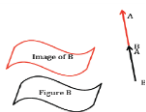


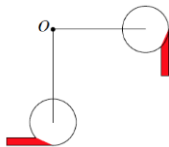
### The Concept of Congruence

In this 16-lesson module, students learn about translations, reflections, and rotations in the plane and how to use them to precisely define the concept of *congruence*. Up to this point, “congruence” has been taken to mean, intuitively, “same size and same shape.” Because this module begins a serious study of geometry, this intuitive definition must be replaced by a precise definition. This module is a first step; its goal is to provide the needed intuitive background for the precise definitions that are introduced in this module for the first time. Students are also introduced to the Pythagorean Theorem.

#### Translation

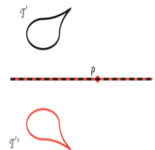


#### Rotation



### Transformations in this module

#### Reflection



### Key Words

#### Transformation

A rule, to be denoted by  $T$ , that assigns each point  $P$  of the plane a unique point, which is denoted by  $T(P)$ .

#### Basic Rigid Motion

A basic rigid motion is a rotation, reflection, or translation of the plane.

#### Translation

A basic rigid motion that moves a figure along a given vector.

#### Rotation

A basic rigid motion that moves a figure around a point,  $d$  degrees.

#### Reflection

A basic rigid motion that moves a figure across a line.

#### Image of a point, image of a figure

Image refers to the location of a point, or figure, after it has been transformed.

#### Sequence (Composition) of Transformations

More than one transformation.

#### Vector

→

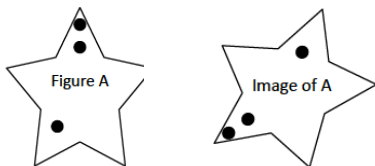
A Euclidean vector (or directed segment)  $AB$  is the line segment  $AB$  together with a direction given by connecting an initial point  $A$  to a terminal point  $B$ .

#### Congruence

A congruence is a sequence of basic rigid motions (rotations, reflections, translations) of the plane.

**Transversal** (Given a pair of lines  $L$  and  $M$  in a plane, a third line  $T$  is a transversal if it intersects  $L$  at a single point and intersects  $M$  at a single but different point.

Describe, intuitively, what kind of transformation will be required to move Figure A on the left to its image on the right.



**What Came Before this Module:** Students used their knowledge of operations on numbers to include integer exponents and transformed expressions. Students made conjectures about how zero and negative exponents of a number should be defined and proved the properties of integer exponents.

**What Comes After this Module:** In Module 3, students learn about dilation and similarity and apply that knowledge to a proof of the Pythagorean Theorem based on the Angle-Angle criterion for similar triangles. Students learn the definition of a dilation, its properties, and how to compose them. One overarching goal of this module is to replace the common idea of “same shape, different sizes” with a definition of similarity that can be applied to shapes that are not polygons, such as ellipses and circles.

### How can you help at home?

- ✓ Every day, ask your child what they learned in school and ask them to show you an example.
- ✓ Be excited to learn new ideas in math! Show your child that math is fun, even when it might be new and challenging!
- ✓ Ask your child to describe a translation; reflection and rotation in his/her own words.
- ✓ Watch the video posted below with your child and discuss each of the transformations.

<http://youtu.be/O2XPy3ZLU7Y>

### Key Common Core Standards:

**Understand congruence and similarity using physical models, transparencies, or geometry software.**

- Verify experimentally the properties of rotations, reflections, and translations:
  - a. Lines are taken to lines, and line segments to line segments of the same length.
  - b. Angles are taken to angles of the same measure.
  - c. Parallel lines are taken to parallel lines.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

**Understand and apply the Pythagorean Theorem.**

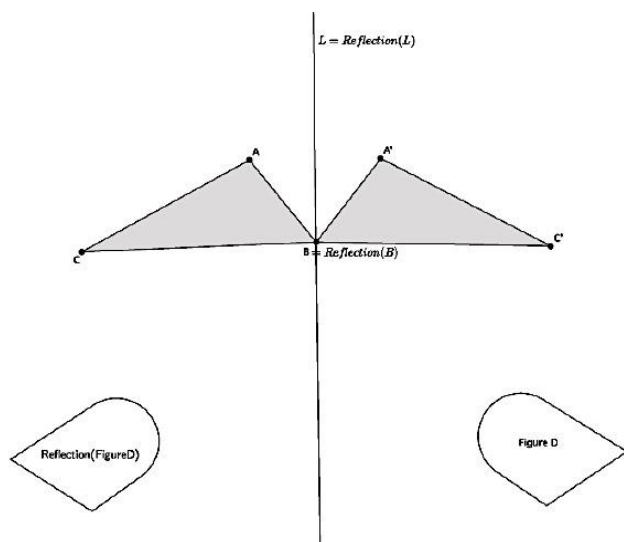
- Explain a proof of the Pythagorean Theorem and its converse.
- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

## Reflections

Below are two examples of the type of problems involving *reflections* your child will see in this unit.

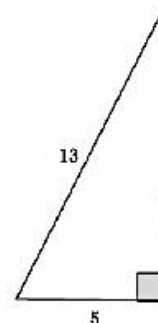
$\triangle ABC$  was reflected across line  $L$ . Notice how Point  $B$  and line  $L$  did not move in this transformation. Also, notice how the image was labeled and how this is different from the original figure.

Figure  $D$  was also reflected across line  $L$  and is shown below.



## Pythagorean Theorem

Given a right triangle with a hypotenuse with length 13 units and a leg with length 5 units, as shown, determine the length of the other leg.



Solution:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + b^2 &= 13^2 \\ 5^2 - 5^2 + b^2 &= 13^2 - 5^2 \\ b^2 &= 13^2 - 5^2 \\ b^2 &= 169 - 25 \\ b^2 &= 144 \\ b &= 12 \end{aligned}$$

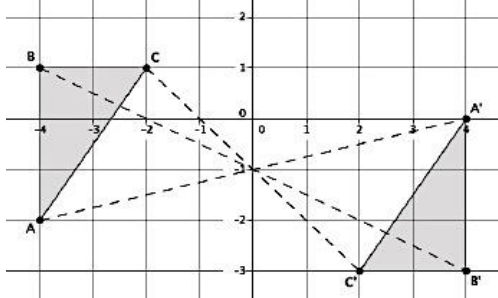
The length of the leg is 12 units.

*A positive attitude towards math is very important in helping your child succeed in school. In the ever-changing world we live in, a strong foundation in math, paired with excellent problem solving skills may open many doors for your child in the years to come!*

Below is a problem taken from this module, which shows a *rotation* (lesson 6).

**Use the following diagram for problems 1–5. Use your transparency, as needed.**

The solution for each problem is also shown below.



4. Connect point  $B$  to point  $B'$ , point  $C$  to point  $C'$ , and point  $A$  to point  $A'$ . What do you notice? What do you think that point is?

*All of the lines intersect at one point. The point is the center of rotation, I checked by using my transparency.*

5. Would a rotation map triangle  $ABC$  onto triangle  $A'B'C'$ ? If so, define the rotation (i.e., degree and center). If not, explain why not.

*Let there be a rotation  $180^\circ$  around point  $(0, -1)$ . Then  $\text{Rotation}(\triangle ABC) = \triangle A'B'C'$ .*

1. Looking only at segment  $BC$ , is it possible that a  $180^\circ$  rotation would map  $BC$  onto  $B'C'$ ? Why or why not?

*It is possible because the segments are parallel.*

2. Looking only at segment  $AB$ , is it possible that a  $180^\circ$  rotation would map  $AB$  onto  $A'B'$ ? Why or why not?

*It is possible because the segments are parallel.*

3. Looking only at segment  $AC$ , is it possible that a  $180^\circ$  rotation would map  $AC$  onto  $A'C'$ ? Why or why not?

*It is possible because the segments are parallel.*