Brockton High School
Content: Math II
Week of: May 25 to May 31, 2020

Greetings Math II Students! We hope you are safe and well with your families!
This assignment is for the week, use your time wisely. **You do not have to complete this in one sitting**. Here is the lesson plan for this week:

Goals for this week: Module 4 Topic 1, 2, and 3 on Mathia
Learning Objectives: Students will be able to: 1. Explore functions including absolute value, linear, and exponential. 2. Use and apply properties of rational exponents. (Standards N.RN.1, N.RN.2, N.RN.3, F.BF.1a, F.IF.8b, F.LE.1c, F.LE.3, F.LE.5)
Literacy Objectives: Students will be able to: 1. to compute, interpret and explain numbers. 2. to read, break down, and solve a word problem. 3. to explain and/or interpret relationships of space and time. *(https://www.bpsma.org/schools/brockton-high-school/about-us/mission-literacy-charts)*

<table>
<tr><td colspan="2">Carnegie Learning (use with Carnegie Resources provided below)
Carnegie Learning (Log-in through Clever)</td></tr>
<tr><td>Instructional Video Links: Please watch these videos to help guide you.
• Absolute Value Equation videos:
1. Graphing Absolute Value Functions
2. Graphing Absolute Value on the Number Line
3. Solving Absolute Value Equations
• Linear and Exponential Model Videos:
1. Growth and Decay
2. Calculating rate of change
3. Modeling with linear equations
4. Comparing exponential functions
• Rational Exponent videos:
1. Exponential Functions and Rational Exponents
2. Basic Fractional Exponents
3. Rewriting roots as rational exponents</td>
<td>Printable Resources:
Skills Practice:
• Module 3 Topic 1 Putting the V in Absolute value
• Module 3 topic 2 Growth and decay functions
• Module 4 Topic 3 Rational Exponents</td></tr>
<tr><td>Practice Activities:
On-Line:
All students now have access to an on-line program called Mathia!
• Mathia- If you are already in Mathia, please continue to work in the program.
• If you are new to Mathia: Please see the log-in information below.</td>
<td>Key Terms:
• Absolute value: The distance between zero and a point on the number line. The absolute value of a number is always positive.
• Simple interest: When interest is paid only as a percent of the principal.</td></tr>
</table>

- **Common ratio:** Non-zero constant by which each term is multiplied by to produce the next term
- **Common difference:** The positive or negative constant added to each term in an arithmetic sequence to produce the next term
- **Asymptote:** A line that is approached by the graph of a function.
- **Radical:** Expression that represents the root of a number.
- **Index:** A number used to indicate what root is to be determined. It is placed above and to the left of the radical sign

Extension Activities:

Stretch

Consider a piece of paper that is 0.1 mm thick. How many times must it be folded so that it reaches the top of the Eiffel Tower? Assume the paper is as large as needed, and it is possible to fold it as many times as required.

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Log-in Information

1. Log-in to Clever
2. Under Math, click on MATHia
3. Username: 6-digit BHS school ID # @bpsma.org
   Password: Date of birth bps 1920
   Example: Student (Michael) with ID #:123456
   Date of birth: January 1st, 2000
   Username: 123456@bpsma.org
   Password: 01012000bps1920

Additional Support

Email:

- Please email your math teacher with specific questions.

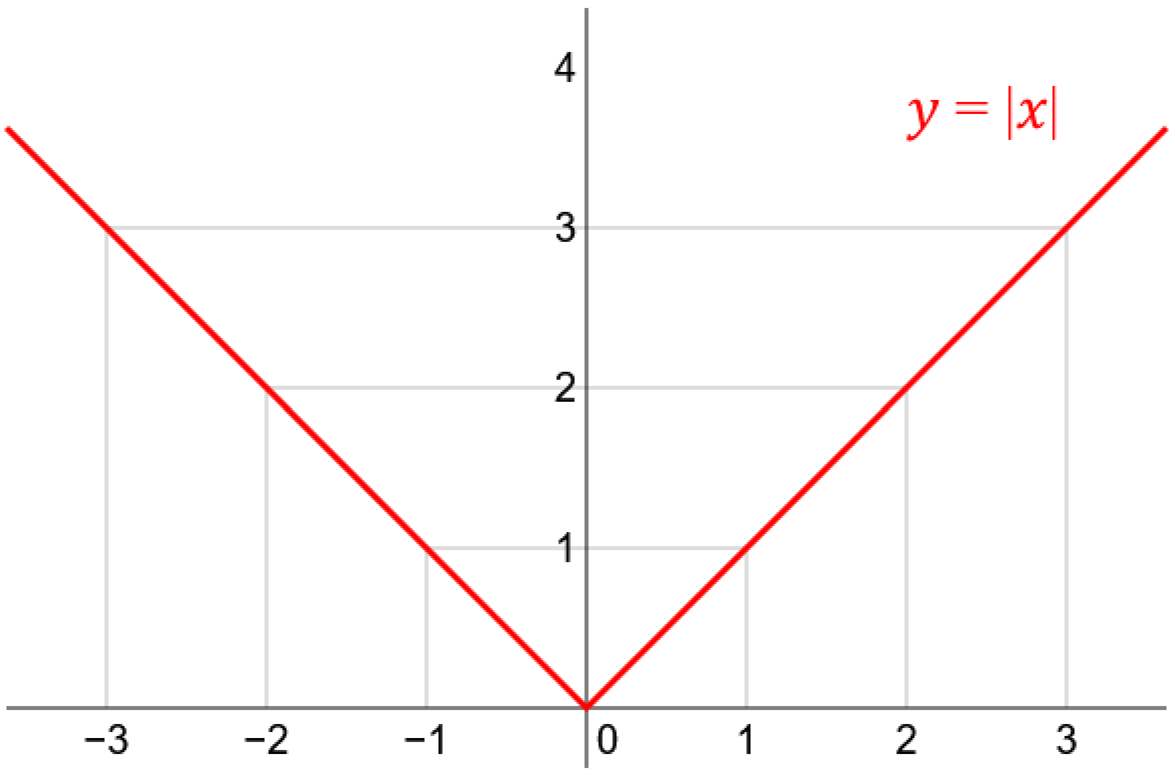
Office Hours:

- Click here for teacher Office hours!

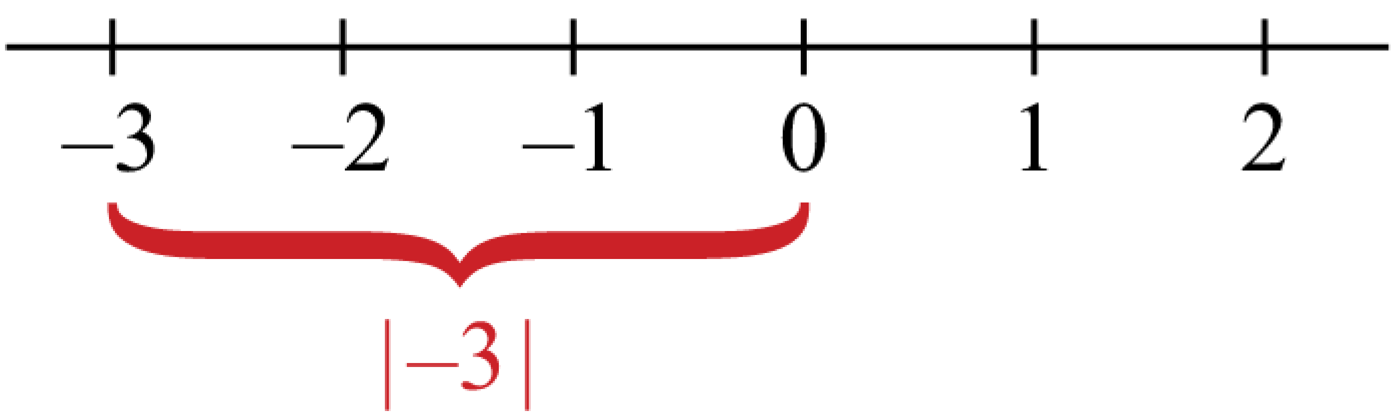
Carnegie Learning Resource

Building Absolute Value Functions:

In mathematics, the absolute value of a real number x, denoted |x|, is the non-negative value of x without regard to its sign. Namely, |x| = x if x is positive, and |x| = −x if x is negative (in which case −x is positive), and |0| = 0. For example, the absolute value of 3 is 3, and the absolute value of −3 is also 3. The absolute value of a number may be thought of as its distance from zero. This is the magnitude of the answer which is the distance from zero regardless of whether the number is positive or negative.



The absolute value of a number may be thought of as its distance from zero.



Solving Absolute Value Equations

Follow the steps below for solving absolute value equations.

Solve Absolute Value Equation

$$|5 - 2x| - 11 = 0$$

$$|5 - 2x| = 11$$ Isolate the absolute value

Split the equation up into two separate equations

$5 - 2x = 11$	$5 - 2x = -11$
$-2x = 6$	$-2x = -16$
$x = -3$	$x = 8$

Solve each of the equations

GETTING STARTED

Distance Is Always Positive

Absolute value is indicated with vertical bars: $|-4|$ is read as "the absolute value of -4."

The **absolute value** of a number is its distance from zero on the number line.

1. Follow your teacher's instructions to model each absolute value expression on the *x*-axis of a classroom coordinate plane. Rewrite each expression without the absolute value symbol.

a. $|-2|$

b. $|2|$

c. $|1 - 2|$

d. $|-3 - (-5)|$

e. $|-2 \cdot 3|$

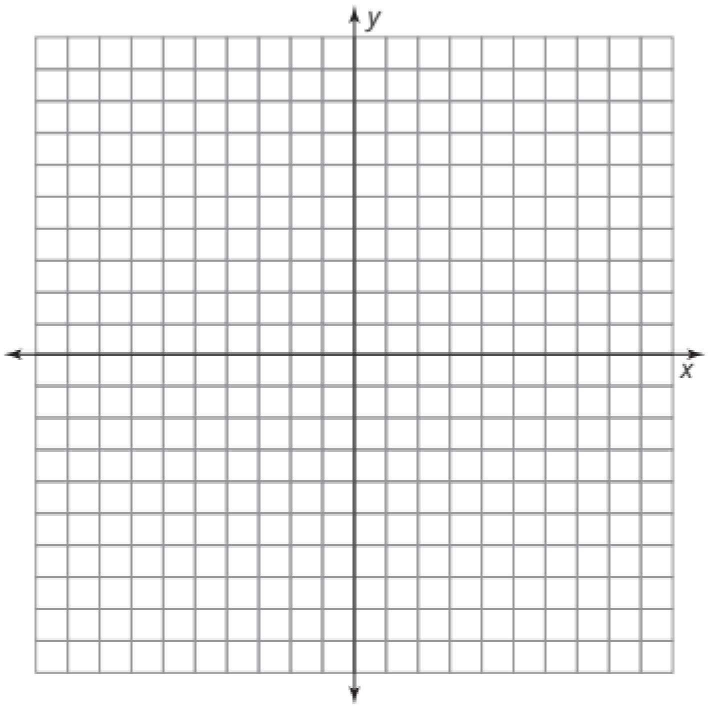
f. $|0 \cdot 4|$

ACTIVITY 1.1 Graphs of Absolute Value Functions

Follow your teacher's instructions to model the function $f(x) = x$ on the classroom coordinate plane with your classmates.

1. Record the coordinates of the plotted points for $f(x) = x$ in the table.

x	y	
	$f(x) = x$	$f(x) = \|x\|$
−9		
−6		
−4		
−1		
0		
3		
5		
8		



2. Change all the plotted points to model the function $f(x) = |x|$. In the table, record the coordinates of the new points for $f(x) = |x|$.

3. Describe how the points move from the graph of $f(x) = x$ to the graph of $f(x) = |x|$.

4. Graph the function $f(x) = |x|$. Describe the characteristics of the function that you notice.

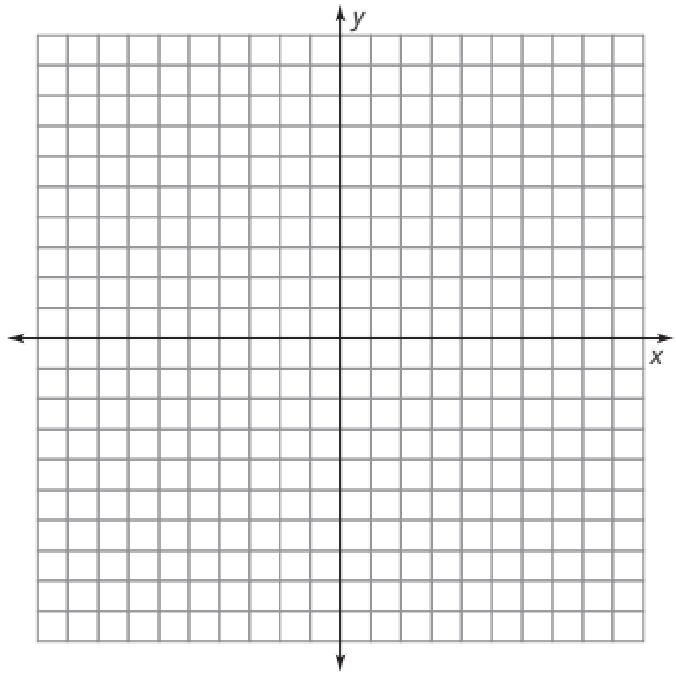
Think about:

What are the domain and range?

Next, consider the function $f(x) = -x$. Model this function on the classroom coordinate plane with your classmates.

5. Record the coordinates of the plotted points for $f(x) = -x$ in the table.

x	y	
	$f(x) = -x$	$f(x) = \lvert -x \rvert$
−9		
−6		
−4		
−1		
0		
3		
5		
8		



6. Change all the plotted points to model the function $f(x) = |-x|$. In the table, record the coordinates of the new points for $f(x) = |-x|$.

Remember:

Use a straightedge to be precise when you graph.

7. Describe how the points move from the graph of $f(x) = -x$ to the graph of $f(x) = |-x|$.

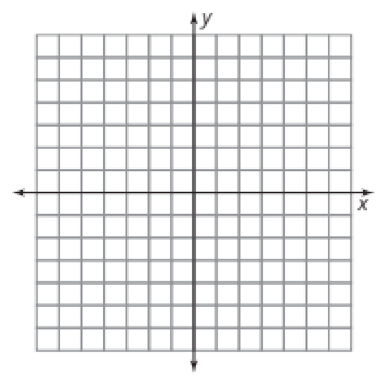
8. Graph the function $f(x) = |-x|$. Compare this function with the function $f(x) = |x|$.

ACTIVITY
1.2
Transformations Inside and Outside the Function

Consider the three absolute value functions shown.

$g(x) = |x|$ $c(x) = |x| + 3$ $d(x) = |x| - 3$

1. Use technology to graph each function. Then, sketch and label the graph of each function.



2. Write the functions $c(x)$ and $d(x)$ in terms of the basic function $g(x)$. Then describe the transformations of each function.

3. Describe the similarities and differences between the three graphs. How do these similarities and differences relate to the equations of the functions $g(x)$, $c(x)$, and $d(x)$?

Recall that a function $t(x)$ of the form $t(x) = f(x) + D$ is a vertical translation of the function $f(x)$. The value $|D|$ describes how many units up or down the graph of the original function is translated.

4. Describe each graph in relation to the basic function $g(x) = |x|$. Then use coordinate notation to represent the vertical translation.

a. $f(x) = g(x) + D$ when $D > 0$

b. $f(x) = g(x) + D$ when $D < 0$

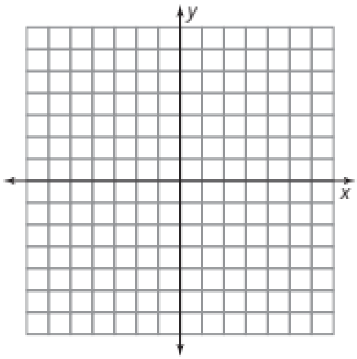
c. Each point (x, y) on the graph of $g(x)$ becomes the point ______________ on $f(x)$.

Consider these absolute value functions.

$g(x) = |x|$ $\qquad$ $k(x) = \frac{1}{2}|x|$

$j(x) = 2|x|$ $\qquad$ $p(x) = -|x|$

5. Use technology to graph each function. Then, sketch and label the graph of each function.



6. Write the functions $j(x)$, $k(x)$, and $p(x)$ in terms of the basic function $g(x)$. Then describe the transformations of each function.

2

Turn That Frown Upside Down

Growth and Decay Functions

Warm Up

Determine the constant ratio for each sequence.

1. 5, 5.25, 5.5125, 5.788125 . . .
2. 100, 20, 4, $\frac{4}{5}$ . . .
3. 1, $\frac{2}{3}$, $\frac{4}{9}$, $\frac{8}{27}$, . . .

Learning Goals

- Classify exponential functions as increasing or decreasing.
- Compare formulas for simple interest and compound interest situations.
- Compare the average rate of change between common intervals of a linear and an exponential relationship.
- Write an exponential function that includes a percent increase or decrease with a *b*-value that is a decimal number.
- Solve exponential equations using graphs.

You know that exponential functions constantly increase or constantly decrease across the entire domain. How can you compare contexts modeled by exponential functions that increase or decrease?

GETTING STARTED

Wakey, Wakey, Eggs and Bakey!

You have studied linear and exponential functions. You know that a linear function increases or decreases by a constant difference, whereas an exponential function increases or decreases by a common ratio.

Remember:

An exponential function can be written as $f(x) = a \cdot b^x$, where b is greater than 0 and not equal to 1.

Consider each function shown.

$f(x) = -3x - 1$	$g(x) = 2^x + 5$	$h(x) = 0.5^x$
$q(x) = 2 \cdot 1.5^{(x+1)}$	$r(x) = -3 \cdot 3^x$	$t(x) = -x + 2^3$
$a(x) = 10 \cdot 1^x$	$b(x) = 5(x - 1) + 4$	$c(x) = 2 \cdot (1 - 0.1)^x$

1. Sort the functions into linear and exponential functions. Justify your choices.

Linear Functions | **Exponential Functions**

Ask yourself:

What does the structure of each equation tell you about its corresponding function?

2. Sort the functions into increasing and decreasing functions. Justify your choices.

Increasing Functions | **Decreasing Functions**

ACTIVITY 2.1

Calculating Interest with Linear and Exponential Functions

Let's consider which function type represents each situation.

Sanjay's family deposited \$20,000 in an interest bearing account for his college fund. Sanjay's account earns simple interest each year.

Chikonde's family deposited \$20,000 in an interest bearing account for her college fund that earns compound interest each year.

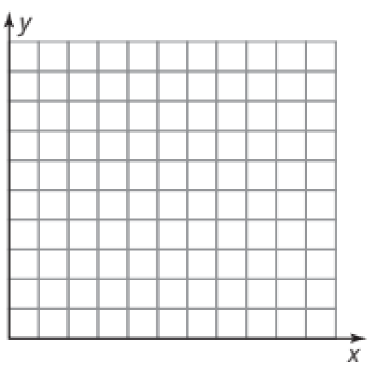
Time (years)	Simple Interest Balance (dollars)	Compound Interest Balance (dollars)
0	20,000	20,000
1	20,800	20,800
2	21,600	21,632
3	22,400	22,497.28
10	28,000	29,604.89

Recall that in a simple interest account, a percent of the starting balance is added to the account at each interval. The formula for simple interest is $I = Prt$, where P represents the starting amount, or principal, r represents the interest rate, t represents time, and I represents the interest earned. In a compound interest account, the balance is multiplied by the same amount at each interval.

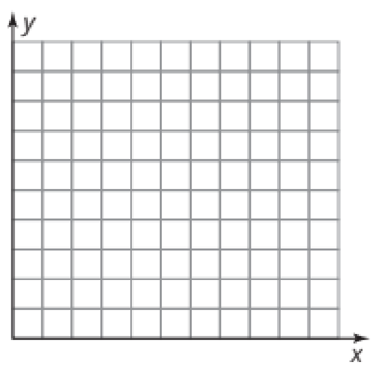
1. Study the table of values.

a. Sketch a graph of each account balance in dollars as a function of the time in years.

Simple Interest Balance



Compound Interest Balance



b. Write a function, $s(t)$, to represent the simple interest account and a function, $c(t)$, to represent the compound interest account.

2. **Use the functions $s(t)$ and $c(t)$ to determine each value.**

a. $s(6)$

b. $c(6)$

c. $c(5)$

d. $s(5)$

3. **Determine the average rate of change between each pair of values given for each relationship.**

Time Intervals (years)	Simple Interest Function (dollars)	Compound Interest Function (dollars)
Between $t = 0$ and $t = 1$		
Between $t = 1$ and $t = 2$		
Between $t = 2$ and $t = 6$		
Between $t = 6$ and $t = 10$		

4. **Compare the average rates of change for the simple and compound interest accounts. What does this tell you about linear and exponential functions?**

ACTIVITY

2.2 Analyzing Exponential Growth and Decay Functions

In 2018, the population of Chicago, Illinois, was about 2.7 million, and the population of Columbus, Ohio, was about 880,000. Chicago's population had decreased from 2010 at a rate of 0.04% each year. At the same time, Columbus's population had grown at a rate of about 0.14% every year.

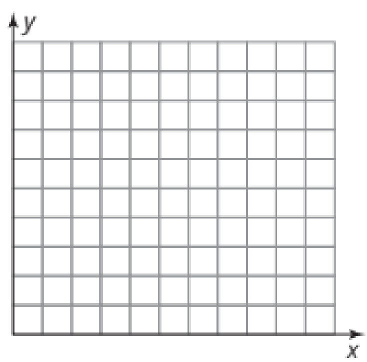
1. Which city's population can be represented as an increasing function, and which can be represented as a decreasing function?

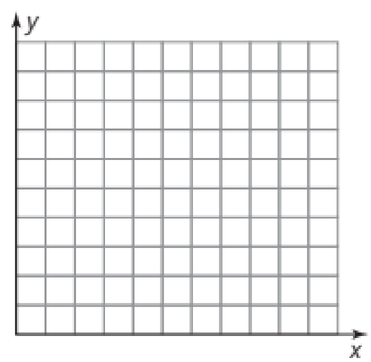
Let's examine the properties of the graphs of the functions for Chicago and Columbus.

Chicago: $G(t) = 2{,}700{,}000(1 - 0.0004)^t$

Columbus: $B(t) = 880{,}000(1 + 0.0014)^t$

2. Sketch a graph of each function. Label key points.





3. The functions $G(t)$ and $B(t)$ can each be written as an exponential function of the form $f(x) = a \cdot b^x$.

a. What is the a-value for each function? What does each a-value mean in terms of this problem situation?

b. What is the b-value for each function? What does each b-value mean in terms of this problem situation?

c. Compare and explain the meanings of the expressions $(1 - 0.0004)^t$ and $(1 + 0.0014)^t$ in terms of this problem situation.

4. Analyze the y-intercepts of each function. Describe how you can determine the y-intercept of each function using only the formula for population increase or decrease.

Think about:

A decreasing exponential function is denoted by a decimal or fractional b-value between 0 and 1, not by a negative b-value.

Consider an exponential function of the form $f(x) = a \cdot b^x$ with $a > 0$. An exponential growth function has a b-value greater than 1 and is of the form $y = a \cdot (1 + r)^x$, where r is the rate of growth. The b-value is $1 + r$. An exponential decay function has a b-value greater than 0 and less than 1 and is of the form $y = a \cdot (1 - r)^x$, where r is the rate of decay. The b-value is $(1 - r)$.

ACTIVITY

2.3 Comparing Increasing and Decreasing Exponential Functions

Consider the six different population scenarios.

1. Match each situation with the appropriate function. Explain your reasoning.

Functions
$f(x) = 17{,}000 \cdot (1 - 0.015)^x$
$f(x) = 17{,}000 \cdot (1 + 0.015)^x$
$f(x) = 17{,}000 \cdot 0.975^x$
$f(x) = 17{,}000 \cdot 1.01^x$
$f(x) = 17{,}000 \cdot 1.025^x$
$f(x) = 17{,}000 \cdot 1.1^x$

Aliso has a population of 17,000. Its population is increasing at a rate of 1.5%.

Charlestown has a population of 17,000. Its population is decreasing at a rate of 2.5%.

Youngstown has a population of 17,000. Its population is decreasing at a rate of 1.5%.

Point Park has a population of 17,000. Its population is increasing at a rate of 2.5%.

North Lake has a population of 17,000. Its population is increasing at a rate of 10%.

Springfield has a population of 17,000. Its population is increasing at a rate of 1%.

Assignment

Write

Explain the difference between simple interest and compound interest.

Remember

An exponential growth function has a b-value greater than 1 and is of the form $y = a \cdot (1 + r)^x$, where r is the rate of growth. An exponential decay function has a b-value greater than 0 and less than 1 and is of the form $y = a \cdot (1 - r)^x$, where r is the rate of decay.

Practice

1. Taylor just received a $2500 bonus check from her employer. She is going to put it into an account that will earn interest. The Level 1 savings account at her bank earns 6% simple interest. The Platinum savings account earns 4.5% compound interest.
   a. Write a function for each account that can be used to determine the balance in the account based on the year, t. Describe each function.
   b. Use your answers to part (a) to create a table of values for each function.
   c. Use technology to graph the functions for the Level 1 and Platinum savings accounts. Then, sketch the graphs.
   d. Into which account would you recommend that Taylor deposit her money? Explain your reasoning.
   e. After reading the pamphlet about the different accounts a little more closely, Taylor realizes that there is a one-time fee of $300 for depositing her money in the Platinum account. Does this change the recommendation you made in part (d)? Why or why not?
   f. Compare the rates of change for the Level 1 and Platinum savings accounts. Explain what the rates of change tell you about the accounts.
   g. What do the rates of change for linear and exponential functions tell you about the graphs of the functions?
2. Brook works for the owners of a bookstore. Her starting salary was $24,500, and she gets a 3% raise
   a. Write an equation in function notation to represent Brook's salary as a function of the number of years she has been working at the bookstore.
   b. What will Brook's salary be when she begins her fourth year working at the bookstore? Show your work.

Module 4
Topic 3

Rational Exponents

rational number

Definition:

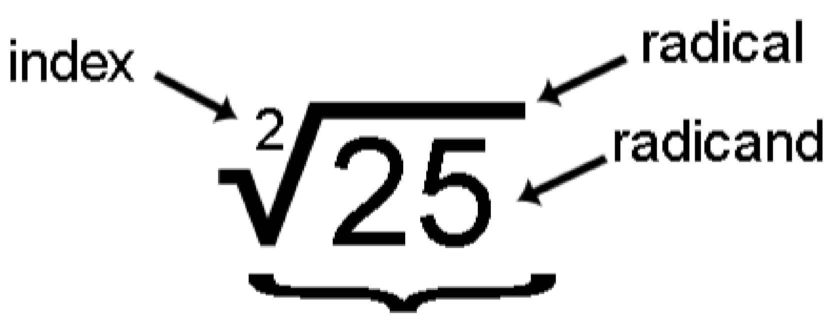
A rational number is a number that can be written in the form $\frac{a}{b}$, where a and b are integers and b is not equal to zero.

Example:

The number -0.5 is a rational number because -0.5 can be written as $-\frac{1}{2} = \frac{-1}{2}$.

Exponent Rule		Example
Power to a Power	$(x^a)^b = x^{ab}$	$(x^2)^4 = x^{2 \cdot 4} = x^8$
Product Rule	$a^b a^c = a^{b+c}$	$(4)^3(4)^5 = (4)^{3+5} = 4^8$
Quotient Rule	$\frac{a^b}{a^c} = a^{b-c}$ for $a \neq 0$.	$\frac{5^6}{5^2} = 5^{6-2} = 5^4$
negative exponent	$x^{-a} = \frac{1}{x^a}$	$2^{-5} = \frac{1}{2^5}$
zero exponent	$x^0 = 1$	$6^0 = 1$

Radicals



This is read as "the square root of 25," or as "radical 25."

You can express a number with rational exponents or in radical form. To change between the two forms, use this rule:

$$x^{\frac{a}{b}} = \sqrt[b]{x^a}$$

Some commonly used equivalents are:

$$x^{\frac{1}{2}} = \sqrt[2]{x^1} = \sqrt{x} \qquad x^{\frac{1}{3}} = \sqrt[3]{x^1} = \sqrt[3]{x}$$

Examples:

Use the appropriate Property of Powers to rewrite each expression.	More Complex Exponential Expressions
$x^6 \cdot x^2 = x^8$ (Product Rule) $(x^6)^2 = x^{12}$ (Power to a Power) $\frac{x^6}{x^2} = x^4$ (Quotient Rule) $\frac{x^2}{x^6} = x^{-4}$ (Quotient Rule) $(x^6)^0 = 1$ (Power to a Power & Zero Exponent)	Exponents may contain algebraic expressions as well as constants. The same Properties of Powers apply when using variables. Exponential expressions must have the same base prior to applying the properties. For example, $\frac{1}{2} \cdot 2^s$, can be rewritten using powers of 2. $\frac{1}{2} \cdot 2^s = 2^{-1} \cdot 2^s$ Negative Exponents Property $= 2^{-1+s}$ Product Rule of Powers $= 2^{s-1}$ Commutative Property of Addition

Skills Practice

Name ______________________________ Date ________________

A. Write each expression as a single power.

1. $\frac{10^5}{10^6}$

2. $\frac{10^0}{10^4}$

3. $\frac{10^2}{10^5}$

4. $\frac{x^4}{x^9}$

5. $\frac{5^3}{5^{10}}$

6. $\frac{y^2}{y^8}$

B. Write each radical using a rational exponent.

1. $\sqrt[4]{15}$

2. $\sqrt[3]{5}$

3. $\sqrt[4]{31}$

4. $\sqrt[3]{x}$

5. $\sqrt[6]{y}$

6. $\sqrt{z}$

C. Write each power in radical form.

1. $12^{\frac{1}{3}}$

2. $7^{\frac{1}{5}}$

3. $18^{\frac{1}{4}}$

4. $a^{\frac{1}{2}}$

5. $d^{\frac{1}{5}}$

6. $c^{\frac{1}{6}}$

7. $5^{\frac{2}{3}}$

8. $8^{\frac{2}{5}}$

9. $18^{\frac{3}{4}}$

10. $x^{\frac{3}{5}}$

11. $y^{\frac{4}{3}}$

12. $m^{\frac{5}{2}}$

D. Write each expression using a rational exponent. Simplify your answer.

1. $\sqrt[4]{6^3}$

2. $\sqrt[5]{8^4}$

3. $\sqrt[3]{12^2}$

4. $\sqrt{n^5}$

5. $\sqrt[4]{p^7}$

6. $\sqrt[5]{m^3}$