved in homework hold less favorable views of productive struggle? To evaluate this question, we tested for a main effect of the parent gender on beliefs on the efficacy of struggle while holding frequency of homework help constant (i.e., as a covariate in the model). In a series of ANCOVAs, we found that the parent gender continues to be a significant factor in predicting responses on the BESQ, $F(1, 194) = 4.607$, $p = .03$, $\eta_p^2 = .023$, as well as the classroom lesson video, $F(1, 194) = 4.20$, $p = .04$, $\eta_p^2 = .021$. The frequency of help covariate for the model predicting BESQ was not significant ($p > .05$), whereas the frequency of help covariate for the model predicting the Classroom Lesson Video was significant, $F(1, 194) = 6.36$, $p = .013$, $\eta_p^2 = .032$. Thus, homework-helping frequency does not fully account for the relationship between the parent gender and ratings on the Classroom Lesson Video measure as well as the BESQ.

Question 3: Is There a Relation Between Parents' Beliefs in the Efficacy of Productive Struggle and How Often They Help with Math Homework?

To assess whether parents' explicit belief in the efficacy of struggle related to how often parents help with math homework, we first ran a correlation between the mean score on the BESQ and parents' self-reports on how often they help with math homework (see Table 4 for all correlations). There was no significant correlation between parents' mean scores on the BESQ and their frequency of math homework help, $r(195) = -.001$, $p > .05$, indicating that the extent to which parents explicitly reported believing in the efficacy of struggle does not relate to the frequency of help around math homework. In contrast, we did find a significant correlation between ratings on the Classroom Lesson Video (our implicit measure of struggle) and math homework help frequency, $r(195) = -.200$, $p < .01$, supporting the hypothesis that parents who believe less in the efficacy of productive struggle at an implicit level will help more often with math homework.

Table 4
Correlations Between Measures

Measure	1	2	3
1. Belief in the Efficacy of Struggle Questionnaire (BESQ)	-		
2. Classroom lesson video	.231**	-	
3. Math homework help frequency	-.001	-.200**	-
4. Perceived ability in math	.228**	.234**	.067

**$p < .01$

Question 4: Do Parents' Perceptions of Their Child's Ability in Math Relate to Parents' Beliefs About the Efficacy of Struggle?

Finally, we were interested in addressing how parent perceptions of their child's ability related to their beliefs about the efficacy of productive struggle. Parents' responses to the item measuring how good they believe their child is at math ($M = 3.76$, $SD = 0.89$) were negatively skewed, with approximately 60% of parents—those who selected 4 and 5 on a 5-point scale—reporting that they believed their child was good at math. Parents' perceptions of their child's math ability significantly correlated with their responses to the BESQ, $r(195) = .228$, $p < .005$, indicating that parents who perceived their child to have a high ability in math were also more likely to explicitly believe in the efficacy of struggle. Conversely, parents who perceived their child to be bad at math were less likely to endorse a strong belief in the efficacy of struggle. Parent's perceptions of math ability were also significantly correlated with parents' responses to the Classroom Lesson Video, $r(195) = .234$, $p < .005$, indicating a positive relationship between a parent's perception of their child's ability in math and how they feel (at an implicit level) about productive struggle in the classroom.

Critically, one might wonder if the aforementioned relationship between homework help frequency and productive struggle beliefs might be accounted for by parents' perceptions of their child's math ability. Parents might step in to help more if they perceive that their child needs greater remediation. However, parents' perceptions of their child's math ability did not relate to how frequently parents helped with math homework. We also ran a correlation between parents' homework help frequency and both explicit and implicit productive struggle beliefs, respectively. The relationship between homework help and explicit productive struggle beliefs was no longer significant after controlling for parent perceptions of their child's current ability, $r(194) = -.02$, $p = .81$. By contrast, we continued to find a significant relationship between homework help frequency and implicit beliefs about productive struggle, $r(194) = -.222$, $p < .01$, even after controlling for parents' perceptions of their child's ability.

Discussion

Research in the areas of education and cognitive psychology provides support that expending effort to make sense of something that is not immediately apparent leads to better retention and understanding of the material being taught (Bjork

& Bjork, 2011; Clark & Bjork, 2014; Hiebert & Grouws, 2007; Kapur, 2008, 2010, 2014). However, because many parents may be unfamiliar with these benefits, it is important to understand parent beliefs regarding productive struggle and whether or not these beliefs relate to how they help their children with math homework. Our main findings indicate that parents' views toward productive struggle are diverse, those views can vary as a function of both the parent's and the child's gender, and there is a relationship between endorsing a negative view toward productive struggle and increased frequency of helping with math homework.

Parents' Beliefs About Productive Struggle

The responses to the BESQ and the Classroom Lesson Video indicate that parents' opinions on productive struggle are very diverse. Within our own data, we found that approximately 30% of parents expressed an explicit belief that productive struggle is not something that would benefit their children, and a little less than half of the parents rated the video featuring productive struggle below average with regard to aiding student learning. Both figures are quite surprising given that the basic idea of productive struggle (i.e., growth arises from encountering novel challenges) is evidenced in a host of noneducational domains.

One possible reason for this is that parents may associate productive struggle with relatively new school reform policies such as the CCSSM. The first of eight Standards for Mathematical Practice in the CCSSM states that students ought to be able to "Make sense of problems and persevere in solving them" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 6). Of the 1,001 adults polled for the 46th edition of the PDK/Gallup Poll of the Public's Attitudes Toward the Public Schools, 80% had heard about Common Core and 60% opposed it (Camera, 2014). These findings are backed up by reports from popular media outlets and on social media websites (Kircher, 2015; Mead, 2014; Summers, 2014; Torres, 2014). Parents may object to Common Core reforms and practices such as productive struggle because they do not cohere with how they were taught math. We surmised that the explicit and implicit beliefs that parents hold around productive struggle may relate to how they interact with their own children around homework in the home.

Homework Help Frequency and Productive Struggle

Our main outcome variable, which was used to evaluate the role of productive struggle beliefs in the home, was parents' frequency of help with math homework. Over 70% of parents within our sample reported helping their children with math homework at least two to three times per week. Parents' frequent homework help was not related to their perceptions of their child's ability, which we interpret to suggest that parents believe that frequent forms of support are efficacious for enhancing learning and not simply remediating poor understanding (Hoover-Dempsey & Sandler, 1995, 1997; Hoover-Dempsey, Walker, Jones, & Reed, 2002; Patrikakou & Weissberg, 2000). These findings are perhaps not surprising considering the current climate that encourages parents' involvement in their child's education (Epstein, 2005; Kohl et al., 2000). In addition, the sample for this study focused on parents with children between the first and fifth grades because that

is a time period during which parental involvement is still normative in a child's development (Brumariu & Kerns, 2010). Green and colleagues found that parent's self-efficacy about their involvement was a predictor of the parental level of involvement in elementary students' schoolwork (Green, Walker, Hoover-Dempsey, & Sandler, 2007), suggesting that parents get involved when they feel that their involvement is helpful. In addition, the level of difficulty of the math encountered during elementary school is low for most parents, thereby increasing their sense of efficacy.

Vukovic, Roberts, and Wright (2013) have suggested that lower order math problems are more likely to elicit help from parents because they are easier to comprehend for both the parent and child, whereas higher order math problems can be difficult for both the parent and child because they involve conceptual understanding. In brief, parents frequently become involved in their child's math homework, perhaps under the premise that they are deepening their child's learning by doing so. Unfortunately, parents may be preventing important productive struggle opportunities from taking place when they become too involved. We wondered whether parents who held strong beliefs in the efficacy of struggle would be less likely to help their child with math homework.

Data from the Classroom Lesson Video but not the BESQ supported the hypothesis that a belief in the efficacy of productive struggle will relate to parent's support around homework. Overall, parents who rated the Classroom Lesson Video to be an effective way of teaching children math also reported helping less often with math homework. Why did parents show a different pattern of results for the BESQ than for the Classroom Lesson Video? When an individual reports a change in his or her belief, it is still possible for their original belief to persist implicitly and to be exhibited indirectly. New educational reforms may lead many parents to explicitly subscribe to the new beliefs about productive struggle being efficacious; however, they may have difficulty shedding their original belief when presented with a less abstract representation of what productive struggle looks like via our video measure. The use of the BESQ was meant to assess parents' explicit beliefs, whereas the use of the Classroom Lesson Video was meant to assess parents' implicit and more enduring beliefs. We found that the BESQ and Classroom Lesson Video were only mildly correlated, which is well in line with a prior meta-analysis (Hofmann, Gawronski, Gschwendner, Le, & Schmitt, 2005) examining the relationship between implicit and explicit belief measures.

There is also the potential that other aspects of the homework-helping interaction can be influencing parents' beliefs about productive struggle. For instance, Pomerantz and colleagues (2005) implicated both the affective experience of the homework-helping interaction and the parents' perceptions of their child's ability to perform well. We did find a significant relationship between parents' perceptions of their child's ability at math and parents' belief in productive struggle—on both the BESQ and the responses to the Classroom Lesson Video. However, we continued to find an inverse relationship between the Classroom Lesson Video ratings and homework-frequency help even after accounting for parents' perception of their child's ability in math.

Parents who perceive their child to be good at math may find that productive struggle is an effective method for their child to learn because the child has the

necessary prior knowledge to persist during difficult math problems. Some studies support this interpretation, finding that having relevant prior knowledge prepares students to infer unstated information when faced with challenging problems (McNamara, Kintsch, Songer, & Kintsch, 1996). This interpretation is also consistent with the notion that for productive struggle to be effective, tasks must fall within a child's zone of proximal development. Any math problem that fails to meet this basic requirement may discourage the child from persisting with the task. As a result, parents who perceive their child to not be good at math may be more inclined to avoid struggle altogether out of fear that struggle will lead the child to feel frustrated or disengaged. These parents would be justified in avoiding a struggle-oriented curriculum if the problems it presents fall outside of their child's zone of proximal development. Parents who have positive experiences when helping their children with math homework, however, may be more receptive to productive struggle because they know that their child will generally maintain a positive attitude, positive belief, and good behavior even while facing a challenging math problem.

Gender Differences

Our data analysis revealed significant differences between fathers and mothers on the BESQ, the Classroom Lesson Video, and the frequency of math homework help. The results from both the BESQ and the Classroom Lesson Video, for instance, indicate that fathers may value productive struggle more than mothers. It is difficult to say exactly why this may be the case, but this pattern of results is consistent with meta-analyses showing that men generally take more risks than women (Byrnes, Miller, & Schafer, 1999) and that fathers provide more leeway during risky situations (Paquette & Bigras, 2010), which may encourage their children to explore their world (Pruett, 2000). However, a more skeptical interpretation of this pattern of results is that fathers may simply have a vaguer understanding of the thinking involved when doing homework because they are generally less involved in school matters. We evaluated this hypothesis and found that the effect of the parent gender on struggle beliefs remained even after covarying out the frequency of homework help.

We also found that parents with sons reported a stronger explicit belief in productive struggle than parents with daughters. These differences may be the result of parents' endorsement of gender stereotypes about math, which allow for productive struggle as a suitable learning experience for boys. The synonyms of the word *struggle* include words like *fight*, *grapple*, *wrestle*, and *brawl*, all of which may be more often associated with boys than with girls. Parents' gender stereotypes may influence how they interact with their children during homework (Bhanot & Jovanovic, 2005). This is especially relevant with mathematics, which is still stereotypically classified as a male domain, and parents are known to enforce gender-typed activities (Lytton & Romney, 1991). If parents are carrying these associations with them when being assessed about their views of productive struggle for their own children, they may be more likely to view productive struggle as more appropriate for males than females.

Knowing how beneficial productive struggle can be for understanding math, it is important that future studies attempt to disentangle in what way mothers and

fathers are treating their sons and daughters differently when it comes to their math homework. If encouraging daughters to persist and struggle with math can lead to a better understanding, productive struggle may be a useful tool for closing the gender gap in math and perhaps other STEM fields as well.

Implications

Overall, the results reported here should serve as positive news for education reformers. The majority of parents are trying to accommodate a productive struggle belief in the home (as revealed by the BESQ), but it will take some time for them to undergo the deep conceptual change required for their explicit and implicit beliefs to become aligned. We do not make a blanket recommendation that parents reduce the frequency of homework help that they provide; instead, we focus on the quality of help. For productive struggle to be effective, parents must intentionally facilitate these experiences as demonstrated in the use of exploratory or delayed instruction, having student-led question-and-answer sessions, and having students reason through failed attempts at problem solving (DeCaro & Rittle-Johnson, 2012; Kapur, 2010, 2014; Reinhart, 2000; Smith, 2009). Although parents are typically not trained teachers, they are often the only person that children can depend on when it comes to homework. A student-centered approach to homework might be an efficacious approach toward ensuring that frequent interactions between parents and children can lead to beneficial academic outcomes for the child.

In general, there is a great need for research-based programs and resources that provide guidance to parents on involvement in math homework. Existing policy and recommendations that broadly emphasize increased parental involvement are based on general assumptions about the efficacy of parental engagement, whereas, in fact, much of the research on parental involvement in the home shows limited or no benefits for student achievement. For example, Robinson and Harris (2014) studied 18 measures of parental involvement in student homework and found no significant influence on grades or test scores compared to students who have less interaction with parents around homework. Consistent with our findings, some measures also showed a negative influence on scores which the authors suggested might be a result of parents who have forgotten or never completely understood the homework material. In addition, evidence for the effectiveness of other forms of involvement—such as home support and parental expectations—suggests that there are less intrusive ways that parents can have a positive impact on their children's math achievement (Vukovic, Roberts, & Wright, 2013). More research and guidance are needed on the specific types of involvement that are most productive and least productive or even harmful for students. Policy and programs for parents should increasingly focus on specific activities and behaviors that are supported by evidence rather than broad generalizations about increased involvement with homework.

We also encourage parents to be mindful of the implicit messages that they send around productive struggle. If more enduring beliefs toward productive struggle are indeed implicit, then parents need to work to counteract these messages that they may be sending by explicitly modeling how struggle and failure are an important part of learning. Parents clearly play a role in how children interpret disfluent learning experiences and make meaning of their broader educational encounters.

Children are well attuned to cues from their environment and are likely to infer what they should believe about productive struggle from interactions with their parents and teachers (Ambady & Gray, 2002; Ambady & Rosenthal, 1993).

In addition to offering some insight into parent perceptions of productive struggle, the findings of this study also have potential implications for schools and educators. As the TIMSS video studies revealed, U.S. teaching of mathematics has traditionally focused on drills and procedures rather than open-ended problems that require reasoning from prior knowledge. In fact, not a single lesson from the 83 U.S. videos collected in these studies included opportunities for productive struggle, even when students were presented with challenging problems. Teachers consistently converted "making connections" problems into "using procedures" by providing rules or formulas that simplified the task (Hiebert et al., 2005). Other studies such as Ermeling (2010) confirm that teachers have difficulty supporting productive struggle even when they are deliberately working to adopt a more inquiry-centered approach.

Teachers who remain skeptical about the value of productive struggle might benefit from reflecting on the findings from the parent perception results and the influence of parent-helping behaviors on student's math performance. Teachers who are already advocates of productive struggle might also benefit as they consider ways to educate parents on the purpose and efficacy of this approach for helping students make connections with math concepts. Schools and educators could also provide useful guidance for parents to increase awareness of gender stereotypes, explain the potential influence of parents' own math anxiety on student learning, and teach parents how to limit homework interaction while students grapple with difficult problems.

Limitations

Our study has several limitations that are important to consider. The online nature of our recruitment procedure prevented us from completely verifying several important study parameters such as parent and marriage status. Also, although participants recruited through MTurk tend to be diverse, they are not representative of the U.S. population as a whole (Ross, Irani, Silberman, Zaldivar, & Tomlinson, 2010). Participants on MTurk tend to be younger, have more years of education, and be underemployed (Paolacci & Chandler, 2014) as well as more anxious and depressed than the general population, which could have introduced bias in our study (Arditte, Çek, Shaw, & Timpano, 2016). Our compensation rate of $2.00 for 30 minutes also falls below the typical preferred rate for requesters. Our method of asking American parents to evaluate a video of Japanese children could have also influenced parents' responses as well. Even though we asked parents to imagine their own child receiving the video lesson, the fact that Asian students tend to be stereotyped as being "good at math" may have influenced parent's perceptions about the efficacy of struggle.

Finally, it is important to highlight that our study relied on parent self-report and perceptions rather than direct behaviors around homework. Future studies should focus on using observation methods for assessing parental endorsement of productive struggle. For instance, some studies have examined parents' intervention behavioral responses when their children are put in socially risky versus

physically risky situations (Paquette & Bigras, 2010). A similar procedure could be adapted to ask how the intrusiveness of homework-helping behaviors might validate video perception and self-report measures like those reported here. It may also be relevant to know how parents help, given that the literature on productive struggle focuses on different techniques such as exploration with problem sets and using open-ended questions that encourage students to explain how they solved a math problem. Some parents, for instance, may be relatively hands off and promote productive struggle using feedback and verbal encouragement. Other parents may be hands on by probing their children with questions that assess for conceptual and procedural understanding. Future studies could gather data about specific strategies that parents use when helping with math homework to assess any differences between parents who support productive struggle and those that do not.

Conclusion

The goal of this study was to learn more about what parents think about productive struggle in math. Prior research supports the efficacy of productive struggle in math, but some parents remain skeptical about its use with their children. This study suggests a relationship between parents' beliefs in productive struggle and how often they help with homework. There is also an indication that mothers and fathers do not agree on the efficacy of productive struggle, and this could be influencing how often boys and girls are helped with math homework. Finally, there is a need to investigate other factors that may influence how parents feel about productive struggle. Future studies should examine the role of gender stereotypes, parents' perceptions of their child's ability in math, and the affective nature of the homework-helping experience to learn more about how parents can change their belief toward productive struggle and learn to incorporate struggle in their homework-helping routines.

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APPENDIX

Belief in the Efficacy of Struggle Questionnaire (BESQ):

1. Children will learn more if they try to make sense of their math homework on their own even if they do not get many of the problems correct.
2. Children will learn more if they are allowed to make mistakes and receive delayed feedback.
3. Children will learn more if they are encouraged to stick with a math problem even when they are not sure how to solve it.
4. Children will learn more if they are asked to wrestle with challenging problems even if they do not remember how to solve them.
5. Children will learn more if they are given difficult math problems to solve even if they did not get a chance to practice them in class.
6. Children will learn more if they expend extra effort trying to understand something they initially find confusing.
7. Children will learn more if their teacher gives them math problems that are a little too difficult for them to solve.
8. Children will learn more if they attempt to solve demanding math problems on their own.
9. Children will learn more if they attempt to come up with additional math solving strategies without any help from their teachers.
10. Children will learn more if they are asked to use unfamiliar methods for solving math problems.
11. Children will learn more if they are given math problems that take a long time to think through and solve.
12. Children who do not yet have a complete understanding of the material can learn more from attempting difficult math problems.
13. Children will learn more if they are taught math solution strategies that are different from what their parents were taught.