Brockton High School
Content: Math III
Week of: June 1 to June 7, 2020
Greetings Math III Students! We hope you are safe and well with your families!
This assignment is for the week, use your time wisely. You do not have to complete this in one sitting. Here is the lesson plan for this week:

Goals for this week

Learning Objectives:
Students will be able to:

- Solve polynomials by factoring
- Use polynomial models to expand polynomials
- Solve polynomial inequalities graphically
- Simplify sums, differences, products and quotients of rational expressions
- Graph square root and inverse functions
- Calculate inverses of linear functions

(Standards A.APR. 2/5, A.CED, 1, A.REI.2, F.BF. 4, F.IF. C 7b, S.ID.6a)

Literacy Objectives:
Students will be able to:

1. to create, interpret and explain a table, chart or graph
2. to compute, interpret and explain numbers
3. to generate a response from what one has read, viewed or heard.

(https://www.bpsma.org/schools/brockton-high-school/about-us/mission-literacy-charts)

Instructional Video Links: Please watch at least one of these videos to help guide you.

- https://www.youtube.com/watch?v=YUqHdxxdbyM
- https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:polynomials/x9e81a4f98389efdf:binomial/v/pascals-triangle-binomial-theorem
- https://www.youtube.com/watch?v=z_hEVWSkQHo
- https://www.youtube.com/watch?v=U9UtNsXLg5E
- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:cancel-common-factor/v/simplifying-rational-expressions-introduction
- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:rational-add-sub-intro/v/adding-rational-expression-w-unlike-denominators
- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:rational-mul-div/v/multiplying-and-dividing-rational-expressions-2
- https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:functions/x2f8bb11595b61c86:inverse-functions-intro/v/introduction-to-function-inverses
- https://www.khanacademy.org/math/algebra-home/alg-radical-eq-func/alg-graphs-of-radical-functions/v/graphs-of-square-root-functions
- https://www.youtube.com/watch?v=Dd1iy_BPfog
- https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:functions/x2f8bb11595b61c86:inverse-functions-intro/v/understanding-function-inverses-example
- https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:composite/x9e81a4f98389efdf:verifying-inverse/v/verifying-function-inverses-by-composition

Your teacher also has a list of additional instructional videos. Feel free to also search for your own tutorial videos to guide you.

Key Terms

- **Polynomial -** an expression of the form where the coefficients are real numbers and the exponents are nonnegative integers.
- **Zero of a polynomial** – A zero of a polynomial function *f* is a value of x for which f(x) = 0
- **Term-** a number, a variable or a product of a number and a variable or variables.
- **Rational expression-** an expression that can be written as the quotient of two nonzero polynomials.
- **Common factor-** a whole number that is a factor of two or more integers or expressions.
- **Trinomial-** a polynomial that consists of three terms
- **Inverse of a function-** a function that results from exchanging the independent and dependent variables.

Practice Activities:
<u>On-Line:</u>
All students now have access to an on-line program called **Mathia (Carnegie Learning)**

- If you are **already in Mathia**, please continue to work in the program. The suggested pacing is to complete Module 4 Unit 1 by the end of the week.
- If you are **new to Mathia**: Please see the log-in information below and begin. The suggested pacing is to complete Module 4 Unit 1 by the end of the week.

Practice Activities:
<u>Printable resources</u>: Optional resources if technology is not available

Math 3 Printable Resources: Pages M2 -91 through M2- 101(through Practice activities)

Extension Activities: Complete the STRETCH activity on M2-101, showing all work. Please forward to your teacher.

Log-in Information

1. Log-in to Clever(if you need to sign in follow number 2 then 3; if you automatically log in go to 3)
2. If you need to sign in to Clever: Username: 6-digit BHS school ID # @bpsma.org
   Password: Date of birth bps 1920
   Example: Student with ID #:123456
   Date of birth: January 1st, 2000
   Username: 123456@bpsma.org
   Password: 01012000bps1920
3. Under Math (in the left- hand column) click on Carnegie Learning (MATHia)
4. Pick up where you left off in Mathia

Additional Support

- Please email your math teacher with specific questions.
- Here is a list of math teachers' office hours:
- https://brocktonpublicschools-my.sharepoint.com/:x:/g/personal/danielcorbett_bpsma_org/EWk_ij9UwjpPtRAHBUkEpS4B3vue-_IG8VYz0AwG9ovJjQ?e=4%3arkcL1r&at=9&CT=1588698277992&OR=OWA-NT&CID=8d0078f2-9a27-460f-e632-64578875ee60

2

Elegant Simplicity

Pascal's Triangle and the Binomial Theorem

Warm Up

Evaluate each expression.

1. 5!

2. 2!3!

3. $\frac{5!}{3!}$

Learning Goals

- Identify patterns in Pascal's Triangle.
- Use Pascal's Triangle to expand powers of binomials.
- Use the Binomial Theorem to expand powers of binomials.
- Extend the Binomial Theorem to expand binomials of the form $(ax + by)^n$.

Key Term

- Binomial Theorem

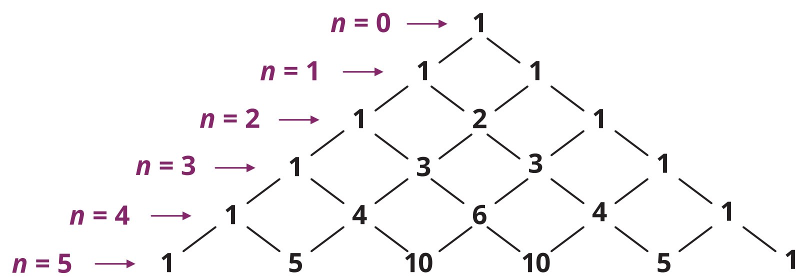
You know how to expand binomials in the form $(a + b)^n$ using the Distributive Property. How can you expand binomials when n is a large number and the Distributive Property would be difficult to use?

GETTING STARTED

In a Blaise of Glory

There is an interesting pattern of numbers that makes up what is referred to as Pascal's Triangle.

The first six rows of Pascal's Triangle are shown, where $n = 0$ represents the first row, $n = 1$ represents the second row, and so on.



1. Analyze the patterns in Pascal's Triangle.

a. Describe all the patterns you see in Pascal's Triangle.

b. Complete the rows for $n = 6$ and $n = 7$ in the diagram of Pascal's Triangle. Describe the pattern you used.

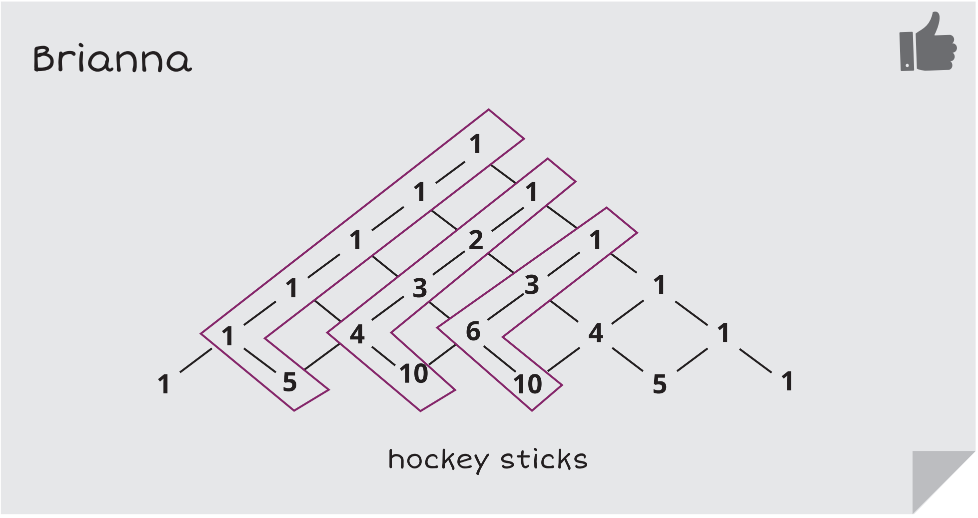
ACTIVITY 2.1 Exploring Patterns in Pascal's Triangle

Mathematicians have discovered countless interesting patterns and relationships within Pascal's Triangle. Despite the fact that Pascal's Triangle has been around for centuries, even before the time of mathematician Blaise Pascal, for whom it is named, new patterns are still being discovered. Because the triangle is infinite, there may yet be patterns that have not been explored.

Think about:

What is the relationship between the numbers along the longer part of the "stick" and the lone number at the end of the shorter part of the stick?

1. Brianna loves hockey. In fact, Brianna is so obsessed with hockey that she drew "hockey sticks" around the numbers in Pascal's Triangle. Lo and behold, she found a pattern! Her work is shown.



a. Describe the pattern shown by the numbers inside the hockey sticks that Brianna drew.

b. Sketch two more hockey sticks that include numbers that have the same pattern described in part (a).

2. **Drew and Latasha analyzed Pascal's Triangle, and each described a pattern.**

Drew

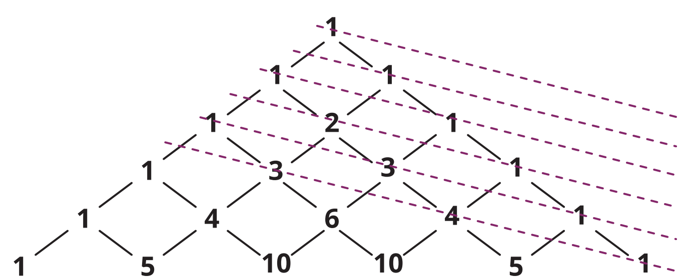
The sum of the numbers in each row is equal to 2^n, where $n = 0$ represents the first row.

Latasha

If I alternate the signs of the numbers in any row after the first row and then add them together, their sum is 0.

Who's correct? Either verify or disprove each student's work.

3. **Consider the numbers along the dashed lines shown.**



a. **Write the sequence for the sum of the numbers along each dashed line.**

b. **Explain how the sum of numbers along the dashed lines in Pascal's Triangle can be linked to a well known sequence of numbers.**

The patterns shown in Pascal's Triangle have many uses. You may have used Pascal's Triangle to calculate probabilities. Let's explore how you can use Pascal's Triangle to raise a binomial to a positive integer.

4. Multiply to expand each binomial. Write your final answer so that the powers of *a* are in descending order.

a. $(a + b)^0 =$

b. $(a + b)^1 =$

c. $(a + b)^2 =$

d. $(a + b)^3 =$

e. $(a + b)^4 =$

5. Analyze your answers to Question 4.

a. Compare the coefficients of each product with the numbers shown in Pascal's Triangle. What do you notice?

b. What do you notice about the exponents of the *a*- and *b*-variables in each expansion?

c. What do you notice about the sum of the exponents of the *a*- and *b*-variables in each expansion?

6. Use Pascal's Triangle to expand each binomial.

a. $(a + b)^5 =$

b. $(a + b)^6 =$

c. $(a + b)^7 =$

ACTIVITY 2.2

Binomial Theorem

Remember:

Recall that the factorial of a whole number n, represented as $n!$, is the product of all numbers from 1 to n.

What if you want to expand a binomial such as $(a + b)^{15}$? You could take the time to draw that many rows of Pascal's Triangle, but there is a more efficient way.

You may have seen the notation $\binom{n}{k}$ or ${}_nC_k$ when calculating probabilities in another course. Both notations represent the formula for a *combination*. Recall that a combination is a selection of objects from a collection in which each selection contains different objects. The formula for a combination of k objects from a set of n objects for $n \geq k$ is shown.

$$\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}$$

Mathematicians define 0! as $0! = 1$.

Worked Example

Calculate $\binom{4}{2}$, or ${}_4C_2$.

$\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}$ Write the formula for a combination.

$n = 4$ and $k = 2$ Identify n and k.

$\binom{4}{2} = \frac{4!}{2!(4-2)!}$ Substitute the values for n and k into the formula.

$= \frac{4 \cdot 3 \cdot (2 \cdot 1)}{(2 \cdot 1)(2 \cdot 1)}$ Write each factorial as a product.

$= \frac{4 \cdot 3 \cdot \cancel{(2 \cdot 1)}}{(2 \cdot 1)\cancel{(2 \cdot 1)}}$ Divide out common factors and evaluate.

$= \frac{12}{2} = 6$

1. Explain why *n* must be greater than or equal to *k* in the formula for a combination.

2. **Perform each calculation.**

a. $\binom{5}{1} =$

b. ${}_7C_4 =$

Your graphing calculator can compute factorials and combinations.

3. **Sarah and Montel's teacher asks each student to use Pascal's Triangle to calculate ${}_6C_3$. Their answers and explanations are shown.**

Sarah

I can calculate ${}_nC_k$ by looking at the kth number (from left to right) in the nth row of Pascal's Triangle. So, ${}_6C_3$ is equal to 20.

Montel

I can calculate ${}_nC_k$ by looking at the $(k + 1)$th number (from left to right) in the $(n + 1)$th row of Pascal's Triangle. So, ${}_6C_3$ is equal to 35.

Who's correct? Explain your reasoning.

The **Binomial Theorem** states that it is possible to extend any power of $(a + b)$ into a sum of the form shown.

$$(a + b)^n = \binom{n}{0}a^nb^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n-1}a^1b^{n-1} + \binom{n}{n}a^0b^n$$

4. **Use the Binomial Theorem to expand $(a + b)^{15}$. You can use your calculator to determine the coefficients.**

Suppose you have a binomial with coefficients other than 1, such as $(2x + 3y)^5$. You can use substitution along with the Binomial Theorem to expand the binomial.

Worked Example

You can use the Binomial Theorem to expand $(a + b)^5$, as shown.

$$(a + b)^5 = \binom{5}{0}a^5b^0 + \binom{5}{1}a^4b^1 + \binom{5}{2}a^3b^2 + \binom{5}{3}a^2b^3 + \binom{5}{4}a^1b^4 + \binom{5}{5}a^0b^5$$
$$= a^5 + 5a^4b^1 + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + b^5$$

Now consider $(2x + 3y)^5$.

Let $2x = a$ and let $3y = b$.

You can substitute $2x$ for a and $3y$ for b into the expansion for $(a + b)^5$.

$$(2x + 3y)^5 = (2x)^5 + 5(2x)^4(3y)^1 + 10(2x)^3(3y)^2 + 10(2x)^2(3y)^3 + 5(2x)^1(3y)^4 + (3y)^5$$
$$= 32x^5 + 5(16x^4)(3y) + 10(8x^3)(9y^2) + 10(4x^2)(27y^3) + 5(2x)(81y^4) + 243y^5$$
$$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$$

Ask Yourself...

What do I substitute for *a* and *b*? Is the term positive or negative?

5. Use the Binomial Theorem and substitution to expand each binomial.

a. $(3x + y)^4$

b. $(x - 2y)^6$

NOTES

TALK the TALK

Give Me a Sign

1. Consider the expressions $(x + y)^n$ and $(x - y)^n$.

a. How many terms does the expanded form of each expression have? Explain how this number relates to Pascal's Triangle.

b. What generalization can you make about the sign of each term in the expanded form of each expression?

c. Aparna says that when n is an odd integer, there are $\frac{(n + 1)}{2}$ different coefficients for the terms in expanded form. Luke says that there are $n + 1$ different coefficients. Who is correct? Explain your reasoning.

NOTES

2. **Use the Binomial Theorem and your generalization to determine each term.**

   a. **the third term of $(x + y)^{20}$**

   b. **the third term of $(x - y)^{20}$**

   c. **the fifth term of $(x + y)^{12}$**

   d. **the fifth term of $(x - y)^{12}$**

   e. **the 100^{th} term of $(x + y)^{100}$**

   f. **the 100^{th} term of $(x - y)^{100}$**

Assignment

Write

Describe how the Binomial Theorem is related to Pascal's Triangle.

Remember

The Binomial Theorem states that it is possible to extend any power of $(a + b)$ into a sum of the form:

$(a + b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{n-1} a^1 b^{n-1} + \binom{n}{n} a^0 b^n$

The formula for a combination of k objects from a set of n objects for $n \geq k$ is:

$\binom{n}{k} = {}_nC_k = \frac{n!}{K!(n-k)!}$

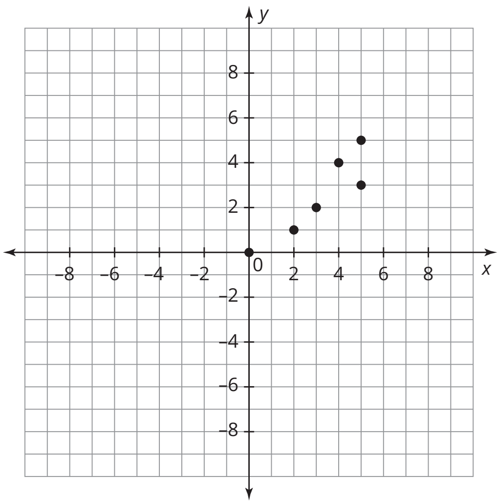
Practice

1. Consider $(v + w)^8$.
   a. Use Pascal's Triangle to expand $(v + w)^8$.
   b. Determine the coefficient of v^5w^3 in the expansion of $(v + w)^8$.
   c. Determine the coefficient of v^5w^3 in the expansion of $(2v + w)^8$.
   d. Determine the coefficient of v^4w^4 in the expansion of $(2v + 3w)^8$.
2. Expand $(4x + 2y)^5$.
3. Expand $(3m - n)^6$.
4. Expand $(-5x - 3y)^4$.
5. Determine the coefficient of c^5d^4 in the expansion of $(2c + 3d)^9$.
6. Determine the coefficient of j^7k^3 in the expansion of $(2j - k)^{10}$.

Stretch

1. Determine the coefficient of $x^{10}y^5$ in th expansion of $(-7x - 3y)^{15}$.

2. Consider the scatter plot shown.



   a. The equation $y = 0.8761x - 0.2743$ represents the line that best fits the points on the graph, and the equation $y = 0.0433x^2 + 0.649x - 0.125$ represents the curve that best fits the points on the graph. Graph each equation on the scatterplot.
   b. Which curve appears to fit the data the best? Explain your reasoning.
   c. The coefficient of determination is a value that measures the strength of the relationship between the original data and the regression equation. The closer the value is to 1, the stronger the relationship. The coefficient of determination for the linear equation is 0.8761, and the coefficient of determination for the quadratic equation is 0.8312. Do these values match your answer from part (b)?

Review

1. Use polynomial identities and number properties to calculate 42^3.
2. Arjun measures the side lengths of a triangular piece of glass. The side lengths are 120 in., 209 in., and 241 in.
   a. Verify that the triangular piece of glass is a right triangle.
   b. Use Euclid's Formula to determine the positive integers r and s, where $r > s$, that will generate these three side lengths.
3. Completely factor each expression over the set of real numbers.
   a. $2x^3 - 12x^2 + 16x$
   b. $8x^3 - 64$
4. Use long division to determine whether $x + 4$ is a factor of $5x^4 + 16x^3 - 15x^2 + 8x + 16$. Show your work.