Calculus Summer Review

(for students who have **completed** Precalculus and are **entering** Calculus)

Reviewing key concepts from Algebra 2 and Precalculus is an excellent way to be fully prepared for the new concepts in Calculus that require prerequisite skills. The following packet will help you practice and also self-assess any concepts that you may want to spend extra time on before the start of school. You will take a low-stakes diagnostic quiz on this material to identify any gaps in critical concepts. You will be given additional assignments to help you learn the material until you can demonstrate mastery.

A breakdown of the skills covered in the packet by item number is as follows:

#1-3 Transformations using function notation
#4-6 Graphing piece-wise functions
#7-12 Evaluating composite functions
#13-14 End behavior and zeros of polynomial functions
#15-16 Rational functions, holes, intercepts, limit behavior at vertical & horizontal asymptotes
#17-18 Properties of logarithms
#19-20 Solving trigonometric equations
#21-22 Evaluating inverse trigonometric expressions
#23-29 Evaluating limits using direct substitution and simplification techniques

If you would like additional resources to support your practice, we recommend Khan Academy as a great first step. For in-person support, consider a peer tutor or a more structured option such as Mathnasium. For a list of peer tutors who are willing to tutor over the summer for community service hours, please contact Linda Graham, Math Department Chair.

Revised 6/4/2020

You are to be able to recite these following facts from memory:

1. To find x-intercepts, set $y = 0$ and solve the equation for x.
2. To find y-intercepts, set $x = 0$ and solve the equation for y.
3. Tests for symmetry
   a. The graph of an equation in x and y is symmetric with respect to the y-axis if replacing x by -x yields an equivalent equation.
   b. The graph of an equation in x and y is symmetric with respect to the x-axis if replacing y by -y yields an equivalent equation.
   c. The graph of an equation in x and y is symmetric with respect to the origin if replacing x by -x and y by -y yields an equivalent equation.
4. The slope m of a non-vertical line passing through the points (x_1,y_1) and (x_2,y_2) is $m = \frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1}$, $x_1 \neq x_2$
5. Summary of Equations of Lines
   a. General form: $Ax + By + C = 0$
   b. Vertical line: $x = a$
   c. Horizontal line: $y = b$
   d. Point-slope form: $y - y_1 = m(x - x_1)$
   e. Slope-intercept form: $y = mx + b$
6. Functions can be specified in a variety of ways:
   a. Implicitly: $x^2 + 2y = 2$
   b. Explicitly: (y by itself) $y = (2 - x^2)/2$
   c. Function notation: $f(x) = (2 - x^2)/2$
7. A function is one-to-one if to each y-value in the range there corresponds exactly one x-value in the domain.
8. Basic Types of Transformations ($c > 0$)

   Original graph: $y = f(x)$
   a. Horizontal shift c units to the right: $y = f(x - c)$
   b. Horizontal shift c units to the left: $y = f(x + c)$
   c. Vertical shift c units downward: $y = f(x) - c$
   d. Vertical shift c units upward: $y = f(x) + c$
   e. Reflection about the x-axis: $y = - f(x)$
   f. Reflection about the y-axis: $y = f(-x)$
   g. Reflection about the origin: $y = - f(-x)$
9. Terms: polynomial, degree of a polynomial, coefficients, leading coefficient, constant term, rational function, algebraic function, transcendental function
10. Definition of a Composite Function: Let f and g be functions. The function given by $(f \circ g)(x) = f(g(x))$ is called the composite of f with g. The domain of $f \circ g$ is the set of all x in the domain of g such that g(x) is in the domain of f.
11. Even and odd functions
    a. A function is even if its graph is symmetric with respect to the y-axis. The function $y = f(x)$ is even if $f(-x) = f(x)$.
    b. A function is odd if its graph is symmetric with respect to the origin. The function $y = f(x)$ is odd if $f(-x) = - f(x)$.
12. Definition of a Limit: Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement $\lim_{x \to c} f(x) = L$ means that for each $\varepsilon > 0$ there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.
13. Some Basic Limits: Let b and c be real numbers and let n be a positive integer.

    a. $\lim_{x \to c} b = b$ b. $\lim_{x \to c} x = c$ c. $\lim_{x \to c} x^n = c^n$
14. Properties of Limits: Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the following limits.

    $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = K$

    a. Scalar multiple: $\lim_{x \to c}[bf(x)] = bL$
    b. Sum or difference: $\lim_{x \to c}[f(x) \pm g(x)] = L \pm K$
    c. Product: $\lim_{x \to c}[f(x)g(x)] = LK$
    d. Quotient: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, provided $K \neq 0$
    e. Power: $\lim_{x \to c}[f(x)]^n = L^n$

15. Limits of Polynomial and Rational Functions

a. If p is a polynomial function and c is a real number, then $\lim_{x\to c} p(x) = p(c)$.

b. If r is a rational function given by r(x) = p(x)/q(x) and c is a real number such that q(c) ≠ 0, then $\lim_{x\to c} r(x) = r(c) = \frac{p(c)}{q(c)}$.

16. The Limit of a Function Involving a Radical: Let n be a positive integer

The following limit is valid for all c if n is odd, and is valid for c> 0 if n is even. $\lim_{x\to c} \sqrt[n]{x} = \sqrt[n]{c}$

17. The Limit of a Composite Function. If f and g are functions such that $\lim_{x\to c} g(x) = L$ and $\lim_{x\to L} f(x) = f(L)$, then $\lim_{x\to c} f(g(x)) = f(L)$.

18. Limits of Trigonometric Functions. Let c be a real number in the domain of the given trigonometric function.

a. $\lim_{x\to c} \sin x = \sin c$ b. $\lim_{x\to c} \cos x = \cos c$ c. $\lim_{x\to c} \tan x = \tan c$

d. $\lim_{x\to c} \cot x = \cot c$ e. $\lim_{x\to c} \sec x = \sec c$ f. $\lim_{x\to c} \csc x = \csc c$

19. Functions that agree at All But One Point

Let c be a real number and let f(x) = g(x) for all x ≠ c in an open interval containing c. If the limit of g(x) as x approaches c exists, then the limit of f(x) also exists and $\lim_{x\to c} f(x) = \lim_{x\to c} g(x)$.

20. Intervals on the real line

a. Bounded open interval: (a,b) $\{x : a < x < b\}$

b. Bounded closed interval: $[a,b]$ $\{x : a \le x \le b\}$

c. Unbounded open interval: $(-\infty, a)$ $\{x : x < a\}$

d. Unbounded closed interval: $[a,\infty)$ $\{x : x \ge a\}$ etc.

21. The distance between points (x_1,y_1) and (x_2,y_2) in a plane is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

22. Midpoint formula for points (x_1,y_1) and (x_2,y_2) in a plane is given by $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$.

23. Standard Form of the Equation of a Circle. The point (x,y) lies on the circle of radius r and center (h,k) if and only if $(x - h)^2 + (y - k)^2 = r^2$.

24. General Form of the Equation of a Circle. $Ax^2 + Ay^2 + Dx + Ey + F = 0$, $A \neq 0$

25. Definitions of the Six Trigonometric Functions.

a. For an acute angle in a right triangle: SohCahToa

b. Circular functions where θ is any angle $r = \sqrt{x^2 + y^2}$

$\sin\theta = y/r$ $\cos\theta = x/r$ $\tan\theta = y/x$ $\csc\theta = r/y$ $\sec\theta = r/x$ $\cot\theta = x/y$

26. Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $\cot^2\theta + 1 = \csc^2\theta$

27. Reduction Formulas:

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$ $\sin(\pi - \theta) = \sin\theta$ $\cos(\pi - \theta) = -\cos\theta$ $\tan(\pi - \theta) = -\tan\theta$

28. Sum of Difference of Two Angles:

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$ $\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

29. Double Angles:

$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\cos 2\theta = 2\cos^2\theta - 1$ $\cos 2\theta = 1 - 2\sin^2\theta$

30. Reciprocal Identities: $\csc\theta = 1/\sin\theta$ $\sec\theta = 1/\cos\theta$ $\cot\theta = 1/\tan\theta$

31. Quotient Identities: $\tan\theta = \sin\theta / \cos\theta$ $\cot\theta = \cos\theta / \sin\theta$

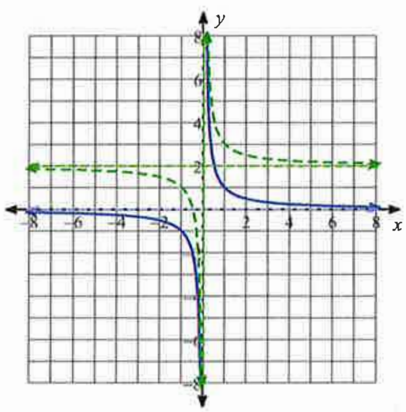
32. Derivative of a function (slope function): $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

AP Calculus AB Name________________________

Summer Review

Write $g(x)$ (dashed line) in terms of $f(x)$ (solid line).

1)



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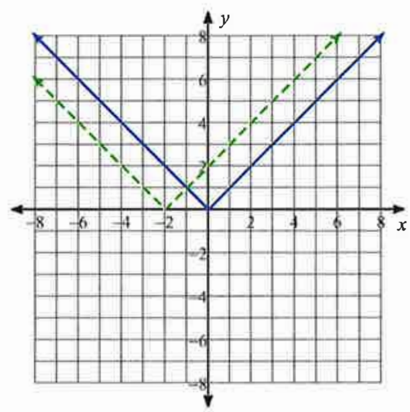

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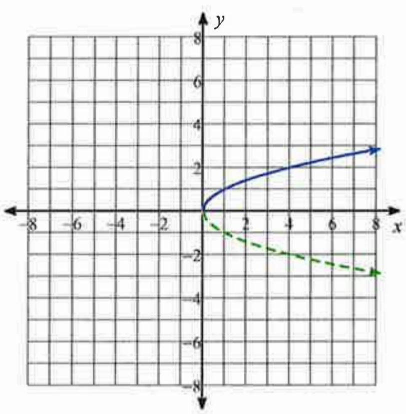
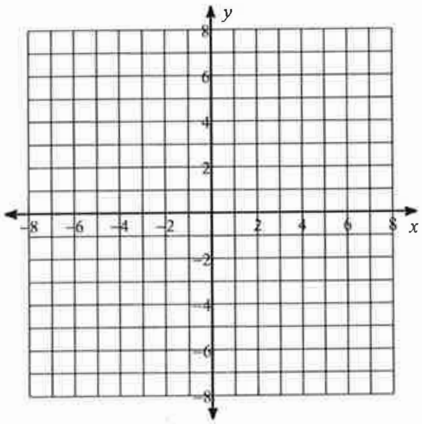
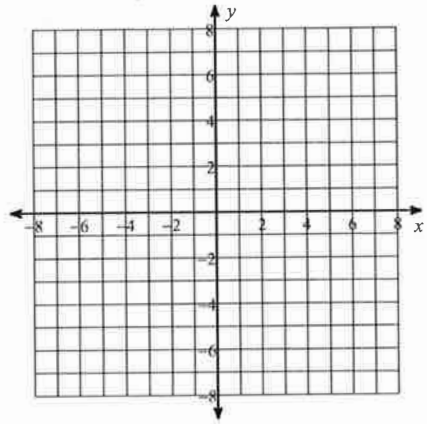


Sketch the graph of each function.

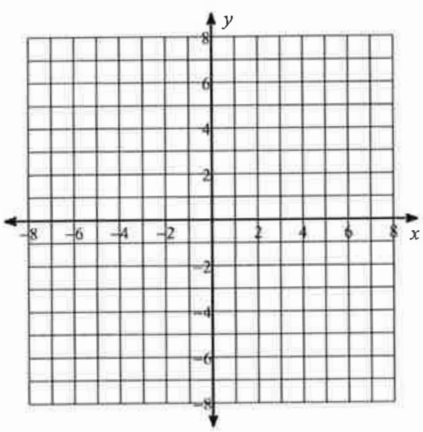
4) $g(x) = \begin{cases} (x+2)^2, & x \le -2 \\ -4 + \sqrt{x}, & -2 < x \le 4 \\ \sqrt{3x}, & x > 4 \end{cases}$



5) $f(x) = \begin{cases} \dfrac{|x|}{2}, & x \le -3 \\ -2|x|, & x > -3 \end{cases}$



6) $f(x) = \begin{cases} (x+1)^2, & x \le -1 \\ 4 - x^2, & x > -1 \end{cases}$



Perform the indicated operation.

7) $g(x) = x^2 - 4$
$f(x) = x - 1$
Find $(g + f)(x)$

8) $g(n) = 3n - 4$
$f(n) = -3n$
Find $(g \circ f)(n)$

9) $g(x) = 4x + 2$
Find $(g \circ g)(x)$

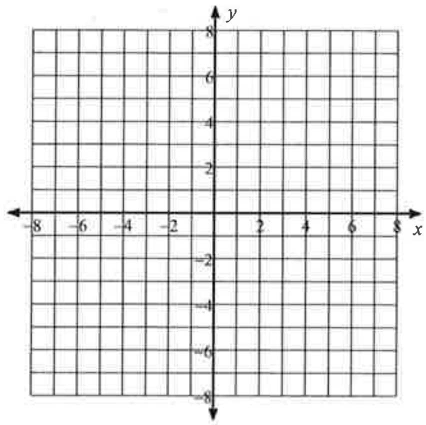
10) $h(t) = 4t + 2$
$g(t) = t^2 + 4t$
Find $(h - g)(-6)$

11) $h(n) = -2n + 3$
$g(n) = n^2 + 2n$
Find $h(-3) + g(-3)$

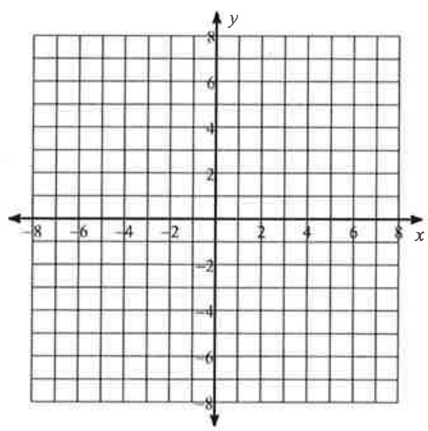
12) $g(x) = x - 3$
$f(x) = x + 5$
Find $g(f(10))$

For each function: (1) determine the real zeros and state the multiplicity of any repeated zeros, (2) describe the end behavior, and (3) sketch the graph.

13) $f(x) = -x^2 + 2x - 1$

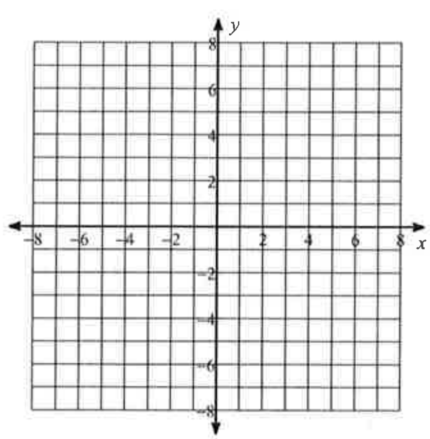


14) $f(x) = x^3 + 4x^2 + 4x$

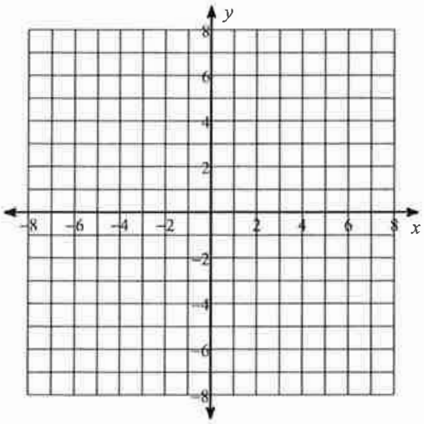


For each function, identify the holes, intercepts, horizontal asymptote, limit behavior at all vertical asymptotes, and end behavior asymptote. Then sketch the graph.

15) $f(x) = \dfrac{x^2 + 4x}{4x^2 - 36}$



16) $f(x) = \dfrac{x^3 - x^2 - 12x}{4x^2 - 4x - 8}$



Condense each expression to a single logarithm.

17) $\ln u - 2\ln v$

18) $5\ln u - \ln v$

Solve each equation for $0 \le \theta < 2\pi$.

19) $-2 + \tan\theta = -3$

20) $\dfrac{-10 - \sqrt{2}}{2} = -5 + \cos\theta$

Find the exact value of each expression.

21) $\cos^{-1} \dfrac{\sqrt{2}}{2}$

22) $\sin^{-1} -\dfrac{1}{2}$

Evaluate each limit.

23) $\lim_{x \to -4} 1$

24) $\lim_{x \to 2} \left(\dfrac{x^2}{2} - 3 \right)$

25) $\lim_{x \to 2} -\frac{x - 2}{x^2 - 6x + 8}$

26) $\lim_{x \to 1} \frac{x^2 - 5x + 4}{x - 1}$

27) $\lim_{x \to -\infty} \frac{2x}{x + 2}$

28) $\lim_{x \to -\infty} -\frac{x^4}{2x^2 - 1}$

29) $\lim_{x \to \infty} -\frac{3x^3}{2x^2 + 1}$

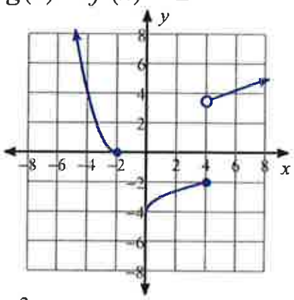
Answers to Summer Review

1) $g(x) = f(x) + 2$

2) $g(x) = f(x + 2)$

3) $g(x) = -f(x)$

4)



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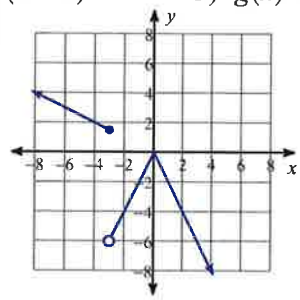

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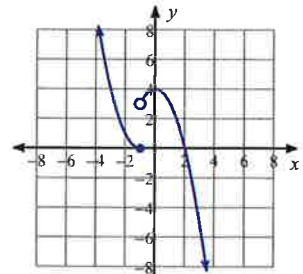


7) $x^2 + x - 5$

8) $-9n - 4$

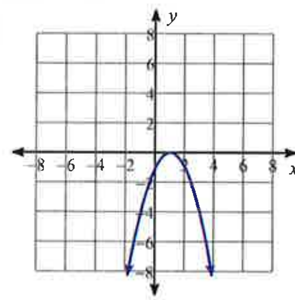
9) $16x + 10$

10) -34

11) 12

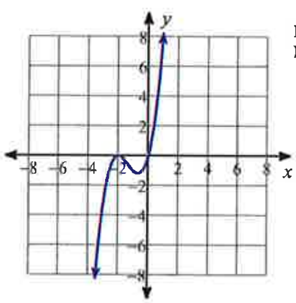
12) 12

13)



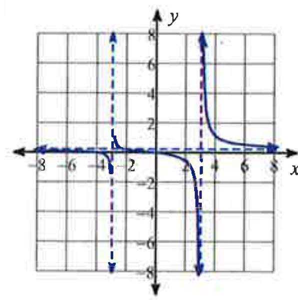
Real zeros: {1 mult. 2}
End behavior:
$\lim_{x \to -\infty} f(x) = -\infty$
$\lim_{x \to \infty} f(x) = -\infty$

14)



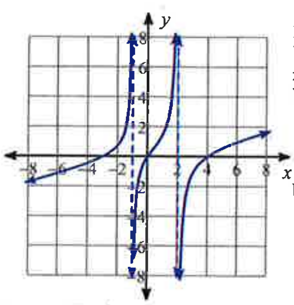
Real zeros: {0, −2 mult. 2}
End behavior:
$\lim_{x \to -\infty} f(x) = -\infty$
$\lim_{x \to \infty} f(x) = \infty$

15)



Holes: None
Horz. Asym.: $y = \frac{1}{4}$
x-intercepts: 0, −4, y-intercept: 0
Vert. Asym. behavior:
$\lim_{x \to -3^-} f(x) = -\infty$, $\lim_{x \to -3^+} f(x) = \infty$
$\lim_{x \to 3^-} f(x) = -\infty$, $\lim_{x \to 3^+} f(x) = \infty$
End behavior asym.: $y = \frac{1}{4}$

16)



Holes: None
Horz. Asym.: None
x-intercepts: 0, 4, −3, y-intercept: 0
Vert. Asym. behavior:
$\lim_{x \to -1^-} f(x) = \infty$, $\lim_{x \to -1^+} f(x) = -\infty$
$\lim_{x \to 2^-} f(x) = \infty$, $\lim_{x \to 2^+} f(x) = -\infty$
End behavior asym.: $y = \frac{x}{4}$

17) $\ln \frac{u}{v^2}$

18) $\ln \frac{u^5}{v}$

19) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

20) $\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}$

21) $\frac{\pi}{4}$

22) $-\frac{\pi}{6}$

23) 1

24) −1

25) $\frac{1}{2}$

26) −3

27) 2

28) $-\infty$

29) $-\infty$