

Dear Students of AP Calculus BC in Fall 2020,

We hope you are having a restful summer! This short set of problems is suggested practice to refresh key skills for AP Calculus BC in the fall. Proficiency with these topics from Precalculus HN will support your success in BC. Rest assured that any shortened or omitted material from PCH is not included below.

The best way to use this packet is to do the work independently and for accuracy. Focus on understanding, not speed! If you need help, you can access your previous notes from Precalculus HN or use helpful sites like Khan Academy. You may find it helpful to work with a friend. If this works for you, great! However, please keep in mind those topics that you needed help on so that you can get additional practice when the year begins. You can also get help from your teacher or the ILC once school begins. The answers are provided for you to check your solutions.

We suggest that you complete this packet a week or two before school begins. It helps you know what you have retained over time and what you will need to refresh. If you feel that you'd like to do this packet closer to the end of this current school year, please plan to re-visit the packet prior to the start of the year.

We are looking forward to a great year together getting to know you and exploring Calculus together!

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No Calculator unless otherwise stated. When using a calculator, round answers to at least 3 decimal places.

1. Quickie Oil raises its prices 5% each year. If an oil change currently costs \$25, in how many years will the cost be \$40? Justify. Calculator allowed.

NO CALCULATOR. Solve over the given interval showing all work leading to exact answers.

2. $\sin(x) - \cos(2x) = 0, -\frac{\pi}{2} \leq x \leq \pi$

3. Solve the following inequalities graphically – no calculator. Show work and write your answer in interval notation.

- a. $x^2 e^x > 4e^x$

- b. $x^2(x-3)(x-4)^2 \leq 0$

4. Sketch the polar curve $r = 2 - 4 \sin \theta$ and determine the values of θ which generate the inner loop. Give these angles in exact radian measure and show work leading to your conclusion.
5. **Show all equations, but use a graphing calculator to solve this problem.**
A closed can in the shape of a right circular cylinder is designed to hold 12 oz. soda. If the height of the cylinder is x in., write an equation in terms of x to represent the amount of material needed to construct the **closed** can. (1 fl. oz. = 1.805 in^3) Find the minimum amount of material needed.
6. Two particles move in the x-y plane. For $t \geq 0$ the position of particle A is $x = (t - 2)^2$ and $y = t - 2$ and the position of particle B is $x = \frac{3}{2}t - 2$ and $y = \frac{3}{2}t - 4$. At what time do the particles collide? Show work to justify your answer.

7. a. Sketch the closed region bounded by $y = 5$, $y = 2x - 1$, and y -axis.

b. Find the exact volume of the solid formed if the region is rotated about the y -axis.

8. Given $r = 4 \cos \theta$

a. Sketch the curve $r = 4 \cos \theta$.

b. Find the area inside the curve.

c. Write the equation in terms of only x and y .

$$9. \sum_{n=0}^{\infty} \frac{(-1)^n (2x-3)^n}{4^n}$$

a. Write out, do not multiply or simplify, the first three terms of the series.

b. Find the values of x which make the series converge. (Interval of convergence)

c. If $x = 2$, find the sum of the series. Show work to justify your answer.

For #10 - 16, evaluate the following limits without using a calculator:

$$10. \lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$$

$$11. \lim_{x \rightarrow 3} \frac{x^2}{x - 3}$$

$$12. \lim_{x \rightarrow \infty} \frac{5x^2}{(2x-1)^2}$$

$$13. \lim_{x \rightarrow \infty} \frac{\sin x}{x}$$

$$14. \lim_{x \rightarrow 0^-} \frac{|x|}{x}$$

$$15. \lim_{x \rightarrow \infty} \cos x$$

$$16. h(x) = \begin{cases} \sin x, & x < \frac{\pi}{2} \\ 1, & \frac{\pi}{2} \leq x \leq 2 \\ 1-x, & x > 2 \end{cases}$$

$$\text{a. } \lim_{x \rightarrow \frac{\pi}{2}} h(x)$$

$$\text{b. } \lim_{x \rightarrow 2} h(x)$$

$$\text{c. } \lim_{x \rightarrow -\frac{\pi}{2}} h(x)$$

For #17-18 Sketch a graph for a function with the given characteristics.

17. $\lim_{x \rightarrow 2} f(x)$ does not exist and $f(2) = 3$.

18. $\lim_{x \rightarrow 2} f(x) = 3$ and $f(2) \neq 3$

19. If $f(x) = \frac{1}{\sqrt{x}}$ find $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ (Show all work, no “short cuts”.)

20. If the sum of an infinite geometric series is $\frac{2}{1+4x}$, find the common ratio r and the 4th term of the series.

KNOW the following graphs and identities:

Identities:

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x$$

$$\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\log_a x + \log_a y = \log_a xy$$

$$\log_a x - \log_a y = \log_a \frac{x}{y}$$

$$\log_a x^y = y \log_a x$$

$$|x - a| = \begin{cases} x - a, & x \geq a \\ -(x - a), & x < a \end{cases}$$

Graphs: Know asymptotes and critical points for the graphs of:

$$y = \sin x, y = \cos x, y = \tan x, y = \ln x, y = e^x, \text{ and } y = e^{-x}$$

Selected Answers

1. 9.6 years

2. $x = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{\pi}{2}$

3. a. $(-\infty, -2) \cup (2, \infty)$

b. $(-\infty, 3] \cup [4, 4]$

4. $\frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6}$

5. 43.015

6. $t = 4$

7b. 18π

8b. 4π

9b. $-\frac{1}{2} < x < \frac{7}{2}$ 9c. $\frac{4}{5}$

10. 12

12. $\frac{5}{4}$

13. 0

14. -1

16b. does not exist c. -1

19a. $-\frac{1}{2x^{\frac{3}{2}}}$

20. $a_4 = -128x^3$