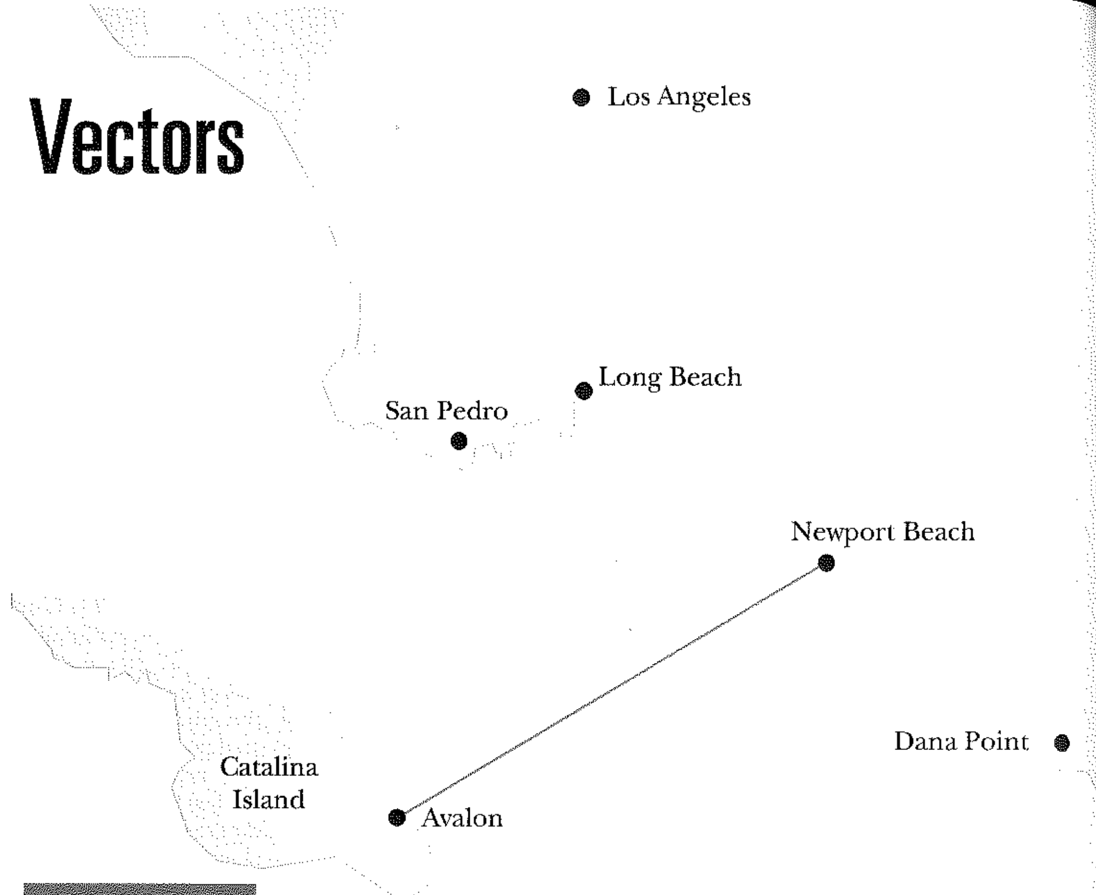
3 Vectors



Catalina Island can be reached from different starting points along the Los Angeles–Orange County coast. The opening storyline refers to a trip to Avalon beginning in Newport Beach.

STORYLINE **Your road trip in Chapter 2 takes you toward the ocean.** You end up in Newport Beach, California. Your friend who was driving the car in Chapter 2 owns a sailboat and asks you to pilot the boat from Newport Beach to Catalina Island, which is 26 miles off the coast. Your friend challenges you to pilot the boat along a perfectly straight line. Always up for a challenge, you agree, settle into the captain's chair, and then panic. You know you have to travel 26 miles in a straight line, but what should you set as the heading for the boat? The distance of 26 miles is not sufficient information to allow you to travel to Catalina Island in a straight line. You realize that your trip will require both the distance to Catalina Island and the *direction* in which you must travel. You ask your friend the appropriate direction to Catalina Island and he gives you a heading as an angle south of due west. You open the compass app on your smartphone, find the appropriate direction, and set sail!

CONNECTIONS If you move only along a straight line, as in the previous chapter, then a single number (with a positive or negative sign) can be used to specify your position with respect to the origin. In this chapter, we will study the positions of objects or points in two- or three-dimensional space that require two types of information: distance from a reference point and direction relative to a reference axis. Quantities that require these two types of information are called *vectors*. We will learn various properties of vectors and will see how to add and subtract vectors. Vector quantities are used throughout this text. In addition to the position vectors studied in this chapter, we will see other vector quantities in subsequent chapters, such as velocity, acceleration, force, and electric field. Therefore, it is imperative that you master the techniques discussed in this chapter.

3.1 Coordinate Systems

Many aspects of physics involve a description of a location in space. In Chapter 2, for example, we saw that the mathematical description of an object's motion requires a method for describing the object's position at various times. In two dimensions, this description is accomplished with the use of the Cartesian coordinate system, in which perpendicular axes intersect at a point defined as the origin O (Fig. 3.1). Cartesian coordinates of a point in space, representing the x and y values of the point, and expressed as (x, y), are also called *rectangular coordinates.*

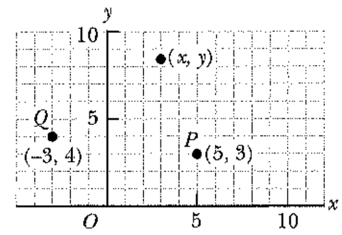


Figure 3.1 Designation of points in a Cartesian coordinate system. Every point is labeled with coordinates (x, y).

Sometimes it is more convenient to represent a point in a plane by its *plane polar coordinates* (r, θ) as shown in Figure 3.2a. In this *polar coordinate system,* r is the distance from the origin to the point having Cartesian coordinates (x, y) and θ is the angle between a fixed axis and a line drawn from the origin to the point. The fixed axis is often the positive x axis, and θ is usually measured counterclockwise from it. From the right triangle in Figure 3.2b, we find that $\sin\theta = y/r$ and that $\cos\theta = x/r$. (A review of trigonometric functions is given in Appendix B.4.) Therefore, starting with the plane polar coordinates of any point, we can obtain the Cartesian coordinates by using the equations

Cartesian coordinates in terms of polar coordinates

$$x = r\cos\theta \tag{3.1}$$

$$y = r\sin\theta \tag{3.2}$$

Conversely, if we know the Cartesian coordinates, the definitions of trigonometry tell us that the polar coordinates are given by

Polar coordinates in terms of Cartesian coordinates

$$\tan\theta = \frac{y}{x} \tag{3.3}$$

$$r = \sqrt{x^2 + y^2} \tag{3.4}$$

Equation 3.4 is the familiar Pythagorean theorem.

These four expressions relating the coordinates (x, y) to the coordinates (r, θ) apply only when θ is defined as shown in Figure 3.2a—in other words, when positive θ is an angle measured counterclockwise from the positive x axis. (Some scientific calculators perform conversions between Cartesian and polar coordinates based on these standard conventions.) If the reference axis for the polar angle θ is chosen to be one other than the positive x axis or if the sense of increasing θ is chosen differently, the expressions relating the two sets of coordinates will be different from those above.

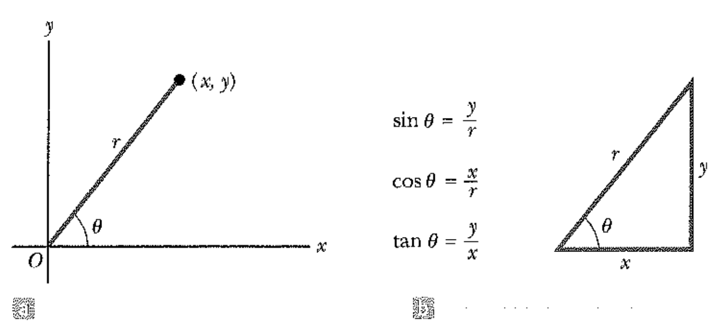


Figure 3.2 (a) The plane polar coordinates of a point are represented by the distance r and the angle θ, where θ is measured counterclockwise from the positive x axis. (b) The right triangle used to relate (x, y) to (r, θ).

Example 3.1 Polar Coordinates

The Cartesian coordinates of a point in the xy plane are $(x, y) = (-3.50, -2.50)$ m as shown in Figure 3.3. Find the polar coordinates of this point.

SOLUTION

Conceptualize The drawing in Figure 3.3 helps us conceptualize the problem. We wish to find r and θ. Based on the figure and the data given in the problem statement, we expect r to be a few meters and θ to be between 180° and 270°.

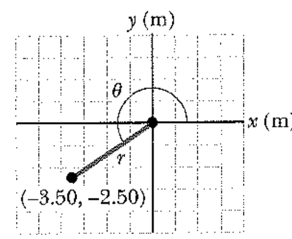


Figure 3.3 (Example 3.1) Finding polar coordinates when Cartesian coordinates are given.

Categorize Based on the statement of the problem and the Conceptualize step, we recognize that we are simply converting from Cartesian coordinates to polar coordinates. We therefore categorize this example as a substitution problem. As mentioned in Section 2.4, substitution problems generally do not have an extensive Analyze step other than the substitution of numbers into a given equation. Similarly, the Finalize step consists primarily of checking the units and making sure that the answer is reasonable and consistent with our expectations. Therefore, for substitution problems, we will not label Analyze or Finalize steps.

Use Equation 3.4 to find r:

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2} = 4.30\text{ m}$$

Use Equation 3.3 to find θ:

$$\tan\theta = \frac{y}{x} = \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$

$$\theta = 216°$$

Notice that you must use the signs of x and y to find that the point lies in the third quadrant of the coordinate system. That is, $\theta = 216°$, not 35.5°, whose tangent is also 0.714. Answers to both r and θ agree with our expectations in the Conceptualize step.

3.2 Vector and Scalar Quantities

We now formally describe the difference between scalar quantities and vector quantities. When you want to know the temperature outside so that you will know how to dress, the only information you need is a number and the unit "degrees C" or "degrees F." Temperature is therefore an example of a *scalar quantity:*

A **scalar quantity** is completely specified by a single value with an appropriate unit and has no direction.

Other examples of scalar quantities are volume, mass, speed, time, and time intervals. Some scalars are always positive, such as mass and speed. Others, such as temperature, can have either positive or negative values. The rules of ordinary arithmetic are used to manipulate scalar quantities.

If you are preparing to pilot a small plane and need to know the wind velocity, you must know both the speed of the wind and its direction. Because direction is important for its complete specification, velocity is a *vector quantity:*

A **vector quantity** is completely specified by a number with an appropriate unit (the *magnitude* of the vector) plus a direction.

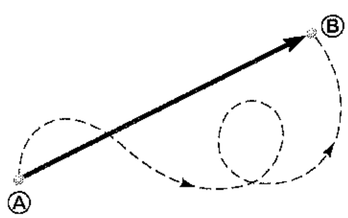


Figure 3.4 As a particle moves from Ⓐ to Ⓑ either along the straight line *or* along an arbitrary path represented by the broken line, its displacement is a vector quantity shown by the arrow drawn from Ⓐ to Ⓑ.

Another example of a vector quantity is displacement, as you know from Chapter 2. Suppose a particle moves from some point Ⓐ to some point Ⓑ along a straight path as shown in Figure 3.4. We represent this displacement by drawing an arrow from Ⓐ to Ⓑ, with the tip of the arrow pointing away from the starting point. The direction of the arrowhead represents the direction of the displacement, and the length of the arrow represents the magnitude of the displacement. If the particle travels along some other path from Ⓐ to Ⓑ such as shown by the broken line in Figure 3.4, its displacement is still the arrow drawn from Ⓐ to Ⓑ. Displacement depends only on the initial and final positions, so the displacement vector is independent of the path taken by the particle between these two points.

In this text, we use a boldface letter with an arrow over the letter, such as $\vec{\mathbf{A}}$, to represent a vector. Another common notation for vectors with which you should be familiar is a simple boldface character: **A**. The magnitude of the vector $\vec{\mathbf{A}}$ is written either A or $|\vec{\mathbf{A}}|$. The magnitude of a vector has physical units, such as meters for displacement or meters per second for velocity. The magnitude of a vector is *always* a positive number.

What about the vector to follow in our opening storyline? What heading did your friend give you to Catalina Island? You can use a latitude and longitude finder online to find the coordinates for the opening of Newport Harbor and for Avalon Harbor. Then, putting these coordinates into a distance and azimuth calculator online, you find that the distance is 30.7 mi, with a heading of 236.2° relative to due east. (Note that Catalina is described as "26 miles across the sea" in a popular song from the 1950s, but we need to travel a bit farther to make this trip. An online calculation shows the distance between San Pedro and Avalon to be 27 miles, which might be the origin of the song.)

QUICK QUIZ 3.1 Which of the following are vector quantities and which are scalar quantities? **(a)** your age **(b)** acceleration **(c)** velocity **(d)** speed **(e)** mass

3.3 Basic Vector Arithmetic

For many purposes, two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ may be defined to be *equal* if they have the same magnitude and if they point in the same direction. That is, $\vec{\mathbf{A}} = \vec{\mathbf{B}}$ only if $A = B$ and if $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ point in the same direction along parallel lines. For example, all the vectors in Figure 3.5 are equal even though they have different starting points. This property allows us to move a vector to a position parallel to itself in a diagram without affecting the vector.

The rules for **vector addition** are conveniently described by a graphical method. To add vector $\vec{\mathbf{B}}$ to vector $\vec{\mathbf{A}}$, first draw vector $\vec{\mathbf{A}}$ on graph paper, with its magnitude represented by a convenient length scale, and then draw vector $\vec{\mathbf{B}}$ to the same scale, with its tail starting from the tip of $\vec{\mathbf{A}}$, as shown in Figure 3.6. The **resultant vector** $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ is the vector drawn from the tail of $\vec{\mathbf{A}}$ to the tip of $\vec{\mathbf{B}}$.

A geometric construction can also be used to add more than two vectors as shown in Figure 3.7 for the case of three vectors. The resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$ is the vector that completes the polygon. In other words, $\vec{\mathbf{R}}$ is the vector drawn from the tail of the first vector to the tip of the last vector. This technique for adding vectors is often called the "head to tail method."

PITFALL PREVENTION 3.1
Vector Addition Versus Scalar Addition Notice that $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{C}}$ is very different from $A + B = C$. The first equation is a vector sum, which must be handled carefully, such as with the graphical method. The second equation is a simple algebraic addition of numbers that is handled with the normal rules of arithmetic.

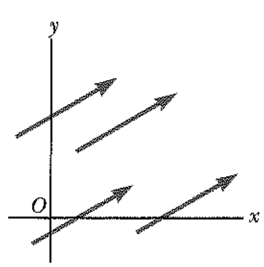


Figure 3.5 These four vectors are equal because they have equal lengths and point in the same direction.

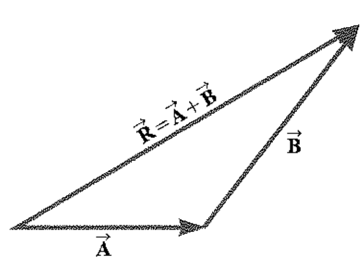


Figure 3.6 When vector $\vec{\mathbf{B}}$ is added to vector $\vec{\mathbf{A}}$ the resultant $\vec{\mathbf{R}}$ is the vector that runs from the tail of $\vec{\mathbf{A}}$ to the tip of $\vec{\mathbf{B}}$.

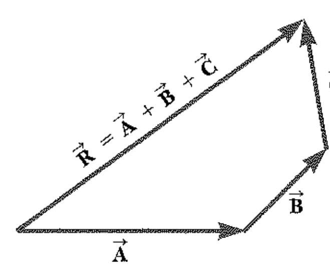


Figure 3.7 Geometric construction for summing three vectors. The resultant vector $\vec{\mathbf{R}}$ is by definition the one that completes the polygon.

When two vectors are added, the sum is independent of the order of the addition. (This fact may seem trivial, but as you will see in Chapter 11, the order is important when vectors are multiplied. Procedures for multiplying vectors are discussed in Chapters 7 and 11.) This property, which can be seen from the geometric construction in Figure 3.8, is known as the **commutative law of addition:**

Commutative law of addition ▶

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}} \tag{3.5}$$

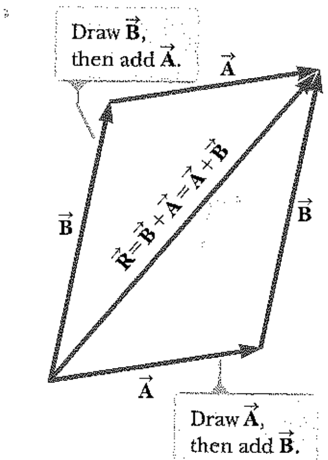


Figure 3.8 This construction shows that $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$ or, in other words, that vector addition is commutative.

When three or more vectors are added, their sum is independent of the way in which the individual vectors are grouped together. A geometric proof of this rule for three vectors is given in Figure 3.9, where two ways of adding the same three vectors are shown. This property is called the **associative law of addition:**

$$\vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = (\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}} \tag{3.6}$$

We have described adding displacement vectors in this section because these types of vectors are easy to visualize. We can also add other types of vectors, such as velocity, force, and electric field vectors, which we will do in later chapters. When two or more vectors are added together, they must all have the same units and they must all be the same type of quantity. It would be meaningless to add a velocity vector (for example, 60 km/h to the east) to a displacement vector (for example, 200 km to the north) because these vectors represent different physical quantities. The same rule also applies to scalars. For example, it would be meaningless to add time intervals to temperatures.

The operation of **vector subtraction** makes use of the definition of the negative of a vector. The negative of the vector $\vec{\mathbf{A}}$ is defined as the vector that when added to $\vec{\mathbf{A}}$ gives zero for the vector sum. That is, $\vec{\mathbf{A}} + (-\vec{\mathbf{A}}) = 0$. The vectors $\vec{\mathbf{A}}$ and $-\vec{\mathbf{A}}$ have the same magnitude but point in opposite directions. We define the operation $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ as vector $-\vec{\mathbf{B}}$ added to vector $\vec{\mathbf{A}}$:

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}}) \tag{3.7}$$

The geometric construction for subtracting two vectors in this way is illustrated in Figure 3.10a.

Another way of looking at vector subtraction is to notice that the difference $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ between two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is what you have to add to the second vector to obtain the first. In this case, as Figure 3.10b shows, the vector $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ points from the tip of the second vector to the tip of the first.

Scalar multiplication of vectors is straightforward. If vector $\vec{\mathbf{A}}$ is multiplied by a positive scalar quantity m, the product $m\vec{\mathbf{A}}$ is a vector that has the same direction as $\vec{\mathbf{A}}$ and magnitude mA. If vector $\vec{\mathbf{A}}$ is multiplied by a negative scalar quantity $-m$,

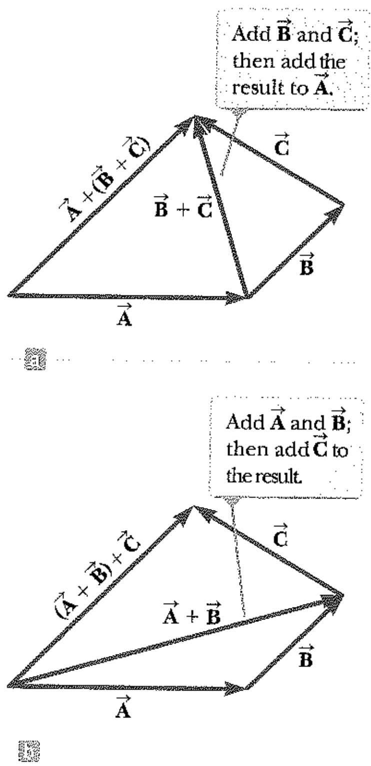


Figure 3.9 Geometric constructions for verifying the associative law of addition. (a) Vectors $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$ are added first and added to $\vec{\mathbf{A}}$. (b) Vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are added first, and then $\vec{\mathbf{C}}$ is added.

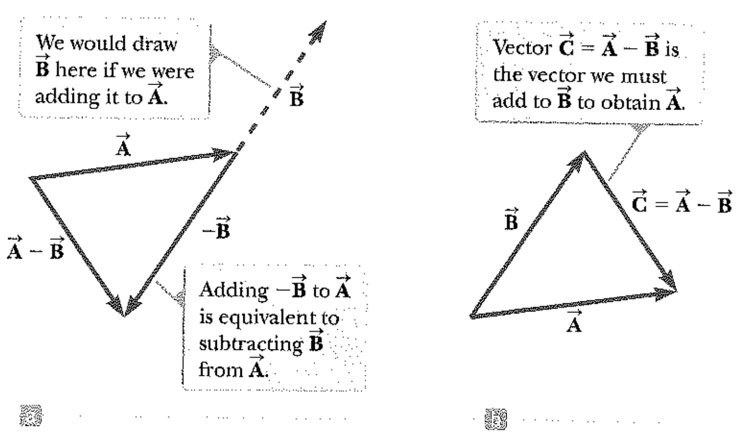


Figure 3.10 (a) Subtracting vector $\vec{\mathbf{B}}$ from vector $\vec{\mathbf{A}}$. The vector $-\vec{\mathbf{B}}$ is equal in magnitude to vector $\vec{\mathbf{B}}$ and points in the opposite direction. (b) A second way of looking at vector subtraction.

the product $-m\vec{\mathbf{A}}$ is directed opposite $\vec{\mathbf{A}}$. For example, the vector $5\vec{\mathbf{A}}$ is five times as long as $\vec{\mathbf{A}}$ and points in the same direction as $\vec{\mathbf{A}}$; the vector $-\frac{1}{3}\vec{\mathbf{A}}$ is one-third the length of $\vec{\mathbf{A}}$ and points in the direction opposite $\vec{\mathbf{A}}$.

QUICK QUIZ 3.2 The magnitudes of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are $A = 12$ units and $B = 8$ units. Which pair of numbers represents the *largest* and *smallest* possible values for the magnitude of the resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$? **(a)** 14.4 units, 4 units **(b)** 12 units, 8 units **(c)** 20 units, 4 units **(d)** none of these answers

QUICK QUIZ 3.3 If vector $\vec{\mathbf{B}}$ is added to vector $\vec{\mathbf{A}}$, which *two* of the following choices must be true for the resultant vector to be equal to zero? **(a)** $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are parallel and in the same direction. **(b)** $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are parallel and in opposite directions. **(c)** $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitude. **(d)** $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular.

Example 3.2 A Vacation Trip

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north as shown in Figure 3.11a. Find the magnitude and direction of the car's resultant displacement.

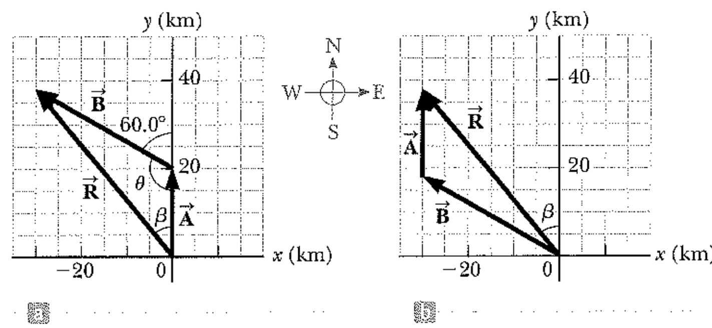


Figure 3.11 (Example 3.2) (a) Graphical method for finding the resultant displacement vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. (b) Adding the vectors in reverse order ($\vec{\mathbf{B}} + \vec{\mathbf{A}}$) gives the same result for $\vec{\mathbf{R}}$.

SOLUTION

Conceptualize The two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ that appear in Figure 3.11a help us conceptualize the problem. The resultant vector $\vec{\mathbf{R}}$ has also been drawn. We expect its magnitude to be a few tens of kilometers. The angle β that the resultant vector makes with the y axis is expected to be less than 60°, the angle that vector $\vec{\mathbf{B}}$ makes with the y axis.

Categorize We can categorize this example as a simple analysis problem in vector addition. The displacement $\vec{\mathbf{R}}$ is the resultant when the two individual displacements $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are added. We can further categorize it as a problem about the analysis of triangles, so we appeal to our expertise in geometry and trigonometry.

Analyze In this example, we show two ways to analyze the problem of finding the resultant of two vectors. The first way is to solve the problem geometrically, using graph paper and a protractor to measure the magnitude of $\vec{\mathbf{R}}$ and its direction in Figure 3.11a. (In fact, even when you know you are going to be carrying out a calculation, you should sketch the vectors to check your results.) With an ordinary ruler and protractor, a large diagram typically gives answers to two-digit but not to three-digit precision. Try using these tools on $\vec{\mathbf{R}}$ in Figure 3.11a and compare to the trigonometric analysis below!

The second way to solve the problem is to analyze it using algebra and trigonometry. The magnitude of $\vec{\mathbf{R}}$ can be obtained from the law of cosines as applied to the triangle in Figure 3.11a (see Appendix B.4).

Use $R^2 = A^2 + B^2 - 2AB\cos\theta$ from the law of cosines to find R:

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

Substitute numerical values, noting that $\theta = 180° - 60° = 120°$:

$$R = \sqrt{(20.0\text{ km})^2 + (35.0\text{ km})^2 - 2(20.0\text{ km})(35.0\text{ km})\cos 120°}$$
$$= 48.2\text{ km}$$

Use the law of sines (Appendix B.4) to find the direction of $\vec{\mathbf{R}}$ measured from the northerly direction:

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta = \frac{35.0\text{ km}}{48.2\text{ km}}\sin 120° = 0.629$$

$$\beta = 38.9°$$

The resultant displacement of the car is 48.2 km in a direction 38.9° west of north.

continued

3.2 continued

Finalize Does the angle β that we calculated agree with an estimate made by looking at Figure 3.11a or with an actual angle measured from the diagram using the graphical method? Is it reasonable that the magnitude of $\vec{\mathbf{R}}$ is larger than that of both $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$? Are the units of $\vec{\mathbf{R}}$ correct?

Although the head to tail method of adding vectors works well, it suffers from two disadvantages. First, some people find using the laws of cosines and sines to be awkward. Second, a triangle only results if you are adding two vectors. If you are adding three or more vectors, the resulting geometric shape is usually not a triangle. In Section 3.4, we explore a new method of adding vectors that will address both of these disadvantages.

WHAT IF? Suppose the trip were taken with the two vectors in reverse order: 35.0 km at 60.0° west of north first and then 20.0 km due north. How would the magnitude and the direction of the resultant vector change?

Answer They would not change. The commutative law for vector addition tells us that the order of vectors in an addition is irrelevant. Graphically, Figure 3.11b shows that the vectors added in the reverse order give us the same resultant vector.

3.4 Components of a Vector and Unit Vectors

The graphical method of adding vectors is not recommended whenever high accuracy is required or in three-dimensional problems. In this section, we describe a method of adding vectors that makes use of the projections of vectors along coordinate axes. These projections are called the **components** of the vector or its **rectangular components.** Any vector can be completely described by its components.

Consider a vector $\vec{\mathbf{A}}$ lying in the xy plane and making an arbitrary angle θ with the positive x axis as shown in Figure 3.12a. This vector can be expressed as the sum of two other *component vectors* $\vec{\mathbf{A}}_x$, which is parallel to the x axis, and $\vec{\mathbf{A}}_y$, which is parallel to the y axis. From the figure, we see that the three vectors form a right triangle and that $\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$. We shall often refer to the "components of a vector $\vec{\mathbf{A}}$," written A_x and A_y (without the boldface notation). Figure 3.12b shows the component vector $\vec{\mathbf{A}}_y$ moved to the left so that it lies along the y axis. We see that the component A_x represents the projection of $\vec{\mathbf{A}}$ along the x axis, and the component A_y represents the projection of $\vec{\mathbf{A}}$ along the y axis. These components can be positive or negative. The component A_x is positive if the component vector $\vec{\mathbf{A}}_x$ points in the positive x direction and is negative if $\vec{\mathbf{A}}_x$ points in the negative x direction. A similar statement is made for the component A_y.

PITFALL PREVENTION 3.2

***x* and *y* Components** Equations 3.8 and 3.9 associate the cosine of the angle with the x component and the sine of the angle with the y component. This association is true *only* because we measured the angle θ with respect to the x axis, so do not memorize these equations. If θ is measured with respect to the y axis (as in some problems), these equations will be incorrect. Think about which side of the triangle containing the components is adjacent to the angle and which side is opposite and then assign the cosine and sine accordingly.

From Figure 3.12 and the definition of sine and cosine, we see that $\cos\theta = A_x/A$ and that $\sin\theta = A_y/A$. Hence, the components of $\vec{\mathbf{A}}$ are

$$A_x = A\cos\theta \tag{3.8}$$

$$A_y = A\sin\theta \tag{3.9}$$

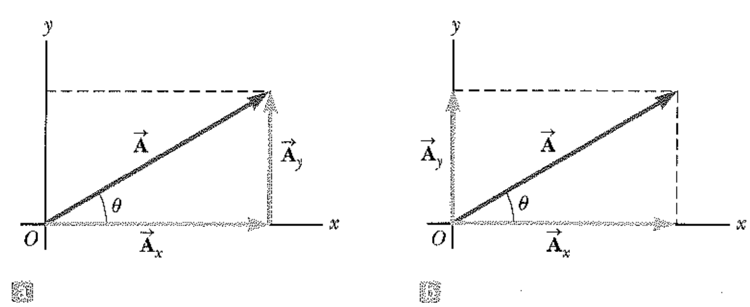


Figure 3.12 (a) A vector $\vec{\mathbf{A}}$ lying in the xy plane can be represented as a vector sum of its component vectors $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$. These three vectors form a right triangle. (b) The y component vector $\vec{\mathbf{A}}_y$ can be moved to the left so that it lies along the y axis.

The magnitudes of these components are the lengths of the two sides of a right triangle with a hypotenuse of length A. Therefore, the magnitude and direction of $\vec{\mathbf{A}}$ are related to its components through the expressions

$$A = \sqrt{A_x^2 + A_y^2} \tag{3.10}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \tag{3.11}$$

Notice that the signs of the components A_x and A_y depend on the angle θ. For example, if $\theta = 120°$, A_x is negative and A_y is positive. If $\theta = 225°$, both A_x and A_y are negative. Figure 3.13 summarizes the directions of the component vectors and signs of the components when $\vec{\mathbf{A}}$ lies in the various quadrants.

When solving problems in two dimensions, you can specify a vector $\vec{\mathbf{A}}$ either with its components A_x and A_y or with its magnitude and direction A and θ.

In many applications, it is convenient to express the components in a coordinate system having axes that are not horizontal and vertical but that are still perpendicular to each other. For example, we will consider the motion of objects sliding down inclined planes. For these examples, it is often convenient to orient the x axis parallel to the plane and the y axis perpendicular to the plane.

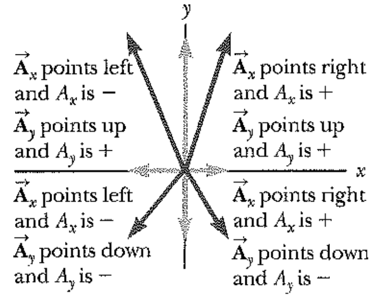


Figure 3.13 The signs of the components of a vector $\vec{\mathbf{A}}$ depend on the quadrant in which the vector is located.

QUICK QUIZ 3.4 Choose the correct response to make the sentence true: A component of a vector is **(a)** always, **(b)** never, or **(c)** sometimes larger than the magnitude of the vector.

Vector quantities often are expressed in terms of unit vectors. A **unit vector** is a dimensionless vector having a magnitude of exactly 1. Unit vectors are used to specify a given direction and have no other physical significance. They are used solely as a bookkeeping convenience in describing a direction in space. We shall use the symbols $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ to represent unit vectors pointing in the positive x, y, and z directions, respectively. (The "hats," or circumflexes, on the symbols are a standard notation for unit vectors.) The unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ form a set of mutually perpendicular vectors in a right-handed coordinate system as shown in Figure 3.14a. The magnitude of each unit vector equals 1; that is, $|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$.

Consider a vector $\vec{\mathbf{A}}$ lying in the xy plane as shown in Figure 3.14b. The product of the component A_x and the unit vector $\hat{\mathbf{i}}$ is the component vector $\vec{\mathbf{A}}_x = A_x\hat{\mathbf{i}}$, which lies on the x axis and has magnitude $|A_x|$. Likewise, $\vec{\mathbf{A}}_y = A_y\hat{\mathbf{j}}$ is the component vector of magnitude $|A_y|$ lying on the y axis. Therefore, the unit-vector notation for the vector $\vec{\mathbf{A}}$ is

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} \tag{3.12}$$

Consider now the polar coordinates shown for the point in Figure 3.2. The point in the first quadrant in that figure is reproduced in Figure 3.15. Notice that we can

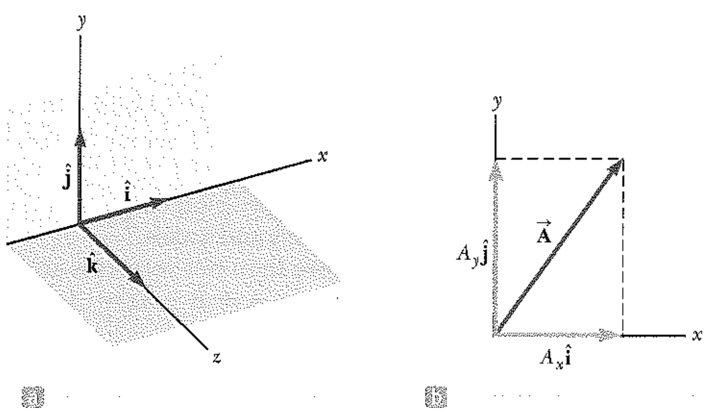


Figure 3.14 (a) The unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are directed along the x, y, and z axes, respectively. (b) Vector $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$ lying in the xy plane has components A_x and A_y.

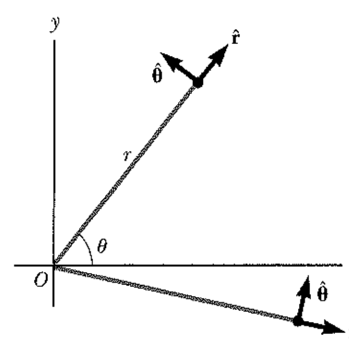


Figure 3.15 Unit vectors for a point described by polar coordinates.

identify radial and angular unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$. Just like for rectangular coordinates, these vectors are of unit length. Unlike rectangular coordinates, however, the directions of radial and angular unit vectors depend on the point, as shown by the point in the fourth quadrant in Figure 3.15.

Now let us see how to use components to add vectors when the graphical method is not sufficiently accurate. Suppose we wish to add vector $\vec{\mathbf{B}}$ to vector $\vec{\mathbf{A}}$ in Equation 3.12, where vector $\vec{\mathbf{B}}$ has components B_x and B_y. Because of the bookkeeping convenience of the unit vectors, all we do is add the x and y components separately. The resultant vector $\vec{\mathbf{R}}$ is

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}) + (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}})$$

or, rearranging terms,

$$\vec{\mathbf{R}} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} \tag{3.13}$$

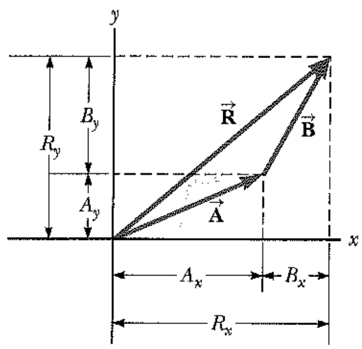


Figure 3.16 This geometric construction for the sum of two vectors shows the relationship between the components of the resultant $\vec{\mathbf{R}}$ and the components of the individual vectors.

Because $\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$, we see that the components of the resultant vector are

$$R_x = A_x + B_x$$
$$R_y = A_y + B_y \tag{3.14}$$

Therefore, we see that in the component method of adding vectors, we add all the x components together to find the x component of the resultant vector and use the same process for the y components. We can check this addition by components with a geometric construction as shown in Figure 3.16.

The magnitude of $\vec{\mathbf{R}}$ and the angle it makes with the x axis are obtained from its components using the relationships

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \tag{3.15}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x} \tag{3.16}$$

PITFALL PREVENTION 3.3

Tangents on Calculators

Equation 3.16 involves the calculation of an angle by means of a tangent function. Generally, the inverse tangent function on calculators provides an angle between $-90°$ and $+90°$. As a consequence, if the vector you are studying lies in the second or third quadrant, the angle measured from the positive x axis will be the angle your calculator returns plus 180°.

At times, we need to consider situations involving motion in three component directions. The extension of our methods to three-dimensional vectors is straightforward. If $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ both have x, y, and z components, they can be expressed in the form

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}} \tag{3.17}$$

$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} \tag{3.18}$$

The sum of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is

$$\vec{\mathbf{R}} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} + (A_z + B_z)\hat{\mathbf{k}} \tag{3.19}$$

Notice that Equation 3.19 differs from Equation 3.13: in Equation 3.19, the resultant vector also has a z component $R_z = A_z + B_z$. If a vector $\vec{\mathbf{R}}$ has x, y, and z components, the magnitude of the vector is $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$. The angle θ_x that $\vec{\mathbf{R}}$ makes with the x axis is found from the expression $\cos\theta_x = R_x/R$, with similar expressions for the angles with respect to the y and z axes.

The extension of our method to adding more than two vectors is also straightforward using the component method. For example, $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = (A_x + B_x + C_x)\hat{\mathbf{i}} + (A_y + B_y + C_y)\hat{\mathbf{j}} + (A_z + B_z + C_z)\hat{\mathbf{k}}$.

QUICK QUIZ 3.5 For which of the following vectors is the magnitude of the vector equal to one of the components of the vector? **(a)** $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$ **(b)** $\vec{\mathbf{B}} = -3\hat{\mathbf{j}}$ **(c)** $\vec{\mathbf{C}} = +5\hat{\mathbf{k}}$

Example 3.3 The Sum of Two Vectors

Find the sum of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ lying in the *xy* plane and given by

$$\vec{\mathbf{A}} = (2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}}) \quad \text{and} \quad \vec{\mathbf{B}} = (2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}})$$

SOLUTION

Conceptualize You can conceptualize the situation by drawing the vectors on graph paper. Do this and then draw an approximation of the expected resultant vector.

Categorize We categorize this example as a simple substitution problem. Comparing this expression for $\vec{\mathbf{A}}$ with the general expression $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$, we see that $A_x = 2.0$, $A_y = 2.0$, and $A_z = 0$. Likewise, $B_x = 2.0$, $B_y = -4.0$, and $B_z = 0$. We can use a two-dimensional approach because there are no *z* components.

Use Equation 3.13 to obtain the resultant vector $\vec{\mathbf{R}}$:

$$\vec{\mathbf{R}} = (A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} = (2.0 + 2.0)\hat{\mathbf{i}} + (2.0 - 4.0)\hat{\mathbf{j}}$$
$$= 4.0\hat{\mathbf{i}} - 2.0\hat{\mathbf{j}}$$

Use Equation 3.15 to find the magnitude of $\vec{\mathbf{R}}$:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0)^2 + (-2.0)^2} = \sqrt{20} = 4.5$$

Find the direction of $\vec{\mathbf{R}}$ from Equation 3.16:

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2.0}{4.0} = -0.50$$

Your calculator likely gives the answer $-27°$ for $\theta = \tan^{-1}(-0.50)$. This answer is correct if we interpret it to mean 27° *clockwise* from the *x* axis. Our standard form has been to quote the angles measured *counterclockwise* from the $+x$ axis, and that angle for this vector is $\theta = 333°$.

Example 3.4 The Resultant Displacement

A particle undergoes three consecutive displacements: $\Delta\vec{\mathbf{r}}_1 = (15\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$ cm, $\Delta\vec{\mathbf{r}}_2 = (23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}})$ cm, and $\Delta\vec{\mathbf{r}}_3 = (-13\hat{\mathbf{i}} + 15\hat{\mathbf{j}})$ cm. Find unit-vector notation for the resultant displacement and its magnitude.

SOLUTION

Conceptualize Although *x* is sufficient to locate a point in one dimension, we need a vector $\vec{\mathbf{r}}$ to locate a point in two or three dimensions. The notation $\Delta\vec{\mathbf{r}}$ is a generalization of the one-dimensional displacement Δx in Equation 2.1. Three-dimensional displacements are more difficult to conceptualize than those in two dimensions because they cannot be drawn on paper like the latter.

For this problem, let us imagine that you start with your pencil at the origin of a piece of graph paper on which you have drawn *x* and *y* axes. Move your pencil 15 cm to the right along the *x* axis, then 30 cm upward along the *y* axis, and then 12 cm *perpendicularly toward you away* from the graph paper. This procedure provides the displacement described by $\Delta\vec{\mathbf{r}}_1$. From this point, move your pencil 23 cm to the right parallel to the *x* axis, then 14 cm parallel to the graph paper in the $-y$ direction, and then 5.0 cm perpendicularly away from you toward the graph paper. You are now at the displacement from the origin described by $\Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2$. From this point, move your pencil 13 cm to the left in the $-x$ direction, and (finally!) 15 cm parallel to the graph paper along the *y* axis. Your final position is at a displacement $\Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_3$ from the origin.

Categorize Despite the difficulty in conceptualizing in three dimensions, we can categorize this problem as a substitution problem because of the careful bookkeeping methods that we have developed for vectors. The mathematical manipulation keeps track of this motion along the three perpendicular axes in an organized, compact way, as we see below.

To find the resultant displacement, add the three vectors:

$$\Delta\vec{\mathbf{r}} = \Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_3$$
$$= (15 + 23 - 13)\hat{\mathbf{i}}\text{ cm} + (30 - 14 + 15)\hat{\mathbf{j}}\text{ cm} + (12 - 5.0 + 0)\hat{\mathbf{k}}\text{ cm}$$
$$= (25\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 7.0\hat{\mathbf{k}})\text{ cm}$$

Find the magnitude of the resultant vector:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$
$$= \sqrt{(25\text{ cm})^2 + (31\text{ cm})^2 + (7.0\text{ cm})^2} = 40\text{ cm}$$

Example 3.5 Taking a Hike

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(A) Determine the components of the hiker's displacement for each day.

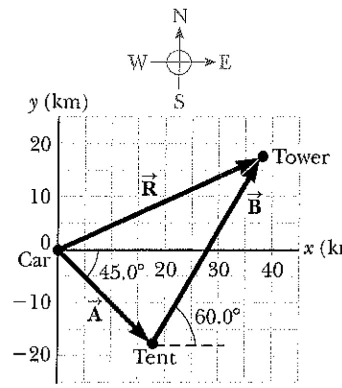


Figure 3.17 (Example 3.5) The total displacement of the hiker is the vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.

SOLUTION

Conceptualize We conceptualize the problem by drawing a sketch as in Figure 3.17. If we denote the displacement vectors on the first and second days by $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, respectively, and use the car as the origin of coordinates, we obtain the vectors shown in Figure 3.17. The sketch allows us to estimate the resultant vector as shown.

Categorize Having drawn the resultant $\vec{\mathbf{R}}$, we can now categorize this problem as one we've solved before: an addition of two vectors. You should now have a hint of the power of categorization in that many new problems are very similar to problems we have already solved if we are careful to conceptualize them. Once we have drawn the displacement vectors and categorized the problem, this problem is no longer about a hiker, a walk, a car, a tent, or a tower. It is a problem about vector addition, one that we have already solved.

Analyze Displacement $\vec{\mathbf{A}}$ has a magnitude of 25.0 km and is directed 45.0° below the positive x axis.

Find the components of $\vec{\mathbf{A}}$ using Equations 3.8 and 3.9:

$$A_x = A\cos(-45.0°) = (25.0 \text{ km})(0.707) = 17.7 \text{ km}$$

$$A_y = A\sin(-45.0°) = (25.0 \text{ km})(-0.707) = -17.7 \text{ km}$$

The negative value of A_y indicates that the hiker ends up below the x axis on the first day. The signs of A_x and A_y also are evident from Figure 3.17.

Find the components of $\vec{\mathbf{B}}$ using Equations 3.8 and 3.9:

$$B_x = B\cos 60.0° = (40.0 \text{ km})(0.500) = 20.0 \text{ km}$$

$$B_y = B\sin 60.0° = (40.0 \text{ km})(0.866) = 34.6 \text{ km}$$

(B) Determine the components of the hiker's resultant displacement $\vec{\mathbf{R}}$ for the trip. Find an expression for $\vec{\mathbf{R}}$ in terms of unit vectors.

SOLUTION

Use Equation 3.14 to find the components of the resultant displacement $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$:

$$R_x = A_x + B_x = 17.7 \text{ km} + 20.0 \text{ km} = 37.7 \text{ km}$$

$$R_y = A_y + B_y = -17.7 \text{ km} + 34.6 \text{ km} = 17.0 \text{ km}$$

Write the total displacement in unit-vector form:

$$\vec{\mathbf{R}} = (37.7\hat{\mathbf{i}} + 17.0\hat{\mathbf{j}}) \text{ km}$$

Finalize Looking at the graphical representation in Figure 3.17, we estimate the position of the tower to be about (38 km, 17 km), which is consistent with the components of $\vec{\mathbf{R}}$ in our result for the final position of the hiker. Also, both components of $\vec{\mathbf{R}}$ are positive, putting the final position in the first quadrant of the coordinate system, which is also consistent with Figure 3.17.

WHAT IF? After reaching the tower, the hiker wishes to return to her car along a single straight line. What are the components of the vector representing this hike? What should the direction of the hike be?

Answer The desired vector $\vec{\mathbf{R}}_{car}$ is the negative of vector $\vec{\mathbf{R}}$:

$$\vec{\mathbf{R}}_{car} = -\vec{\mathbf{R}} = (-37.7\hat{\mathbf{i}} - 17.0\hat{\mathbf{j}}) \text{ km}$$

The direction is found by calculating the angle that the vector makes with the x axis:

$$\tan\theta = \frac{R_{car,y}}{R_{car,x}} = \frac{-17.0 \text{ km}}{-37.7 \text{ km}} = 0.450$$

which gives an angle of $\theta = 204.2°$, or 24.2° south of west.

Summary

Definitions

Scalar quantities are those that have only a numerical value and no associated direction.

Vector quantities have both magnitude and direction and obey the laws of vector addition. The magnitude of a vector is *always* a positive number.

Concepts and Principles

When two or more vectors are added together, they must all have the same units and they all must be the same type of quantity. We can add two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ graphically. In this method (Fig. 3.6), the resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ runs from the tail of $\vec{\mathbf{A}}$ to the tip of $\vec{\mathbf{B}}$.

A second method of adding vectors involves **components** of the vectors. The x component A_x of the vector $\vec{\mathbf{A}}$ is equal to the projection of $\vec{\mathbf{A}}$ along the x axis of a coordinate system, where $A_x = A \cos \theta$. The y component A_y of $\vec{\mathbf{A}}$ is the projection of $\vec{\mathbf{A}}$ along the y axis, where $A_y = A \sin \theta$.

If a vector $\vec{\mathbf{A}}$ has an x component A_x and a y component A_y, the vector can be expressed in unit-vector form as $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$. In this notation, $\hat{\mathbf{i}}$ is a unit vector pointing in the positive x direction and $\hat{\mathbf{j}}$ is a unit vector pointing in the positive y direction. Because $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors, $|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = 1$.

We can find the resultant of two or more vectors by resolving all vectors into their x and y components, adding their resultant x and y components, and then using the Pythagorean theorem to find the magnitude of the resultant vector. We can find the angle that the resultant vector makes with respect to the x axis by using a suitable trigonometric function.

Think–Pair–Share

See the Preface for an explanation of the icons used in this problems set. For additional assessment items for this section, go to WEBASSIGN From Cengage

1. You are working at a radar station for the Coast Guard. While everyone else is out to lunch, you hear a distress call from a sinking ship. The ship is located at a distance of 51.2 km from the station, at a bearing of 36° west of north. On your radar screen, you see the locations of four other ships as follows:

Ship #	Distance from Station (km)	Bearing	Maximum Speed (km/h)
1	36.1	42° W of N	30.0
2	37.3	61° W of N	38.0
3	10.2	36° W of N	32.0
4	51.2	79° W of N	45.0

Quick! Which ship do you contact to help the sinking ship? Which ship will get there in the shortest time interval? Assume that each ship would accelerate quickly to its maximum speed and then maintain that constant speed in a straight line for the entire trip to the sinking ship.

2. ACTIVITY On a paper map of the United States, locate Memphis, Albuquerque, and Chicago. Draw a vector from Albuquerque to Memphis and another vector from Memphis to Chicago. Using the scale on the map, determine the straight-line distances between Albuquerque and Memphis, and between Memphis and Chicago. Use a protractor to measure the angles of your two vectors with respect to latitude and longitude lines. From this information, determine the straight-line distance in miles between Albuquerque and Chicago.

Problems

See the Preface for an explanation of the icons used in this problems set. For additional assessment items for this section, go to WEBASSIGN From Cengage

SECTION 3.1 Coordinate Systems

1. Two points in the xy plane have Cartesian coordinates (2.00, −4.00) m and (−3.00, 3.00) m. Determine (a) the distance between these points and (b) their polar coordinates.

2. Two points in a plane have polar coordinates (2.50 m, 30.0°) and (3.80 m, 120.0°). Determine (a) the Cartesian coordinates of these points and (b) the distance between them.

3. The polar coordinates of a certain point are (r = 4.30 cm, θ = 214°). (a) Find its Cartesian coordinates x and y. Find the polar coordinates of the points with Cartesian coordinates (b) $(-x, y)$, (c) $(-2x, -2y)$, and (d) $(3x, -3y)$.

4. S Let the polar coordinates of the point (x, y) be (r, θ). Determine the polar coordinates for the points (a) $(-x, y)$, (b) $(-2x, -2y)$, and (c) $(3x, -3y)$.

SECTION 3.2 Vector and Scalar Quantities

5. *Why is the following situation impossible?* A skater glides along a circular path. She defines a certain point on the circle as her origin. Later on, she passes through a point at which the distance she has traveled along the path from the origin is smaller than the magnitude of her displacement vector from the origin.

SECTION 3.3 Basic Vector Arithmetic

6. Vector $\vec{\mathbf{A}}$ has a magnitude of 29 units and points in the positive y direction. When vector $\vec{\mathbf{B}}$ is added to $\vec{\mathbf{A}}$, the resultant vector $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ points in the negative y direction with a magnitude of 14 units. Find the magnitude and direction of $\vec{\mathbf{B}}$.

7. A force $\vec{\mathbf{F}}_1$ of magnitude 6.00 units acts on an object at the origin in a direction $\theta = 30.0°$ above the positive x axis (Fig. P3.7). A second force $\vec{\mathbf{F}}_2$ of magnitude 5.00 units acts on the object in the direction of the positive y axis. Find graphically the magnitude and direction of the resultant force $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$.

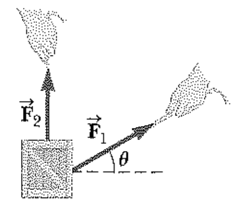


Figure P3.7

8. QIC Three displacements are $\vec{\mathbf{A}} = 200$ m due south, $\vec{\mathbf{B}} = 250$ m due west, and $\vec{\mathbf{C}} = 150$ m at 30.0° east of north. (a) Construct a separate diagram for each of the following possible ways of adding these vectors: $\vec{\mathbf{R}}_1 = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$; $\vec{\mathbf{R}}_2 = \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{A}}$; $\vec{\mathbf{R}}_3 = \vec{\mathbf{C}} + \vec{\mathbf{B}} + \vec{\mathbf{A}}$. (b) Explain what you can conclude from comparing the diagrams.

9. T The displacement vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ shown in Figure P3.9 both have magnitudes of 3.00 m. The direction of vector $\vec{\mathbf{A}}$ is $\theta = 30.0°$. Find graphically (a) $\vec{\mathbf{A}} + \vec{\mathbf{B}}$, (b) $\vec{\mathbf{A}} - \vec{\mathbf{B}}$, (c) $\vec{\mathbf{B}} - \vec{\mathbf{A}}$, and (d) $\vec{\mathbf{A}} - 2\vec{\mathbf{B}}$. (Report all angles counterclockwise from the positive x axis.)

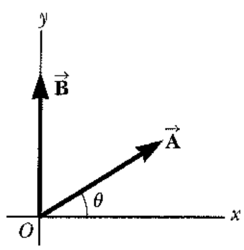


Figure P3.9
Problems 9 and 25.

10. A roller-coaster car moves 200 ft horizontally and then rises 135 ft at an angle of 30.0° above the horizontal. It next travels 135 ft at an angle of 40.0° downward. What is its displacement from its starting point? Use graphical techniques.

SECTION 3.4 Components of a Vector and Unit Vectors

11. QIC A minivan travels straight north in the right lane of a divided highway at 28.0 m/s. A camper passes the minivan and then changes from the left lane into the right lane. As it does so, the camper's path on the road is a straight displacement at 8.50° east of north. To avoid cutting off the minivan, the north–south distance between the camper's back bumper and the minivan's front bumper should not decrease. (a) Can the camper be driven to satisfy this requirement? (b) Explain your answer.

12. A person walks 25.0° north of east for 3.10 km. How far would she have to walk due north and due east to arrive at the same location?

13. T Your dog is running around the grass in your back yard. He undergoes successive displacements 3.50 m south, 8.20 m northeast, and 15.0 m west. What is the resultant displacement?

14. V Given the vectors $\vec{\mathbf{A}} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$, (a) draw the vector sum $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ and the vector difference $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$. (b) Calculate $\vec{\mathbf{C}}$ and $\vec{\mathbf{D}}$, in terms of unit vectors. (c) Calculate $\vec{\mathbf{C}}$ and $\vec{\mathbf{D}}$ in terms of polar coordinates, with angles measured with respect to the positive x axis.

15. V The helicopter view in Fig. P3.15 shows two people pulling on a stubborn mule. The person on the right pulls with a force $\vec{\mathbf{F}}_1$ of magnitude 120 N and direction of $\theta_1 = 60.0°$. The person on the left pulls with a force $\vec{\mathbf{F}}_2$ of magnitude 80.0 N and direction of $\theta_2 = 75.0°$. Find (a) the single force that is equivalent to the two forces shown and (b) the force that a third person would have to exert on the mule to make the resultant force equal to zero. The forces are measured in units of newtons (symbolized N).

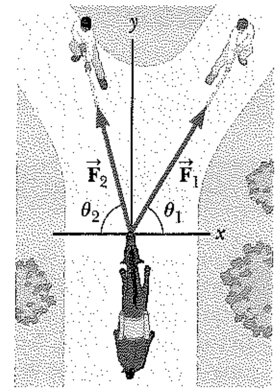


Figure P3.15

16. A snow-covered ski slope makes an angle of 35.0° with the horizontal. When a ski jumper plummets onto the hill, a parcel of splashed snow is thrown up to a maximum displacement of 1.50 m at 16.0° from the vertical in the uphill direction as shown in Figure P3.16. Find the components of its maximum displacement (a) parallel to the surface and (b) perpendicular to the surface.

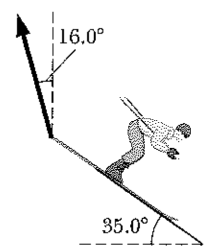


Figure P3.16

17. V Consider the three displacement vectors $\vec{\mathbf{A}} = (3\hat{\mathbf{i}} - 3\hat{\mathbf{j}})$ m, $\vec{\mathbf{B}} = (\hat{\mathbf{i}} - 4\hat{\mathbf{j}})$ m, and $\vec{\mathbf{C}} = (-2\hat{\mathbf{i}} + 5\hat{\mathbf{j}})$ m. Use the component method to determine (a) the magnitude and direction of $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$ and (b) the magnitude and direction of $\vec{\mathbf{E}} = -\vec{\mathbf{A}} - \vec{\mathbf{B}} + \vec{\mathbf{C}}$.

18. V Vector $\vec{\mathbf{A}}$ has x and y components of -8.70 cm and 15.0 cm, respectively; vector $\vec{\mathbf{B}}$ has x and y components of 13.2 cm and -6.60 cm, respectively. If $\vec{\mathbf{A}} - \vec{\mathbf{B}} + 3\vec{\mathbf{C}} = 0$, what are the components of $\vec{\mathbf{C}}$?

19. T The vector $\vec{\mathbf{A}}$ has x, y, and z components of 8.00, 12.0, and -4.00 units, respectively. (a) Write a vector expression for $\vec{\mathbf{A}}$ in unit-vector notation. (b) Obtain a unit-vector expression for a vector $\vec{\mathbf{B}}$ one-fourth the length of $\vec{\mathbf{A}}$ pointing in the same direction as $\vec{\mathbf{A}}$. (c) Obtain a unit-vector expression for a vector $\vec{\mathbf{C}}$ three times the length of $\vec{\mathbf{A}}$ pointing in the direction opposite the direction of $\vec{\mathbf{A}}$.

20. V Given the displacement vectors $\vec{\mathbf{A}} = (3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ m and $\vec{\mathbf{B}} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 7\hat{\mathbf{k}})$ m, find the magnitudes of the following vectors and express each in terms of its rectangular components. (a) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ (b) $\vec{\mathbf{D}} = 2\vec{\mathbf{A}} - \vec{\mathbf{B}}$

21. T Vector $\vec{\mathbf{A}}$ has a negative x component 3.00 units in length and a positive y component 2.00 units in length. (a) Determine an expression for $\vec{\mathbf{A}}$ in unit-vector notation. (b) Determine the magnitude and direction of $\vec{\mathbf{A}}$. (c) What vector $\vec{\mathbf{B}}$ when added to $\vec{\mathbf{A}}$ gives a resultant vector with no x component and a negative y component 4.00 units in length?

22. Three displacement vectors of a croquet ball are shown in Figure P3.22, where $|\vec{\mathbf{A}}| = 20.0$ units, $|\vec{\mathbf{B}}| = 40.0$ units, and $|\vec{\mathbf{C}}| = 30.0$ units. Find (a) the resultant in unit-vector notation and (b) the magnitude and direction of the resultant displacement.

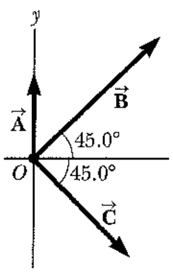


Figure P3.22

23. QIC (a) Taking $\vec{\mathbf{A}} = (6.00\,\hat{\mathbf{i}} - 8.00\,\hat{\mathbf{j}})$ units, $\vec{\mathbf{B}} = (-8.00\,\hat{\mathbf{i}} + 3.00\,\hat{\mathbf{j}})$ units, and $\vec{\mathbf{C}} = (26.0\,\hat{\mathbf{i}} + 19.0\,\hat{\mathbf{j}})$ units, determine a and b such that $a\vec{\mathbf{A}} + b\vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$. (b) A student has learned that a single equation cannot be solved to determine values for more than one unknown in it. How would you explain to him that both a and b can be determined from the single equation used in part (a)?

24. Vector $\vec{\mathbf{B}}$ has x, y, and z components of 4.00, 6.00, and 3.00 units, respectively. Calculate (a) the magnitude of $\vec{\mathbf{B}}$ and (b) the angle that $\vec{\mathbf{B}}$ makes with each coordinate axis.

25. Use the component method to add the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ shown in Figure P3.9. Both vectors have magnitudes of 3.00 m and vector $\vec{\mathbf{A}}$ makes an angle of $\theta = 30.0°$ with the x axis. Express the resultant $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ in unit-vector notation.

26. A girl delivering newspapers covers her route by traveling 3.00 blocks west, 4.00 blocks north, and then 6.00 blocks east. (a) What is her resultant displacement? (b) What is the total distance she travels?

27. T A man pushing a mop across a floor causes it to undergo two displacements. The first has a magnitude of 150 cm and makes an angle of 120° with the positive x axis. The resultant displacement has a magnitude of 140 cm and is directed at an angle of 35.0° to the positive x axis. Find the magnitude and direction of the second displacement.

28. BIO Figure P3.28 illustrates typical proportions of male (m) and female (f) anatomies. The displacements $\vec{\mathbf{d}}_{1m}$ and $\vec{\mathbf{d}}_{1f}$ from the soles of the feet to the navel have magnitudes of 104 cm and 84.0 cm, respectively. The displacements $\vec{\mathbf{d}}_{2m}$ and $\vec{\mathbf{d}}_{2f}$ from the navel to outstretched fingertips have magnitudes of 100 cm and 86.0 cm, respectively. Find the vector sum of these displacements $\vec{\mathbf{d}}_3 = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2$ for both people.

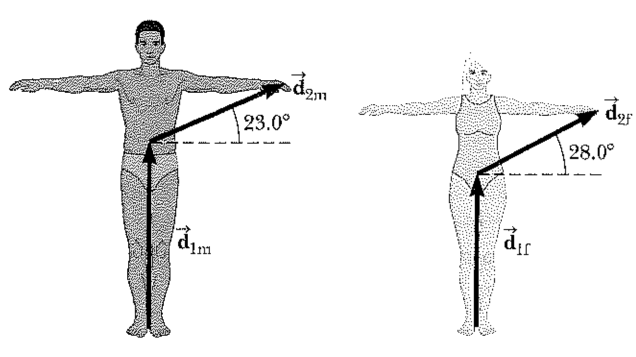


Figure P3.28

29. AMT GP **Review.** As it passes over Grand Bahama Island, the eye of a hurricane is moving in a direction 60.0° north of west with a speed of 41.0 km/h. (a) What is the unit-vector expression for the velocity of the hurricane? It maintains this velocity for 3.00 h, at which time the course of the hurricane suddenly shifts due north, and its speed slows to a constant 25.0 km/h. This new velocity is maintained for 1.50 h. (b) What is the unit-vector expression for the new velocity of the hurricane? (c) What is the unit-vector expression for the displacement of the hurricane during the first 3.00 h? (d) What is the unit-vector expression for the displacement of the hurricane during the latter 1.50 h? (e) How far from Grand Bahama is the eye 4.50 h after it passes over the island?

30. In an assembly operation illustrated in Figure P3.30, a robot moves an object first straight upward and then also to the east, around an arc forming one-quarter of a circle of radius 4.80 cm that lies in an east–west vertical plane. The robot then moves the object upward and to the north, through one-quarter of a circle of radius 3.70 cm that lies in a north–south vertical plane. Find (a) the magnitude of the total displacement of the object and (b) the angle the total displacement makes with the vertical.

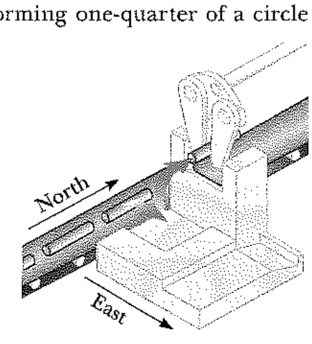


Figure P3.30

31. AMT **Review.** You are standing on the ground at the origin of a coordinate system. An airplane flies over you with constant velocity parallel to the x axis and at a fixed height of 7.60×10^3 m. At time $t = 0$, the airplane is directly above you so that the vector leading from you to it is $\vec{\mathbf{P}}_0 = 7.60 \times 10^3\,\hat{\mathbf{j}}$ m. At $t = 30.0$ s, the position vector leading from you to the airplane is $\vec{\mathbf{P}}_{30} = (8.04 \times 10^3\,\hat{\mathbf{i}} + 7.60 \times 10^3\,\hat{\mathbf{j}})$ m as suggested in Figure P3.31. Determine the magnitude and orientation of the airplane's position vector at $t = 45.0$ s.

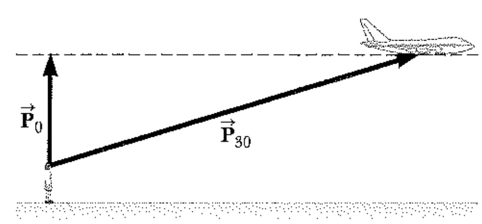


Figure P3.31

32. *Why is the following situation impossible?* A shopper pushing a cart through a market follows directions to the canned goods and moves through a displacement $8.00\,\hat{\mathbf{i}}$ m down one aisle. He then makes a 90.0° turn and moves 3.00 m along the y axis. He then makes another 90.0° turn and moves 4.00 m along the x axis. *Every* shopper who follows these directions correctly ends up 5.00 m from the starting point.

33. In Figure P3.33, the line segment represents a path from the point with position vector $(5\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ m to the point with location $(16\hat{\mathbf{i}} + 12\hat{\mathbf{j}})$ m. Point Ⓐ is along this path, a fraction f of the way to the destination. (a) Find the position vector of point Ⓐ in terms of f. (b) Evaluate the expression from part (a) for $f = 0$. (c) Explain whether the result in part (b) is reasonable. (d) Evaluate the expression for $f = 1$. (e) Explain whether the result in part (d) is reasonable.

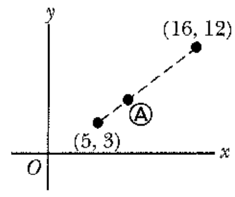


Figure P3.33 Point Ⓐ is a fraction f of the distance from the initial point (5, 3) to the final point (16, 12).

ADDITIONAL PROBLEMS

34. You are spending the summer as an assistant learning how to navigate on a large ship carrying freight across Lake Erie. One day, you and your ship are to travel across the lake a distance of 200 km traveling due north from your origin port to your destination port. Just as you leave your origin port, the navigation electronics go down. The captain continues sailing, claiming he can depend on his years of experience on the water as a guide. The engineers work on the navigation system while the ship continues to sail, and winds and waves push it off course. Eventually, enough of the navigation system comes back up to tell you your location. The system tells you that your current position is 50.0 km north of the origin port and 25.0 km east of the port. The captain is a little embarrassed that his ship is so far off course and barks an order to you to tell him immediately what heading he should set from your current position to the destination port. Give him an appropriate heading angle.

35. A person going for a walk follows the path shown in Figure P3.35. The total trip consists of four straight-line paths. At the end of the walk, what is the person's resultant displacement measured from the starting point?

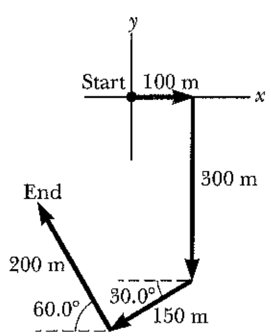


Figure P3.35

36. A ferry transports tourists between three islands. It sails from the first island to the second island, 4.76 km away, in a direction 37.0° north of east. It then sails from the second island to the third island in a direction 69.0° west of north. Finally it returns to the first island, sailing in a direction 28.0° east of south. Calculate the distance between (a) the second and third islands and (b) the first and third islands.

37. Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have precisely equal magnitudes. For the magnitude of $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ to be 100 times larger than the magnitude of $\vec{\mathbf{A}} - \vec{\mathbf{B}}$, what must be the angle between them?

38. Two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have precisely equal magnitudes. For the magnitude of $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ to be larger than the magnitude of $\vec{\mathbf{A}} - \vec{\mathbf{B}}$ by the factor n, what must be the angle between them?

39. **Review.** The biggest stuffed animal in the world is a snake 420 m long, constructed by Norwegian children. Suppose the snake is laid out in a park as shown in Figure P3.39, forming two straight sides of a 105° angle, with one side 240 m long. Olaf and Inge run a race they invent. Inge runs directly from the tail of the snake to its head, and Olaf starts from the same place at the same moment but runs along the snake. (a) If both children run steadily at 12.0 km/h, Inge reaches the head of the snake how much earlier than Olaf? (b) If Inge runs the race again at a constant speed of 12.0 km/h, at what constant speed must Olaf run to reach the end of the snake at the same time as Inge?

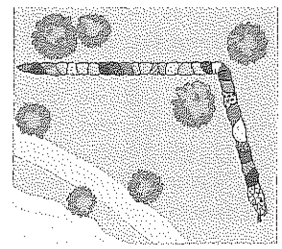


Figure P3.39

40. Ecotourists use their global positioning system indicator to determine their location inside a botanical garden as latitude 0.002 43 degree south of the equator, longitude 75.642 38 degrees west. They wish to visit a tree at latitude 0.001 62 degree north, longitude 75.644 26 degrees west. (a) Determine the straight-line distance and the direction in which they can walk to reach the tree as follows. First model the Earth as a sphere of radius 6.37×10^6 m to determine the westward and northward displacement components required, in meters. Then model the Earth as a flat surface to complete the calculation. (b) Explain why it is possible to use these two geometrical models together to solve the problem.

41. A vector is given by $\vec{\mathbf{R}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$. Find (a) the magnitudes of the x, y, and z components; (b) the magnitude of $\vec{\mathbf{R}}$; and (c) the angles between $\vec{\mathbf{R}}$ and the x, y, and z axes.

42. You are working as an assistant to an air-traffic controller at the local airport, from which small airplanes take off and land. Your job is to make sure that airplanes are not closer to each other than a minimum safe separation distance of 2.00 km. You observe two small aircraft on your radar screen, out over the ocean surface. The first is at altitude 800 m above the surface, horizontal distance 19.2 km, and 25.0° south of west. The second aircraft is at altitude 1 100 m, horizontal distance 17.6 km, and 20.0° south of west. Your supervisor is concerned that the two aircraft are too close together and asks for a separation distance for the two airplanes. (Place the x axis west, the y axis south, and the z axis vertical.)

43. **Review.** The instantaneous position of an object is specified by its position vector leading from a fixed origin to the location of the object, modeled as a particle. Suppose for a certain object the position vector is a function of time given by $\vec{\mathbf{r}} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2t\ \hat{\mathbf{k}}$, where