Brockton High School
Content: Math III
Week of: May 25 to May 31, 2020
Greetings Math III Students! We hope you are safe and well with your families!
This assignment is for the week, use your time wisely. You do not have to complete this in one sitting. Here is the lesson plan for this week:

Goals for this week

Learning Objectives:
Students will be able to:
- Predict end behavior for an odd-degree function and for an even-degree function.
- Infer properties of a polynomial equation from its graph.
- Interpret the graph of a polynomial equation in context.
- Identify characteristics of the graph of a polynomial equation.
- Calculate the average rate of change over a given interval.
- Add, subtract and multiply polynomials.
- Use synthetic division to simplify polynomials

(Standards A.APR.1/3/6, A.REI.4b, F.BF. 1b/4, F.IF. 4/6/B5/C9)

Literacy Objectives:
Students will be able to:
1. to create, interpret and explain a table, chart or graph
2. to compute, interpret and explain numbers
3. to generate a response from what one has read, viewed or heard.

(https://www.bpsma.org/schools/brockton-high-school/about-us/mission-literacy-charts)

Instructional Video Links: Please watch at least one of these videos to help guide you.
Module 2 Unit 5: Graphs of Polynomial Functions
- https://www.youtube.com/watch?v=aElUAe7aYEA
- https://www.youtube.com/watch?v=m7mIkplcCmk
- https://www.youtube.com/watch?v=4OPINzI4dWc
- https://www.youtube.com/watch?v=votVWz-wKeI
- https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:functions/x2f8bb11595b61c86:maximum-and-minimum-points/v/identifying-relative-and-absolute-maxima-and-minima
- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-graphs/x2ec2f6f830c9fb89:poly-intervals/v/polynomial-zero-multiplicity
- https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-arithmetic/x2ec2f6f830c9fb89:poly-avg-rate/v/avg-rate-of-change-of-polynomials

Module 3 Unit 1: Polynomial Operations
- https://www.khanacademy.org/math/algebra-home/alg-polynomials/alg-adding-and-subtracting-polynomials/v/adding-and-subtracting-polynomials-3
- https://www.khanacademy.org/math/algebra-home/alg-polynomials/alg-synthetic-division-of-polynomials/v/synthetic-division

Your teacher also has a list of additional instructional videos. Feel free to also search for your own tutorial videos to guide you.

Key Terms

- **Polynomial -** an expression of the form where the coefficients are real numbers and the exponents are nonnegative integers.
- **Zero of a polynomial** – A zero of a polynomial function *f* is a value of x for which f(x) = 0
- **Average rate of change –** of a nonlinear function is the ration of the change in the dependent variable to the change in the independent variable over a specific interval.
- **Term-** a number, a variable or a product of a number and a variable or variables.
- **Exponent –** the number of times a base is used as a factor of repeated multiplication.

Practice Activities:
<u>On-Line:</u>
All students now have access to an on-line program called **Mathia (Carnegie Learning)**

- If you are **already in Mathia**, please continue to work in the program. The suggested pacing is to complete Module 3 Unit 1 by the end of the week.
- If you are **new to Mathia**: Please see the log-in information below and begin working on Modules 1 and 2. The suggested pacing is to complete Module 3 Unit 1 by the end of the week.

Practice Activities:
<u>Printable resources</u>: Optional resources if technology is not available

Math 3 Printable Resources (To a Greater or Lesser Degree) pages M1 289 through M1 303

Extension Activities:

Complete the **STRETCH** and **REVIEW activity** on the Printable Resource form page M1 304 and send a copy to your teacher.

Log-in Information

1. Log-in to Clever(if you need to sign in follow number 2 then 3; if you automatically log in go to 3)
2. Directions If you need to sign in to Clever: Username: 6-digit BHS school ID # @bpsma.org
   Password: Date of birth bps 1920
   Example: Student with ID #:123456
   Date of birth: January 1st, 2000
   Username: 123456@bpsma.org
   Password: 01012000bps1920
3. Under Math (in the left- hand column) click on Carnegie Learning (MATHia)
4. Pick up where you left off in Mathia

Additional Support

- Please email your math teacher with specific questions.
- Here is a list of math teachers' office hours:
- https://brocktonpublicschools-my.sharepoint.com/:x:/g/personal/danielcorbett_bpsma_org/EWk_ij9UwjpPtRAHBUkEpS4B3vue-_IG8VYz0AwG9ovJjQ?e=4%3arkcL1r&at=9&CT=1588698277992&OR=OWA-NT&CID=8d0078f2-9a27-460f-e632-64578875ee60

6

To a Greater or Lesser Degree

Comparing Polynomial Functions

Warm Up

Use technology to evaluate each polynomial expression for $x = 5$.

1. $x^3 + 10x^2 - 1$

2. $2x^5 - 6x^4 - x + 2$

3. $\frac{2x^5 - 6x^4 - x + 2}{x^3 + 10x^2 - 1}$

Learning Goals

- Compare polynomial functions by their key characteristics.
- Compare polynomial functions using multiple representations.

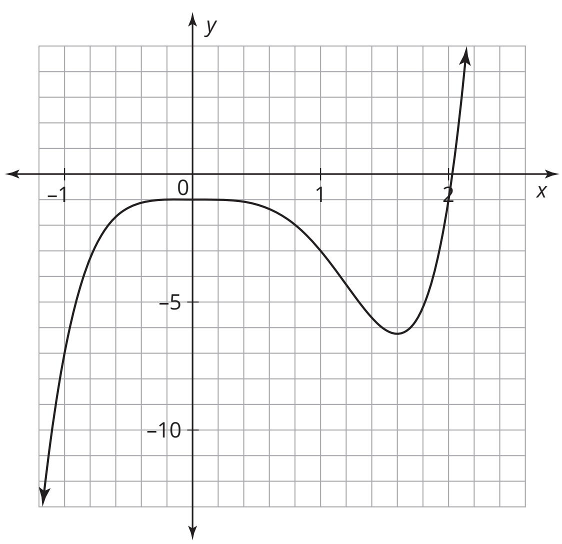
You know how to represent a polynomial function using a graph, a table of values, and key characteristics. How can you compare polynomials using these different representations?

The Lesser Degree

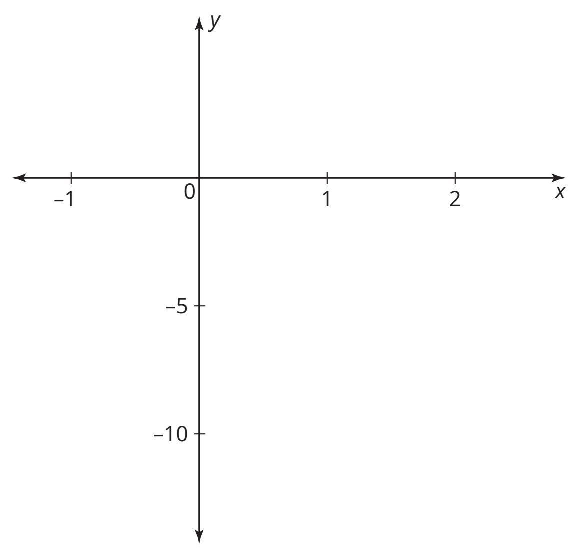
You know that the degree of a polynomial function is the greatest exponent of a term in the polynomial. Let's investigate polynomial functions with greater and lesser degrees.

1. Given each polynomial function and its graph, determine a function of lesser degree using the same coefficients. Write the equation and then use technology to sketch the graph of the function.

a.

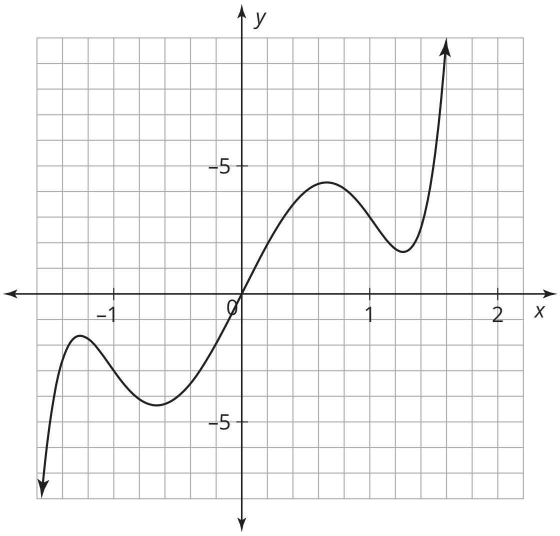


$f(x) = 2x^5 - 4x^4 - 1$

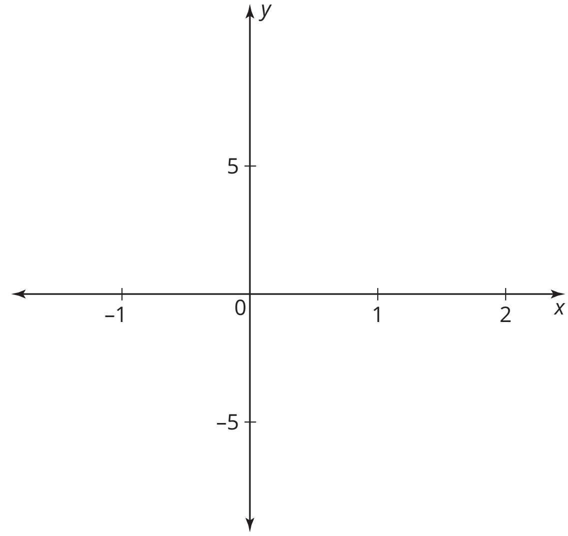


Function of lesser degree: ______________

b.



$g(x) = x^7 - 8x^3 + 10x$



Function of lesser degree: ____________

2. Compare your functions with your classmates' functions. Did everyone write the same equations?

3. Compare your functions with the corresponding functions of lesser degree. What do you notice?

ACTIVITY

6.1

Comparing Polynomial Functions

Recall that you can represent a polynomial using a graph, table of values, equation, or description of its key characteristics. The ability to compare functions using different representations is an important mathematical habit. This skill allows you to model problems in different ways, solve problems using a variety of methods, and more easily identify patterns. At times, you may need to compare functions when they are in different representations.

Metacognition is an important mathematical habit that involves mentally asking yourself a series of questions to determine what you know about a problem and how you can reason your way to a solution.

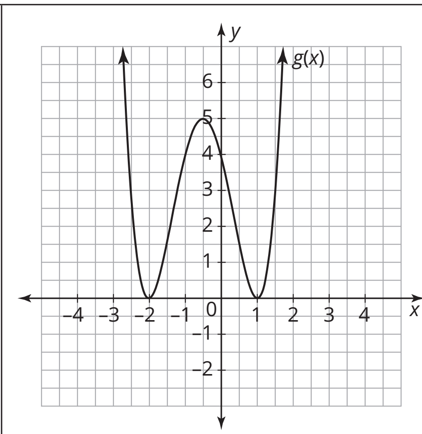
When comparing two functions in different forms, it may be helpful to ask yourself a series of questions. Examples include:

- What information is given?
- What is the degree of each function?
- What do I know about all functions of this degree?
- What key characteristics do I need to know?
- How do the functions compare?

Worked Example

Consider two polynomial functions $f(x)$ and $g(x)$. Which polynomial has a greater number of real zeros? Justify your choice.

$f(x) = -2(x - 1)^3$

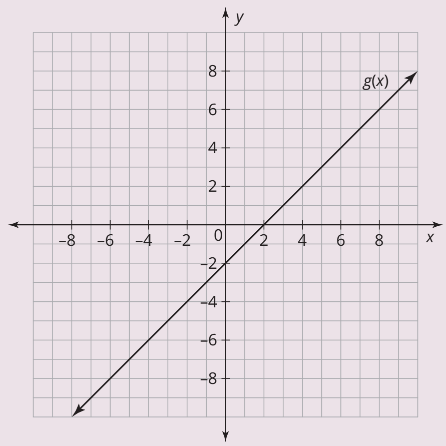


- The Fundamental Theorem of Algebra states that the number of zeros must be equal to the degree of the function. Therefore, $f(x)$ has 3 zeros.
- The function $f(x)$ has a real zero at 1 (multiplicity 3), so all zeros are real.
- The graph of $g(x)$ shows each zero has multiplicity 2, for a total of 4 real zeros.

The function $g(x)$ has 4 real zeros, while $f(x)$ has 3. Therefore, the correct choice is $g(x)$.

1. **Toby compared the table of values for $f(x)$ and the graph of $g(x)$ to determine which polynomial function has the greater number of real zeros.**

x	$f(x)$
−2	3
−1	−2
0	−5
1	−6
2	−5
3	−2
4	3



Toby

Function g(x) has the greater number of real zeros. The graph has 1 zero at x = 2 while the table of values has no output value of 0, and therefore no zeros.

Is Toby correct? Explain your reasoning.

2. **Analyze each pair of representations. Then, answer each question and justify your reasoning.**

   a. Which function has a greater degree?

A polynomial function $h(x)$ has 1 absolute maximum and 1 relative maximum.	$j(x) = -40(x - 7)^2 + 30x^2 - 17x + 1$

b. Which function has a greater degree?

x	$m(x)$	A polynomial function $n(x)$ has a real zero and imaginary zeros.
−2	9	
−1	3	
0	1	
1	3	
2	9	

c. Which function has a degree divisible by 2?

x	$p(x)$	The function $q(x)$ has only imaginary zeros.
−2	2	
−1	4	
0	6	
1	8	
2	10	

d. Determine which function has the greater output as x approaches infinity.

An odd function $r(x)$ with $a < 0$.	$k(x) = x^6 + x^4 + 3x^2 + 5x - 10{,}000$

e. Determine which function has the greater output as x approaches negative infinity.

$t(x) = -3(x - 4)^8 + 130$	A quartic function $s(x)$ with y-intercept (0, 5) and all imaginary roots.

3. Sam and Otis disagree when they compared the two functions shown to determine which one has an odd degree.

The function $f(x)$ has an absolute maximum value.	$g(x) = x^4(3 - x)(2x^2 + 3)(x^4 + 4)$

Sam

The function f(x) has an odd degree because odd functions approach positive infinity as x either increases or decreases. This means f(x) has a maximum value.

Otis

The function g(x) has an odd degree. When I multiplied the factors, I got a term with a highest exponent of 11: $x^4(-x)(2x^2)(x^4) = -2x^{11}$. Therefore, g(x) is odd.

Who is correct? Justify your reasoning.

ACTIVITY 6.2

Multiple Representations of Polynomial Functions

As you consider additional questions in this lesson, it may be helpful to compare the problems to ones that you have already completed.

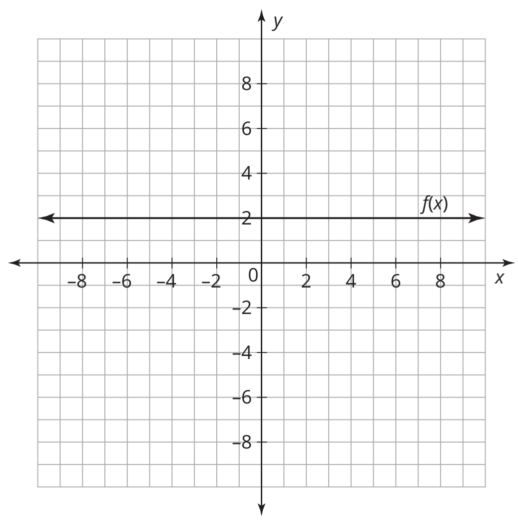
Ask yourself:

How is this problem the same or different than the previous ones that I have already solved?

What do I know about the function that is given? What can I conclude that is not directly stated?

Worked Example

Consider the representations shown. Which function has a greater y-intercept? Justify your reasoning.



A function $g(x)$ has an a-value less than zero and all roots have multiplicity 2.

Solution:

This problem is similar to previous problems in that you must consider functions with restrictions on the a-value and functions with multiple zeros. The problem is also similar in that you must consider an output value for a given input. In this case, the input is 0.

In function $f(x)$, the output value is 2 for any given input. Analyzing function $g(x)$, the multiplicity 2 tells you that the function is even, and the negative a-value indicates that the function opens downward. The multiplicity of the zeros also tells you that the function does not cross the x-axis. Instead, it reflects at a given point where the double zero occurs.

Comparing the two functions, you know that function $g(x)$ is always below the x-axis and function $f(x)$ is above the x-axis. Therefore, $f(x)$ has a greater y-intercept.

Remember:

The a-value is the coefficient of the leading term. For example, in the function $f(x) = 5x^2 + 3x + 4$, the a-value is 5.

1. **Isaac and Tina disagree over which function has a greater *y*-intercept.**

$g(x) = 2(x - 2)(x + 2)(x - 3) - 4$

x	$h(x)$
−2	−2
−1	0
0	4
1	10
2	18

Isaac

Function g(x) has a greater y-intercept. I calculated the y-intercept by substituting 0 for x. This value is greater than (0, 4) shown in the table for the function h(x).

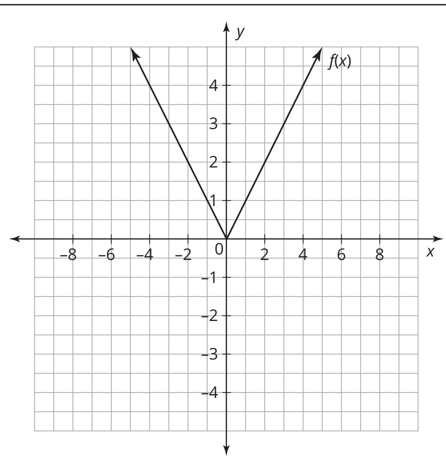
Tina

Function h(x) has a greater y-intercept. The y-intercept of h(x) is (0, 4) and the y-intercept of g(x) is (0, −4).

Who is correct? Justify your reasoning.

2. **Analyze each pair of representations. Then, answer each question and justify your reasoning.**

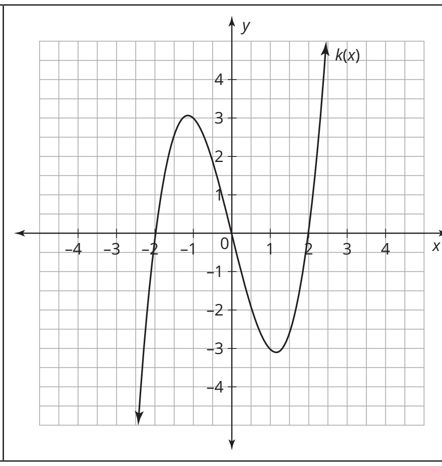
   a. **Which function has a greater average rate of change for the interval (−4, 4)?**



$g(x) = x$

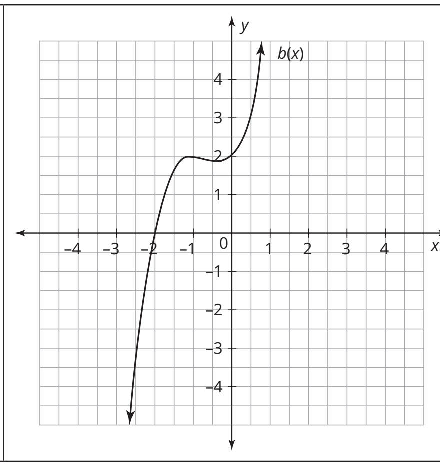
b. Which function has a greater average rate of change for the interval (−1, 1)?

x	$j(x)$
−2	4
−1	1
0	0
1	1
2	4

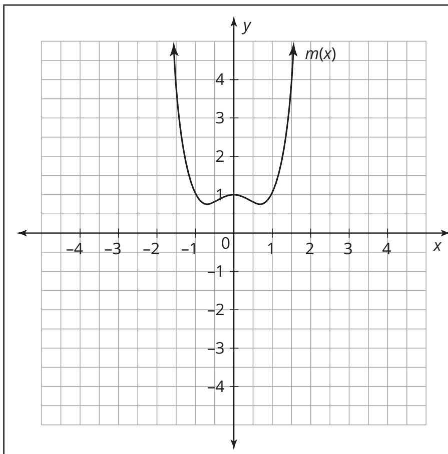


c. Which function has a greater relative minimum?

A cubic function $a(x)$ with $a > 0$ and 3 distinct real roots.



d. Which function's axis of symmetry has a greater *x*-value?



$n(x) = x^2 - 3x + 1$

3. Emilio studied the table of values and description of the key characteristics to determine which function has a greater minimum.

x	$d(x)$
−2	5
−1	2
0	1
1	2
2	5

A quartic function $m(x)$ has $a < 0$ and 2 real zeros (each of multiplicity 2).

Emilio

Function d(x) has a greater minimum. This function is a parabola opening up, with its vertex at (0, 1). Function m(x) opens down because $a < 0$. Since the real zeros have multiplicity 2, I know any real zeros occur when the function reflects off the x-axis. Therefore, the output values of m(x) never reach a point greater than $y = 0$.

Is Emilio correct? Justify your reasoning.

Recall that a basic function is a function in its simplest form. The basic power function is $f(x) = x^n$ for any natural number n. Transformations of a basic function are performed by changing the *A*-, *B*-, *C*-, and *D*-values in the form $g(x) = Af(B(x) - C) + D$. Remember, each value describes different transformations of the graph: the *A*-value vertically stretches or compresses the graph, the *B*-value horizontally stretches or compresses the graph, the *C*-value horizontally shifts the graph right or left, and the *D*-value vertically shifts the graph up or down.

4. Analyze the transformations of the basic functions. Then answer each question and justify your reasoning.

a. Which function has a greater output for a given input?

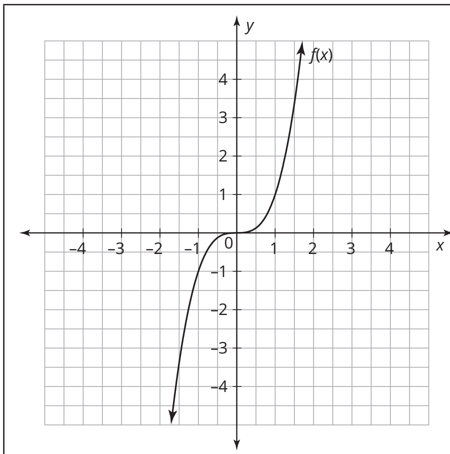
The basic quadratic function $f(x) = x^2$.	$g(x) = f(x - 2) + 1$

b. Which function has a lower minimum?

x	$j(x)$
-2	16
-1	1
0	0
1	1
2	16

$k(x) = 5f(x - 4) + 2$

c. Which function has the greater input for a given output value?



$g(x) = 3f(x - 5) + 1$

NOTES

TALK the TALK

The Greater Degree

1. Which polynomial function has a greater degree?

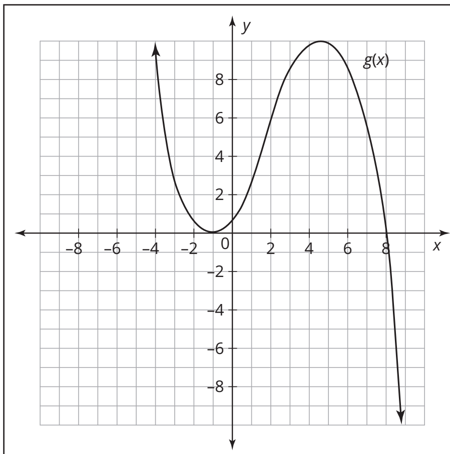
A polynomial function $b(x)$ with 2 absolute minimums and 1 relative maximum.	$c(x) = -2(3 - x^2)(x - 4) + 9$

2. Which polynomial function has a greater number of real zeros?

$d(x) = x^2 - x - 6$

x	$f(x)$
-5	-8
-4	-1
-3	0
-2	1
-1	8
0	27
1	64

3. Which function has an odd degree?



A polynomial function $h(x)$ with 2 real zeros and an imaginary zero.

Assignment

Write

List the key characteristics of a polynomial function.

Remember

The key characteristics of polynomial functions can be compared even when the representations are different.

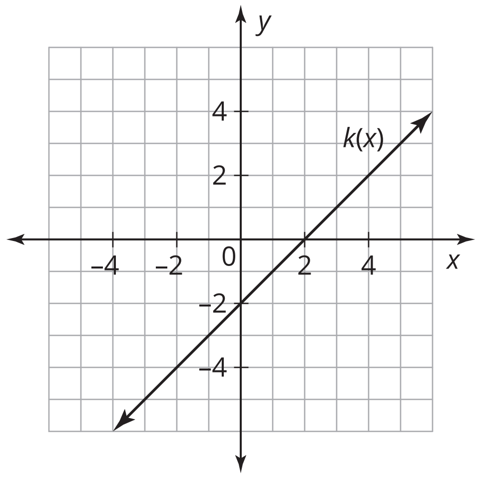
Practice

1. Analyze the given representations of the polynomial functions $f(x)$, $g(x)$, $h(x)$ and $k(x)$. Then, answer each question and justify your reasoning.

The cubic function $f(x)$ with x-intercepts $(-4, 0)$, $(-1, 0)$, and $(2, 0)$ and y-intercept $(0, 8)$.

$$g(x) = (x - 3)^2(x + 2)^2$$

x	$h(x)$
-2	-7
-1	-4
0	-3
1	-4
2	-7



   a. Consider $f(x)$ and $g(x)$. Which function has the smaller output as x approaches infinity?
   b. Consider $f(x)$ and $h(x)$. Which function has the greatest number of imaginary zeros?
   c. Consider $f(x)$ and $h(x)$. Which function has a greater degree?
   d. Consider $g(x)$ and $k(x)$. Which function has the greater y-intercept?
   e. Consider $f(x)$ and $g(x)$. Which function has the lowest relative minimum?
   f. Consider $h(x)$ and $k(x)$. Which function has the greatest average rate of change over the interval $(-2, 0)$?

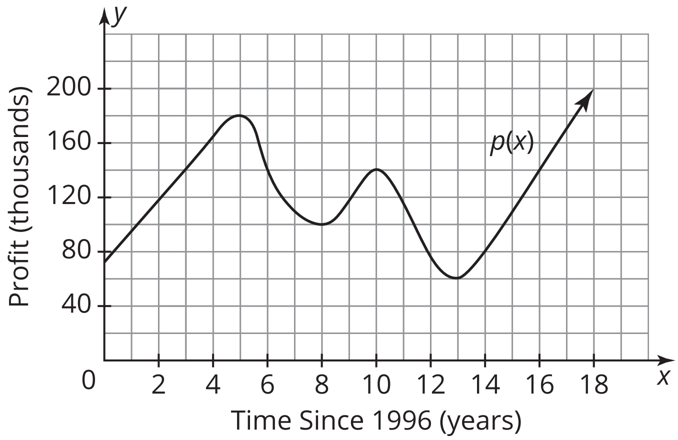
2. Consider the polynomial functions $m(x) = -x^2$ and $n(x) = m(x + 4) - 3$. Which function has the greatest maximum? Explain your reasoning.
3. Consider the polynomial functions $p(x) = x^4$ and $t(x) = p(x - 1) + 7$. Which function's axis of symmetry has a greater x-value? Explain your reasoning.

Stretch

Given $f(x) = x^4 - 625$, determine all the possible polynomial functions for $g(x)$ such that $h(x) = \frac{f(x)}{g(x)}$ is a polynomial function.

Review

1. A manager conducted an 18-year study of the profits of his company. The polynomial function $p(x)$ models the company's profits from the year 1996 (when $x = 0$) to the year 2014 (when $x = 18$).



   a. Estimate when the profit was $140,000. Explain your reasoning.
   b. At what point during the 18-year study was the profit the lowest? What was the profit at that time?
   c. Estimate the average rate of change of the profit over the entire 18-year study. Explain the meaning of your answer in terms of the problem situation.
2. Determine the average rate of change for the function $f(x) = -2x^4 + x^3 - 7x^2 - 2x + 3$ over the interval (4, 8).
3. Add or subtract the expressions given.
   a. $(4x^2 - 2x + 7) + (-8x^2 + 5x - 25)$
   b. $(-9x^2 + 16x - 17) - (-12x^2 - 7x + 3)$