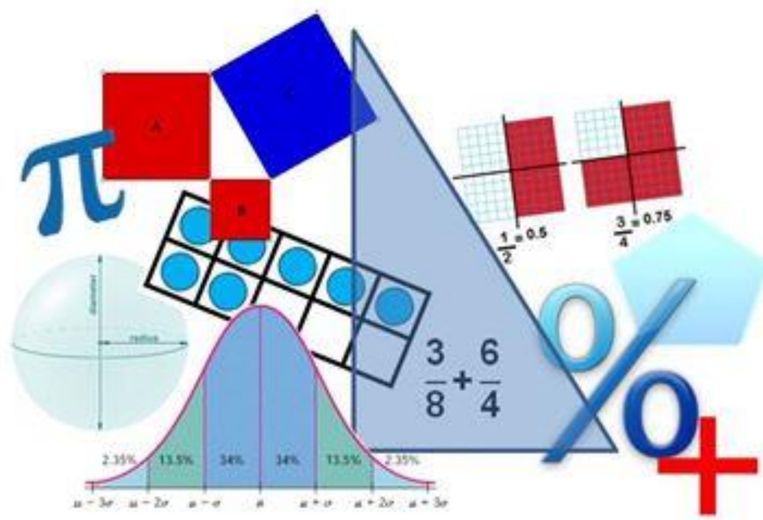
Mathematics

2016 Standards of Learning

Grade 7

Curriculum Framework



Board of Education
Commonwealth of Virginia

Superintendent of Public Instruction
Steven R. Staples

Assistant Superintendent Chief Academic Officer/Assistant Superintendent for Instruction
Steven M. Constantino

Office of Mathematics and Governor's Schools
Debra Delozier, Mathematics Specialist
Tina Mazzacane, Mathematics and Science Specialist
Christa Southall, Mathematics Specialist

Acknowledgements
The Virginia Department of Education wishes to express sincere thanks to Michael Bolling, who assisted in the development of the 2016 *Mathematics Standards of Learning* and 2016 *Mathematics Standards of Learning Curriculum Framework*.

NOTICE
The Virginia Department of Education does not unlawfully discriminate on the basis of race, color, sex, national origin, age, or disability in employment or in its educational programs or services.

Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*

Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for *Standards of Learning* assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the *Standards of Learning*. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and to see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "… the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> "Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
>
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that requires students to think critically, to reason, to develop problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Mathematics instruction in grades six through eight continues to focus on the development of number sense, with emphasis on rational and real numbers. Rational numbers play a critical role in the development of proportional reasoning and advanced mathematical thinking. The study of rational numbers builds on the understanding of whole numbers, fractions, and decimals developed by students in the elementary grades. Proportional reasoning is the key to making connections to many middle school mathematics topics.

Students develop an understanding of integers and rational numbers using concrete, pictorial, and abstract representations. They learn how to use equivalent representations of fractions, decimals, and percents and recognize the advantages and disadvantages of each type of representation.
Flexible thinking about rational number representations is encouraged when students solve problems.

Students develop an understanding of real numbers and the properties of operations on real numbers through experiences with rational and irrational numbers and apply the order of operations.

Students use a variety of concrete, pictorial, and abstract representations to develop proportional reasoning skills. Ratios and proportions are a major focus of mathematics learning in the middle grades.

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) compare and order numbers greater than zero written in scientific notation;*
c) compare and order rational numbers;*
d) determine square roots of perfect squares;* and
e) identify and describe absolute value of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

<table>
<tr><th>Understanding the Standard</th><th>Essential Knowledge and Skills</th></tr>
<tr><td>

- Negative exponents for powers of 10 are used to represent numbers between 0 and 1. (e.g., $10^{-3} = \frac{1}{10^3} = 0.001$).
- Negative exponents for powers of 10 can be investigated through patterns such as:
$$10^2 = 100$$
$$10^1 = 10$$
$$10^0 = 1$$
$$10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$$
$$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$$
- Percent means “per 100” or how many “out of 100”; percent is another name for hundredths.
- A percent is a ratio in which the denominator is 100. A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., $\frac{3}{5} = \frac{60}{100} = 0.60 = 60\%$).
- Scientific notation should be used whenever the situation calls for use of very large or very small numbers.
- A number written in scientific notation is the product of two factors — a decimal greater than or equal to 1 but less than 10, and a power of 10 (e.g., $3.1 \times 10^5 = 310{,}000$ and $2.85 \times 10^{-4} = 0.000285$).
- The set of integers includes the set of whole numbers and their opposites, {…-2, -1, 0, 1, 2…}. Zero has no opposite and is neither positive nor negative.
- The opposite of a positive number is negative and the opposite of a negative number is positive.

</td><td>

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Recognize powers of 10 with negative exponents by examining patterns. (a)
- Represent a power of 10 with a negative exponent in fraction and decimal form. (a)
- Convert between numbers greater than 0 written in scientific notation and decimals. (b)
- Compare and order no more than four numbers greater than 0 written in scientific notation. Ordering may be in ascending or descending order. (b)
- Compare and order no more than four rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions and mixed numbers may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place. Ordering may be in ascending or descending order. (c)
- Identify the perfect squares from 0 to 400. (d)
- Determine the positive square root of a perfect square from 0 to 400. (d)
- Demonstrate absolute value using a number line. (e)

</td></tr>
</table>

7.1 The student will
- **a) investigate and describe the concept of negative exponents for powers of ten;**
- **b) compare and order numbers greater than zero written in scientific notation;***
- **c) compare and order rational numbers;***
- **d) determine square roots of perfect squares;* and**
- **e) identify and describe absolute value of rational numbers.**

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are: $\sqrt{25}$, $\frac{1}{4}$, -2.3, 82, 75%, $4.\overline{59}$. • Rational numbers may be expressed as positive and negative fractions or mixed numbers, positive and negative decimals, integers and percents. • Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). Fractions can be positive or negative. • Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, base ten blocks, fraction circles, colored counters, cubes, decimal squares, shaded figures, shaded grids, number lines and calculators). • Negative numbers lie to the left of zero and positive numbers lie to the right of zero on a number line. • Smaller numbers always lie to the left of larger numbers on the number line. • A perfect square is a whole number whose square root is an integer. Zero (a whole number) is a perfect square. (e.g., $36 = 6 \cdot 6 = 6^2$).	• Determine the absolute value of a rational number. (e) • Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle to solve practical problems. (e)

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) compare and order numbers greater than zero written in scientific notation;*
c) compare and order rational numbers;*
d) determine square roots of perfect squares;* and
e) identify and describe absolute value of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• A square root of a number is a number which, when multiplied by itself, produces the given number (e.g., $\sqrt{121}$ is 11 since $11 \cdot 11 = 121$). • The symbol $\sqrt{}$ may be used to represent a non-negative (principal) square root. Students in grade 8 mathematics will explore the negative square root of a number, denoted $-\sqrt{}$. • The square root of a number can be represented geometrically as the length of a side of a square. • Squaring a number and taking a square root are inverse operations. • The absolute value of a number is the distance from 0 on the number line regardless of direction. Distance is positive (e.g., $\left\|-\frac{1}{2}\right\| = \frac{1}{2}$). • The absolute value of zero is zero.	

The computation and estimation strand in grades six through eight focuses on developing conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring an understanding as to why procedures work and make sense.

Students develop and refine estimation strategies based on an understanding of number concepts, properties and relationships. The development of problem solving, using operations with integers and rational numbers, builds upon the strategies developed in the elementary grades. Students will reinforce these skills and build on the development of proportional reasoning and more advanced mathematical skills.

Students learn to make sense of the mathematical tools available by making valid judgments of the reasonableness of answers. Students will balance the ability to make precise calculations through the application of the order of operations with knowing when calculations may require estimation to obtain appropriate solutions to practical problems.

7.2 The student will solve practical problems involving operations with rational numbers.

Understanding the Standard	Essential Knowledge and Skills
• The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are: $\sqrt{25}$, $\frac{1}{4}$, -2.3, 82, 75%, $4.\overline{59}$. • Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). A fraction can have a positive or negative value. • Solving problems in the context of practical situations enhances interconnectedness and proficiency with estimation strategies. Practical problems involving rational numbers in grade seven provide students the opportunity to use problem solving to apply computation skills involving positive and negative rational numbers expressed as integers, fractions, and decimals, along with the use of percents within practical situations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Solve practical problems involving addition, subtraction, multiplication, and division with rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place.

7.3 The student will solve single-step and multistep practical problems, using proportional reasoning.

Understanding the Standard	Essential Knowledge and Skills
• A proportion is a statement of equality between two ratios. A proportion can be written as $\frac{a}{b}=\frac{c}{d}$, *a*:*b* = *c*:*d*, or *a* is to *b* as *c* is to *d*. • Equivalent ratios arise by multiplying each value in a ratio by the same constant value. For example, the ratio of 3:2 would be equivalent to the ratio 6:4 because each of the values in 3:2 can be multiplied by 2 to get 6:4. • A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios. • A proportion can be solved by determining the product of the means and the product of the extremes. For example, in the proportion *a*:*b* = *c*:*d*, *a* and *d* are the extremes and *b* and *c* are the means. If values are substituted for *a*, *b*, *c*, and *d* such as 5:12 = 10:24, then the product of extremes (5 · 24) is equal to the product of the means (12 · 10). • In a proportional relationship, two quantities increase multiplicatively. One quantity is a constant multiple of the other. • A proportion is an equation which states that two ratios are equal. When solving a proportion, the ratios may first be written as fractions. − Example: A recipe for oatmeal cookies calls for 2 cups of flour for every 3 cups of oatmeal. How much flour is needed for a larger batch of cookies that uses 9 cups of oatmeal? To solve this problem, the ratio of flour to oatmeal could be written as a fraction in the proportion used to determine the amount of flour needed when 9 cups of oatmeal is used. To use a proportion to solve for the unknown cups of flour needed, solve the proportion: $\frac{2}{3}=\frac{x}{9}$. To use a table of equivalent ratios to find the unknown amount, create the table: <table><tr><td>flour (cups)</td><td>2</td><td>4</td><td>?</td></tr><tr><td>oatmeal (cups)</td><td>3</td><td>6</td><td>9</td></tr></table>To complete the table, we must create an equivalent ratio to 2:3, Just as 4:6 is equivalent to 2:3, then 6 cups of flour to 9 cups of oatmeal would create an equivalent ratio. • A proportion can be solved by determining equivalent ratios.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a proportional relationship between two quantities, create and use a ratio table to determine missing values. • Write and solve a proportion that represents a proportional relationship between two quantities to find a missing value. • Apply proportional reasoning to convert units of measurement within and between the U.S. Customary System and the metric system when given the conversion factor. • Apply proportional reasoning to solve practical problems, including scale drawings. Scale factors shall have denominators no greater than 12 and decimals no less than tenths. • Using 10% as a benchmark, compute 5%, 10%, 15%, or 20% of a given whole number. • Using 10% as a benchmark, compute 5%, 10%, 15%, or 20% in a practical situation such as tips, tax, and discounts. • Solve problems involving tips, tax, and discounts. Limit problems to only one percent computation per problem.

7.3 The student will solve single-step and multistep practical problems, using proportional reasoning.

Understanding the Standard	Essential Knowledge and Skills
• A rate is a ratio that compares two quantities measured in different units. A unit rate is a rate with a denominator of 1. Examples of rates include miles/hour and revolutions/minute. • Proportions are used in everyday contexts, such as speed, recipe conversions, scale drawings, map reading, reducing and enlarging, comparison shopping, tips, tax, and discounts, and monetary conversions. • A multistep problem is a problem that requires two or more steps to solve. • Proportions can be used to convert length, weight (mass), and volume (capacity) within and between measurement systems. For example, if 1 inch is about 2.54 cm, how many inches are in 16 cm? $\frac{1\ inch}{2.54\ cm} = \frac{x\ inch}{16\ cm}$ $2.54x = 1 \cdot 16$ $2.54x = 16$ $x = \frac{16}{2.54}$ $x = 6.299$ or about 6.3 inches • Examples of conversions may include, but are not limited to: − Length: between feet and miles; miles and kilometers − Weight: between ounces and pounds; pounds and kilograms − Volume: between cups and fluid ounces; gallons and liters • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes depending on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • When converting measurement units in practical situations, the precision of the conversion factor used will be based on the accuracy required within the context of the problem. For example, when converting from miles to kilometers, we may use a conversion factor of 1 mile ≈ 1.6 km or 1 mile ≈ 1.609 km, depending upon the accuracy needed.	

7.3 The student will solve single-step and multistep practical problems, using proportional reasoning.

Understanding the Standard	Essential Knowledge and Skills
• Estimation may be used prior to calculating conversions to evaluate the reasonableness of a solution. • A percent is a ratio in which the denominator is 100. • Proportions can be used to represent percent problems as follows: $\frac{percent}{100} = \frac{part}{whole}$	

Measurement and geometry in the middle grades provide a natural context and connection among many mathematical concepts. Students expand informal experiences with geometry and measurement in the elementary grades and develop a solid foundation for further exploration of these concepts in high school. Spatial reasoning skills are essential to the formal inductive and deductive reasoning skills required in subsequent mathematics learning.

Students develop measurement skills through exploration and estimation. Physical exploration to determine length, weight/mass, liquid volume/capacity, and angle measure are essential to develop a conceptual understanding of measurement. Students examine perimeter, area, and volume, using concrete materials and practical situations. Students focus their study of surface area and volume on rectangular prisms, cylinders, square-based pyramids, and cones.

Students learn geometric relationships by visualizing, comparing, constructing, sketching, measuring, transforming, and classifying geometric figures. A variety of tools such as geoboards, pattern blocks, dot paper, patty paper, and geometry software provide experiences that help students discover geometric concepts. Students describe, classify, and compare plane and solid figures according to their attributes. They develop and extend understanding of geometric transformations in the coordinate plane.

Students apply their understanding of perimeter and area from the elementary grades in order to build conceptual understanding of the surface area and volume of prisms, cylinders, square-based pyramids, and cones. They use visualization, measurement, and proportional reasoning skills to develop an understanding of the effect of scale change on distance, area, and volume. They develop and reinforce proportional reasoning skills through the study of similar figures.

Students explore and develop an understanding of the Pythagorean Theorem. Understanding how the Pythagorean Theorem can be applied in practical situations has a far-reaching impact on subsequent mathematics learning and life experiences.

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

Level 0: Pre-recognition. Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.

Level 1: Visualization. Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one.)

Level 2: Analysis. Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three.)

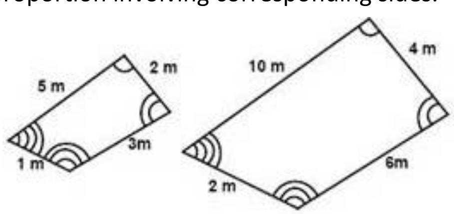
Level 3: Abstraction. Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

Level 4: Deduction. Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students should be able to supply reasons for steps in a proof. (Students should transition to this level before taking geometry.)

7.4 The student will
a) describe and determine the volume and surface area of rectangular prisms and cylinders; and
b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders.

Understanding the Standard	Essential Knowledge and Skills
• A polyhedron is a solid figure whose faces are all polygons. • A rectangular prism is a polyhedron in which all six faces are rectangles. A rectangular prism has eight vertices and 12 edges. • A cylinder is a solid figure formed by two congruent parallel faces called bases joined by a curved surface. In this grade level, cylinders are limited to right circular cylinders. • A face is any flat surface of a solid figure. • The surface area of a prism is the sum of the areas of all 6 faces and is measured in square units. • The volume of a three-dimensional figure is a measure of capacity and is measured in cubic units. • Nets are two-dimensional representations of a three-dimensional figure that can be folded into a model of the three-dimensional figure. • A rectangular prism can be represented on a flat surface as a net that contains six rectangles — two that have measures of the length and width of the base, two others that have measures of the length and height, and two others that have measures of the width and height. The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$). • A cylinder can be represented on a flat surface as a net that contains two circles (the bases of the cylinder) and one rectangular region (the curved surface of the cylinder) whose length is the circumference of the circular base and whose width is the height of the cylinder. The surface area of the cylinder is the sum of the area of the two circles and the rectangle representing the curved surface ($SA = 2\pi r^2 + 2\pi rh$). • The volume of a rectangular prism is computed by multiplying the area of the base, *B*, (length times width) by the height of the prism ($V = lwh = Bh$). • The volume of a cylinder is computed by multiplying the area of the base, *B*, (πr^2) by the height of the cylinder ($V = \pi r^2 h = Bh$). • The calculation of determining surface area and volume may vary depending upon the approximation for pi. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi button on the calculator.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the surface area of rectangular prisms and cylinders using concrete objects, nets, diagrams, and formulas. (a) • Determine the volume of rectangular prisms and cylinders using concrete objects, diagrams, and formulas. (a) • Determine if a practical problem involving a rectangular prism or cylinder represents the application of volume or surface area. (b) • Solve practical problems that require determining the surface area of rectangular prisms and cylinders. (b) • Solve practical problems that require determining the volume of rectangular prisms and cylinders. (b).

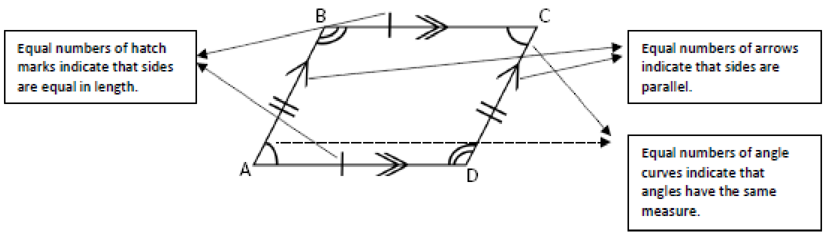
7.5 The student will solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar quadrilaterals and triangles.

Understanding the Standard	Essential Knowledge and Skills
• Similar polygons have corresponding sides that are proportional and corresponding interior angles that are congruent. • Similarity has practical applications in a variety of areas, including art, architecture, and the sciences. • Similarity does not depend on the position or orientation of the figures. • Congruent polygons have the same size and shape. Corresponding angles and sides are congruent. • Congruent polygons are similar polygons for which the ratio of the corresponding sides is 1:1. However, similar polygons are not necessarily congruent. • The symbol ~ is used to represent similarity. For example, $\Delta ABC \sim \Delta DEF$. • The symbol ≅ is used to represent congruence. For example, $\angle A \cong \angle B$ • Similarity statements can be used to determine corresponding parts of similar figures such as: Given: $\Delta ABC \sim \Delta DEF$ $\angle A$ corresponds to $\angle D$ $\overline{AB}$ corresponds to $\overline{DE}$ • A proportion representing corresponding sides of similar figures can be written as $\frac{a}{b} = \frac{c}{d}$. − Example: Given two similar quadrilaterals with corresponding angles labeled, write a proportion involving corresponding sides.  $\frac{5}{10} = \frac{2}{4}$ or $\frac{5}{10} = \frac{3}{6}$ or $\frac{1}{2} = \frac{2}{4}$ are some of the ways to express the proportional relationships that exist.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify corresponding sides and corresponding congruent angles of similar quadrilaterals and triangles. • Given two similar quadrilaterals or triangles, write similarity statements using symbols. • Write proportions to express the relationships between the lengths of corresponding sides of similar quadrilaterals and triangles. • Solve a proportion to determine a missing side length of similar quadrilaterals or triangles. • Given angle measures in a quadrilateral or triangle, determine unknown angle measures in a similar quadrilateral or triangle.

7.5 The student will solve problems, including practical problems, involving the relationship between corresponding sides and corresponding angles of similar quadrilaterals and triangles.

<table>
<tr><th>Understanding the Standard</th><th>Essential Knowledge and Skills</th></tr>
<tr><td>

- The traditional notation for marking congruent angles is to use a curve on each angle. Denote which angles are congruent with the same number of curved lines. For example, if $\angle A$ is congruent to $\angle C$, then both angles will be marked with the same number of curved lines.



- Congruent sides are denoted with the same number of hatch (or hash) marks on each congruent side. For example, a side on a polygon with 2 hatch marks is congruent to the side with 2 hatch marks on a congruent polygon or within the same polygon.

</td><td></td></tr>
</table>

7.6 The student will

a) compare and contrast quadrilaterals based on their properties; and

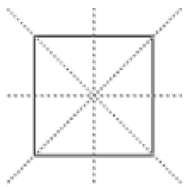
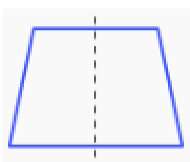
b) determine unknown side lengths or angle measures of quadrilaterals.

Understanding the Standard	Essential Knowledge and Skills
• A polygon is a closed plane figure composed of at least three line segments that do not cross. • A quadrilateral is a polygon with four sides. • Properties of quadrilaterals include: number of parallel sides, angle measures, number of congruent sides, lines of symmetry, and the relationship between the diagonals. • A diagonal is a segment in a polygon that connects two vertices but is not a side. • To bisect means to divide into two equal parts. • A line of symmetry divides a figure into two congruent parts each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines. • A parallelogram is a quadrilateral with both pairs of opposite sides parallel. Properties of a parallelogram include the following: − opposite sides are parallel and congruent; − opposite angles are congruent; and − diagonals bisect each other and one diagonal divides the figure into two congruent triangles. • Parallelograms, with the exception of rectangles and rhombi, have no lines of symmetry. A rectangle and a rhombus have two lines of symmetry, with the exception of a square which has four lines of symmetry. • A rectangle is a quadrilateral with four right angles. Properties of a rectangle include the following: − opposite sides are parallel and congruent; − all four angles are congruent and each angle measures 90°; and − diagonals are congruent and bisect each other. • A square is a quadrilateral that is a regular polygon with four congruent sides and four right angles. Properties of a square include the following: − opposite sides are congruent and parallel; − all four angles are congruent and each angle measures 90°; and − diagonals are congruent and bisect each other at right angles.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast properties of the following quadrilaterals: parallelogram, rectangle, square, rhombus, and trapezoid. (a) • Sort and classify quadrilaterals, as parallelograms, rectangles, trapezoids, rhombi, and/or squares based on their properties. (a) • Given a diagram, determine an unknown angle measure in a quadrilateral, using properties of quadrilaterals. (b) • Given a diagram determine an unknown side length in a quadrilateral using properties of quadrilaterals. (b)

7.6 The student will

a) compare and contrast quadrilaterals based on their properties; and

b) determine unknown side lengths or angle measures of quadrilaterals.

Understanding the Standard	Essential Knowledge and Skills
• A rhombus is a quadrilateral with four congruent sides. Properties of a rhombus include the following: − all sides are congruent; − opposite sides are parallel; − opposite angles are congruent; and − diagonals bisect each other at right angles. • A square has four lines of symmetry. The diagonals of a square coincide with two of the lines of symmetry that can be drawn. Example: Square with lines of symmetry shown:  • A trapezoid is a quadrilateral with exactly one pair of parallel sides. The parallel sides of a trapezoid are called bases. The nonparallel sides of a trapezoid are called legs. • An isosceles trapezoid has legs of equal length and congruent base angles. An isosceles trapezoid has one line of symmetry. Example: Isosceles trapezoid with line of symmetry shown:  • A chart, graphic organizer, or Venn diagram can be made to organize quadrilaterals according to properties such as sides and/or angles. • Quadrilaterals can be classified by the number of parallel sides: parallelogram, rectangle, rhombus, and square each have two pairs of parallel sides; a trapezoid has one pair of parallel sides; other quadrilaterals have no parallel sides.	

7.6 The student will
a) compare and contrast quadrilaterals based on their properties; and
b) determine unknown side lengths or angle measures of quadrilaterals.

Understanding the Standard	Essential Knowledge and Skills
• Quadrilaterals can be classified by the measures of their angles: a rectangle and a square have four 90° angles; a trapezoid may have zero or two 90° angles. • Quadrilaterals can be classified by the number of congruent sides: a rhombus and a square have four congruent sides; a parallelogram and a rectangle each have two pairs of congruent sides, and an isosceles trapezoid has one pair of congruent sides. • A square is a special type of both a rectangle and a rhombus, which are special types of parallelograms, which are special types of quadrilaterals. • Any figure that has the properties of more than one subset of quadrilaterals can belong to more than one subset. • The sum of the measures of the interior angles of a quadrilateral is 360°. Properties of quadrilaterals can be used to find unknown angle measures in a quadrilateral.	

7.7 The student will apply translations and reflections of right triangles or rectangles in the coordinate plane.

Understanding the Standard	Essential Knowledge and Skills
• A transformation of a figure called the preimage changes the size, shape, or position of the figure to a new figure called the image. • Translations and reflections do not change the size or shape of a figure (e.g., the preimage and image are congruent figures). Translations and reflections change the position of a figure. • A translation is a transformation in which an image is formed by moving every point on the preimage the same distance in the same direction. • A reflection is a transformation in which an image is formed by reflecting the preimage over a line called the line of reflection. All corresponding points in the image and preimage are equidistant from the line of reflection. • The image of a polygon is the resulting polygon after the transformation. The preimage is the polygon before the transformation. • A transformation of preimage point *A* can be denoted as the image *A′* (read as "A prime"). • The preimage of a figure that has been translated and then reflected over the *x*- or *y*-axis may result in a different transformation than the preimage of a figure that has been reflected over the *x*- or *y*-axis and then translated.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or rectangle that has been translated either vertically, horizontally, or a combination of a vertical and horizontal translation. • Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or a rectangle that has been reflected over the *x*- or *y*-axis. • Given a preimage in the coordinate plane, identify the coordinates of the image of a right triangle or rectangle that has been translated and reflected over the *x*- or *y*-axis or reflected over the *x*- or *y*-axis and then translated. • Sketch the image of a right triangle or rectangle that has been translated vertically, horizontally, or a combination of both. • Sketch the image of a right triangle or rectangle that has been reflected over the *x*- or *y*-axis. • Sketch the image of a right triangle or rectangle that has been translated and reflected over the *x*- or *y*-axis or reflected over the *x*- or *y*-axis and then translated.

In the middle grades, students develop an awareness of the power of data analysis and the application of probability through fostering their natural curiosity about data and making predictions.

The exploration of various methods of data collection and representation allows students to become effective at using different types of graphs to represent different types of data. Students use measures of center and dispersion to analyze and interpret data.

Students integrate their understanding of rational numbers and proportional reasoning into the study of statistics and probability. Through experiments and simulations, students build on their understanding of the Fundamental Counting Principle from elementary mathematics to learn more about probability in the middle grades.

7.8 The student will

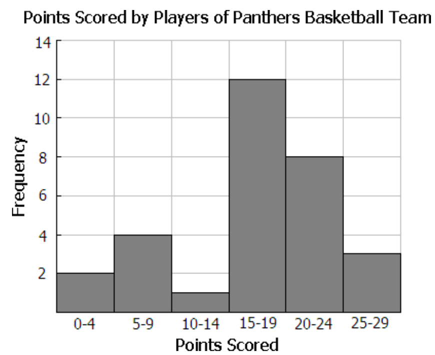
a) determine the theoretical and experimental probabilities of an event; and

b) investigate and describe the difference between the experimental probability and theoretical probability of an event.

Understanding the Standard	Essential Knowledge and Skills

• In general, if all outcomes of an event are equally likely, the probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes in the sample space.
• The probability of an event occurring can be represented as a ratio or equivalent fraction, decimal, or percent.
• The probability of an event occurring is a ratio between 0 and 1.
  – A probability of 0 means the event will never occur.
  – A probability of 1 means the event will always occur.
• The theoretical probability of an event is the expected probability and can be determined with a ratio.
• If all outcomes of an event are equally likely, the theoretical probability of an event = $\frac{\text{number of possible favorable outcomes}}{\text{total number of possible outcomes}}$
• The experimental probability of an event is determined by carrying out a simulation or an experiment.
• The experimental probability of an event = $\frac{\text{number of times desired outcomes occur}}{\text{number of trials in the experiment}}$
• In experimental probability, as the number of trials increases, the experimental probability gets closer to the theoretical probability (Law of Large Numbers).
	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
	• Determine the theoretical probability of an event. (a)
	• Determine the experimental probability of an event. (a)
	• Describe changes in the experimental probability as the number of trials increases. (b)
	• Investigate and describe the difference between the probability of an event found through experiment or simulation versus the theoretical probability of that same event. (b)

7.9 The student, given data in a practical situation, will
a) represent data in a histogram;
b) make observations and inferences about data represented in a histogram; and
c) compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.

Understanding the Standard	Essential Knowledge and Skills
• A histogram is a graph that provides a visual interpretation of numerical data by indicating the number of data points that lie within a range of values, called a class or a bin. The frequency of the data that falls in each class or bin is depicted by the use of a bar. Every element of the data set is not preserved when representing data in a histogram. • All graphs must include a title and labels that describe the data. • Numerical data that can be characterized using consecutive intervals are best displayed in a histogram. • Teachers should be reasonable about the selection of data values. Students should have experiences constructing histograms, but a focus should be placed on the analysis of histograms. • A histogram is a form of bar graph in which the categories are consecutive and equal intervals. The length or height of each bar is determined by the number of data elements (frequency) falling into a particular interval.  • A frequency distribution shows how often an item, a number, or range of numbers occurs. It can be used to construct a histogram.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Collect, organize, and represent data in a histogram. (a) • Make observations and inferences about data represented in a histogram. (b) • Compare data represented in histograms with the same data represented in line plots, circle graphs, and stem-and-leaf plots. (c)

7.9 The student, given data in a practical situation, will
a) represent data in a histogram;
b) make observations and inferences about data represented in a histogram; and
c) compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.

Understanding the Standard	Essential Knowledge and Skills

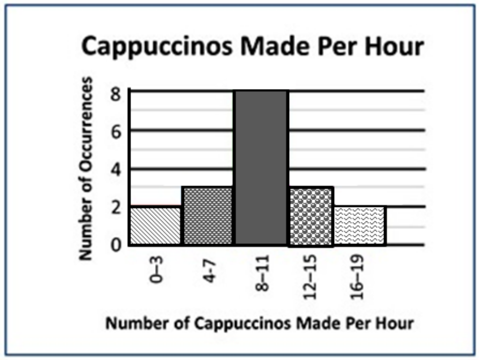
Number of Cappuccinos Made per Hour at the Cafe

Number of Cups of Coffee	Tally	Frequency
0 - 3	||	2
4 – 7	|||	3
8 – 11	~~||||~~ |||	8
12 – 15	|||	3
16 – 19	||	2

To construct a histogram:

- Organize collected data into a table. Create one column for data range categories (bins), divided into equal intervals that will include all of your data (for example, 0-10, 11-20, 21-30), and another column for frequency.
  - Bins should be all the same size.
  - Bins should include all of the data.
  - Boundaries for bins should reflect the data values being represented.
  - Determine the number of bins based upon the data.
  - If possible, the number of bins created should be a factor the number of data values (e.g., a histogram representing 20 data values might have 4 or 5 bins).
- Create a graph. Mark the data range intervals on the *x*-axis (horizontal axis) with no space between the categories. Mark frequency on the *y*-axis (vertical axis), also in equal intervals.
- Plot the data. For each data range category (bin), draw a horizontal line at the appropriate frequency or marker. Then, create a vertical bar for that category reaching up to the marked frequency. Do this for each data range category (bin).

7.9 The student, given data in a practical situation, will
a) represent data in a histogram;
b) make observations and inferences about data represented in a histogram; and
c) compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.

Understanding the Standard	Essential Knowledge and Skills
 – Note: histograms may be drawn so that the bars are horizontal. To do this, interchange the *x*- and *y*-axis. Mark the data range intervals (bins) on the *y*-axis and the frequency on the *x*-axis. Draw the bars horizontally. • Comparisons, predictions and inferences are made by examining characteristics of a data set displayed in a variety of graphical representations to draw conclusions. Data analysis helps describe data, recognize patterns or trends, and make predictions. • There are two types of data: categorical and numerical. Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. While students need to be aware of the differences, they do not have to know the terms for each type of data. • Different types of graphs can be used to display categorical data. The way data is displayed is often dependent on what someone is trying to communicate. • A line plot provides an ordered display of all values in a data set and shows the frequency of data on a number line. Line plots are used to show the spread of the data, to include clusters (groups of data points) and gaps (large spaces between data points), and quickly identify the range, mode, and any extreme data values.	

7.9 The student, given data in a practical situation, will
a) represent data in a histogram;
b) make observations and inferences about data represented in a histogram; and
c) compare histograms with the same data represented in stem-and-leaf plots, line plots, and circle graphs.

Understanding the Standard	Essential Knowledge and Skills
• A circle graph is used for categorical and discrete numerical data. Circle graphs are used to show a relationship of the parts to a whole. Every element of the data set is not preserved when representing data in a circle graph. • A stem and leaf plot is used for discrete numerical data and is used to show frequency of data distribution. A stem and leaf plot displays the entire data set and provides a picture of the distribution of data. • Different situations or contexts warrant different types of graphs, and it helps to have a good knowledge of what graphs are available. Students can determine which graph makes the most sense to use based on the type of data provided and which graph can help them answer questions most easily. • Comparing different types of representations (charts and graphs) provide students an opportunity to learn how different graphs can show different things about the same data. Following construction of graphs, students benefit from discussions around what information each graph provides. • The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or "what could happen if" (inference).	

Patterns, functions and algebra become a larger mathematical focus in the middle grades as students extend their knowledge of patterns developed in the elementary grades.

Students make connections between the numeric concepts of ratio and proportion and the algebraic relationships that exist within a set of equivalent ratios. Students use variable expressions to represent proportional relationships between two quantities and begin to connect the concept of a constant of proportionality to rate of change and slope. Representation of relationships between two quantities using tables, graphs, equations, or verbal descriptions allow students to connect their knowledge of patterns to the concept of functional relationships. Graphing linear equations in two variables in the coordinate plane is a focus of the study of functions which continues in high school mathematics.

Students learn to use algebraic concepts and terms appropriately. These concepts and terms include *variable, term, coefficient, exponent, expression, equation, inequality, domain*, and *range*. Developing a beginning knowledge of algebra is a major focus of mathematics learning in the middle grades. Students learn to solve equations by using concrete materials. They expand their skills from one-step to multistep equations and inequalities through their application in practical situation.

7.10 The student will.

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y = mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y = x + b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y = x + b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- When two quantities, *x* and *y*, vary in such a way that one of them is a constant multiple of the other, the two quantities are "proportional". A model for that situation is $y = mx$ where *m* is the slope or rate of change. Slope may also represent the unit rate of a proportional relationship between two quantities, also referred to as the constant of proportionality or the constant ratio of *y* to *x*.
- The slope of a proportional relationship can be determined by finding the unit rate.

  Example: The ordered pairs (4, 2) and (6, 3) make up points that could be included on the graph of a proportional relationship. Determine the slope, or rate of change, of a line passing through these points. Write an equation of the line representing this proportional relationship.

x	*y*
4	2
6	3

  The slope, or rate of change, would be $\frac{1}{2}$ or 0.5 since the *y*-coordinate of each ordered pair would result by multiplying $\frac{1}{2}$ times the *x*-coordinate. This would also be the unit rate of this proportional relationship. The ratio of *y* to *x* is the same for each ordered pair. That is, $\frac{y}{x} = \frac{2}{4} = \frac{3}{6} = \frac{1}{2} = 0.5$

  The equation of a line representing this proportional relationship of *y* to *x* is $y = \frac{1}{2}x$ or $y = 0.5x$.
- The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change of the line.

$$\text{slope} = \frac{change\ in\ y}{change\ in\ x} = \frac{vertical\ change}{horizontal\ change}$$

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Determine the slope, *m*, as rate of change in a proportional relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form $y = mx$ to represent the relationship. Slope will be limited to positive values. (a)
- Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope, *m*, as rate of change. Slope will be limited to positive values. (b)
- Graph a line representing a proportional relationship between two quantities given the equation of the line in the form $y = mx$, where *m* represents the slope as rate of change. Slope will be limited to positive values. (b)
- Determine the *y*-intercept, *b*, in an additive relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form $y = x + b$, $b \neq 0$, to represent the relationship. (c)

7.10 The student will.

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y = mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y = x + b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y = x + b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- The graph of the line representing a proportional relationship will include the origin (0, 0).
- A proportional relationship between two quantities can be modeled given a practical situation. Representations may include verbal descriptions, tables, equations, or graphs. Students may benefit from an informal discussion about independent and dependent variables when modeling practical situations. Grade eight mathematics formally addresses identifying dependent and independent variables.
  - Example (using a table of values): Cecil walks 2 meters every second (verbal description). If *x* represents the number of seconds and *y* represents the number of meters he walks, this proportional relationship can be represented using a table of values:

x (seconds)	1	2	3	4
y (meters)	2	4	6	8

  This proportional relationship could be represented using the equation $y = 2x$, since he walks 2 meters for each second of time. That is, $\frac{y}{x} = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = 2$, the unit rate (constant of proportionality) is 2 or $\frac{2}{1}$. The same constant ratio of *y* to *x* exists for every ordered pair. This proportional relationship could be represented by the following graph:

Essential Knowledge and Skills

- Graph a line representing an additive relationship ($y = x + b$, $b \neq 0$) between two quantities, given an ordered pair on the line and the *y*-intercept (*b*). The *y*-intercept (*b*) is limited to integer values and slope is limited to 1. (d)
- Graph a line representing an additive relationship between two quantities, given the equation in the form $y = x + b$, $b \neq 0$. The *y*-intercept (*b*) is limited to integer values and slope is limited to 1. (d)
- Make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs. (e)

7.10 The student will.

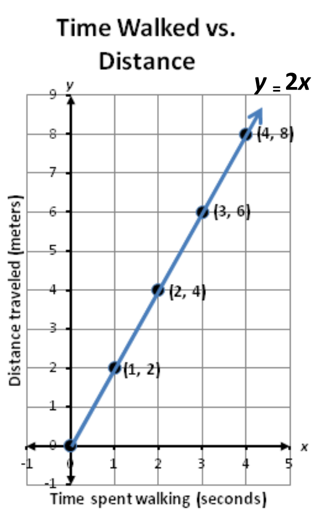
a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard	Essential Knowledge and Skills
 • A graph of a proportional relationship can be created by graphing ordered pairs generated in a table of values (as shown above), or by observing the rate of change or slope of the relationship and using slope triangles to graph ordered pairs that satisfy the relationship given. – Example (using slope triangles): Cecil walks 2 meters every second. If *x* represents the number of seconds and *y* represents the number of meters he walks, this proportional relationship can be represented graphically using slope triangles.	

7.10 The student will.

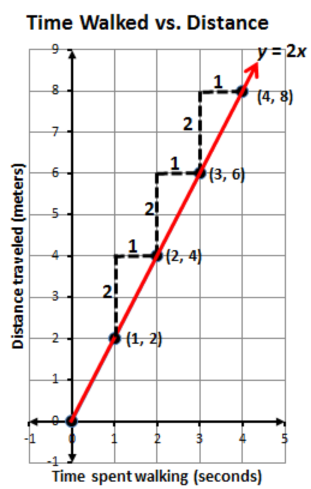
a) **determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form $y = mx$ to represent the relationship;**

b) **graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in $y = mx$ form where *m* represents the slope as rate of change.**

c) **determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form $y = x + b$ to represent the relationship;**

d) **graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form $y = x + b$, where *b* represents the *y*-intercept; and**

e) **make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.**

<table>
<tr><th>Understanding the Standard</th><th>Essential Knowledge and Skills</th></tr>
<tr><td>


The rate of change from (1, 2) to (2, 4) is 2 units up (the change in *y*) and 1 unit to the right (the change in *x*), $\frac{2}{1}$ or 2. Thus, the slope of this line is 2. Slope triangles can be used to generate points on a graph that satisfy this relationship.
</td><td></td></tr>
</table>

7.10 The student will.

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- Proportional thinking requires students to thinking multiplicatively. However, the relationship between two quantities is not always proportional. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e.., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, since context can help students to see the relationship.
  - Example:

Additive relationship:

x		y
2	$\xrightarrow{+8}$	10
3	$\xrightarrow{+8}$	11
4	$\xrightarrow{+8}$	12
5	$\xrightarrow{+8}$	13

Multiplicative relationship:

x		y
2	$\xrightarrow{\cdot 5}$	10
3	$\xrightarrow{\cdot 5}$	15
4	$\xrightarrow{\cdot 5}$	20
5	$\xrightarrow{\cdot 5}$	25

In the additive relationship, *y* is the result of adding 8 to *x.*
In the multiplicative relationship, *y* is the result of multiplying 5 times *x*.
The ordered pair (2, 10) is a quantity in both relationships, however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.

Essential Knowledge and Skills

7.10 The student will.

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y = mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y = x + b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y = x + b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard	Essential Knowledge and Skills
• Two quantities, *x* and *y*, have an additive relationship when a constant value, *b*, exists where $y = x + b$, where $b \neq 0$. An additive relationship is not proportional and its graph does not pass through (0, 0). Note that *b* can be a positive value or a negative value. When *b* is negative, the right side of the equation could be written using a subtraction symbol (e.g., if *b* is −5, then the equation $y = x – 5$ could be used). – Example: Thomas is four years older than his sister, Amanda (verbal description). The following table shows the relationship between their ages at given points in time.	

Amanda's Age	4	5	6	11
Thomas' Age	8	9	10	15
	+4	+4	+4	+4

The equation that represents the relationship between Thomas' age and Amanda's age is $y = x + 4$. A graph of the relationship between their ages is shown below:

7.10 The student will.

a) **determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;**

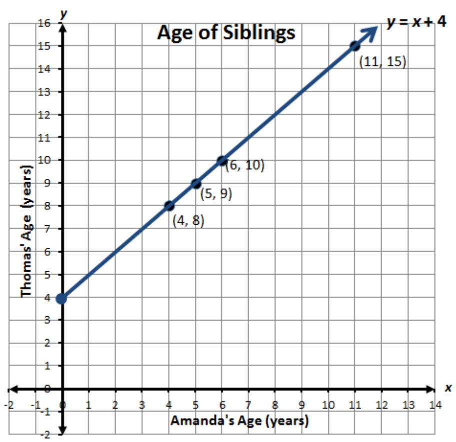
b) **graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form where *m* represents the slope as rate of change.**

c) **determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;**

d) **graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and**

e) **make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.**

Understanding the Standard	Essential Knowledge and Skills



- Graphing a line given an equation can be addressed using different methods. One method involves determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. Another method involves using slope triangles to determine points on the line.
  - Example: Graph the equation $y = x - 1$.
    In order to graph the equation, we can create a table of values by substituting arbitrary values for x to determine coordinating values for y:

7.10 The student will.

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y = mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard	Essential Knowledge and Skills

x	x - 1	y
-1	(-1) - 1	-2
0	(0) - 1	-1
1	(1) - 1	0
2	(2) - 1	1

These values can then be plotted as the points (-1, -2), (0, -1), (1, 0), and (2, 1) on a graph.

An equation written in $y = x + b$ form provides information about the graph. If the equation is $y = x - 1$, then the slope, m, of the line is 1 or $\frac{1}{1}$ and the point where the line crosses the y-axis can be located at (0, -1). We also know,

$$\text{slope} = m = \frac{change\ in\ y-value}{change\ in\ x-value} = \frac{+1}{+1}\ or\ \frac{-1}{-1}$$

So we can plot some other points on the graph using this relationship between *y* and *x* values.

A table of values can be used to determine the graph of a line. The *y*-intercept is located on the *y*-axis which is where the *x*-coordinate is 0. The change in each *y*-value compared to the corresponding *x*-value can be verified by the patterns in the table of values.

	x	y	
	-1	-2	
+1	0	-1	+1
+1	1	0	+1
+1	2	1	+1

7.11 The student will evaluate algebraic expressions for given replacement values of the variables.

<table>
<tr><th>Understanding the Standard</th><th>Essential Knowledge and Skills</th></tr>
<tr><td>

- To evaluate an algebraic expression, substitute a given replacement value for a variable and apply the order of operations. For example, if $a = 3$ and $b = -2$ then $5a + b$ can be evaluated as: 5(3) + (-2) and simplified using the order of operations to equal 15 + (-2) which equals 13.
- Expressions are simplified by using the order of operations.
- The order of operations is a convention that defines the computation order to follow in simplifying an expression. It ensures that there is only one correct value. The order of operations is as follows:
  - First, complete all operations within grouping symbols[1]. If there are grouping symbols within other grouping symbols, do the innermost operations first.
  - Second, evaluate all exponential expressions.
  - Third, multiply and /or divide in order from left to right.
  - Fourth, add and /or subtract in order from left to right.

  [1] Parentheses ( ), brackets [ ], and the division bar should be treated as grouping symbols.
- Expressions are simplified using the order of operations and applying the properties of real numbers. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard).
  - Commutative property of addition: $a + b = b + a$.
  - Commutative property of multiplication: $a \cdot b = b \cdot a$.
  - Associative property of addition: $(a + b) + c = a + (b + c)$.
  - Associative property of multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
  - Subtraction and division are neither commutative nor associative.
  - Distributive property (over addition/subtraction): $a \cdot (b + c) = a \cdot b + a \cdot c$ *and* $a \cdot (b - c) = a \cdot b - a \cdot c$.
  - The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division.

</td><td>

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Represent algebraic expressions using concrete materials and pictorial representations. Concrete materials may include colored chips or algebra tiles.
- Use the order of operations and apply the properties of real numbers to evaluate expressions for given replacement values of the variables. Exponents are limited to 1, 2, 3, or 4 and bases are limited to positive integers. Expressions should not include braces { } but may include brackets [ ] and absolute value | |. Square roots are limited to perfect squares. Limit the number of replacements to no more than three per expression.

</td></tr>
</table>

7.11 The student will evaluate algebraic expressions for given replacement values of the variables.

Understanding the Standard	Essential Knowledge and Skills
– Identity property of addition (additive identity property): $a + 0 = a \ and \ 0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \ and \ 1 \cdot a = a$. – Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$). – Inverse property of addition (additive inverse property): $a + (-a) = 0 \ and \ (-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \ and \ \frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0 \ and \ 0 \cdot a = 0$. – Division by zero is not a possible mathematical operation. It is undefined. – Substitution property: If $a = b$, then *b* can be substituted for *a* in any expression, equation, or inequality.	

7.12 The student will solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
• An equation is a mathematical sentence that states that two expressions are equal. • The solution to an equation is the value(s) that make it a true statement. Many equations have one solution and can be represented as a point on a number line. • A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable. • The inverse operation for addition is subtraction, and the inverse operation for multiplication is division. • A two-step equation may include, but not be limited to equations such as the following: $2x + \frac{1}{2} = -5$; $-25 = 7.2x + 1$; $\frac{x-7}{-3} = 4$; $\frac{3}{4}x - 2 = 10$. • An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an "equal sign (=)" (e.g., $\frac{3}{4}$, $5x$, $140 - 38.2$, $18 \cdot 21$, $5 + x$). • An expression that contains a variable is a variable expression. A variable expression is like a phrase: as a phrase does not have a verb, so an expression does not have an "equal sign (=)." An expression cannot be solved. • A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. For example, the verbal expression "a number multiplied by 5" could be represented by "$n \cdot 5$" or "$5n$". • An algebraic expression is a variable expression that contains at least one variable (e.g., $2x – 3$). • A verbal sentence is a complete word statement (e.g., "The sum of twice a number and two is fifteen." could be represented by "$2n + 2 = 15$"). • An algebraic equation is a mathematical statement that says that two expressions are equal (e.g., $2x – 8 = 7$). • Properties of real numbers and properties of equality can be applied when solving equations, and justifying solutions. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard):	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent and solve two-step linear equations in one variable using a variety of concrete materials and pictorial representations. • Apply properties of real numbers and properties of equality to solve two-step linear equations in one variable. Coefficients and numeric terms will be rational. • Confirm algebraic solutions to linear equations in one variable. • Write verbal expressions and sentences as algebraic expressions and equations. • Write algebraic expressions and equations as verbal expressions and sentences. • Solve practical problems that require the solution of a two-step linear equation.

7.12 The student will solve two-step linear equations in one variable, including practical problems that require the solution of a two-step linear equation in one variable.

Understanding the Standard	Essential Knowledge and Skills
– Commutative property of addition: $a + b = b + a$. – Commutative property of multiplication: $a \cdot b = b \cdot a$. – Subtraction and division are not commutative. – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division. – Identity property of addition (additive identity property): $a + 0 = a \text{ and } 0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a \text{ and } 1 \cdot a = a$. – Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$). – Inverse property of addition (additive inverse property): $a + (-a) = 0 \text{ and } (-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \text{ and } \frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0 \text{ and } 0 \cdot a = 0$. – Division by zero is not a possible mathematical operation. It is undefined. – Substitution property: If $a = b$, then *b* can be substituted for *a* in any expression, equation, or inequality. – Addition property of equality: If $a = b$, then $a + c = b + c$. – Subtraction property of equality: If $a = b$, then $a - c = b - c$. – Multiplication property of equality: If $a = b$, then $a \cdot c = b \cdot c$. – Division property of equality: If $a = b \text{ and } c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.	

7.13 The student will solve one- and two-step linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and graph the solution on a number line.

Understanding the Standard	Essential Knowledge and Skills
• A one-step inequality may include, but not be limited to, inequalities such as the following: $2x > 5$; $y - \frac{2}{3} \leq -6$; $\frac{1}{5}x < -3$; $a - (-4) \geq \frac{11}{2}$. • A two-step inequality may include, but not be limited to inequalities such as the following: $2x + 1 < -25$; $2x + \frac{1}{2} \geq -5$; $-25 > 7.2x + 1$; $\frac{x-7}{-3} \leq 4$; $\frac{3}{4}x - 2 \leq 10$. • The solution set to an inequality is the set of all numbers that make the inequality true. • The inverse operation for addition is subtraction, and the inverse operation for multiplication is division. • The procedures for solving inequalities are the same as those to solve equations except for the case when an inequality is multiplied or divided on both sides by a negative number. Then the inequality sign is changed from less than to greater than, or greater than to less than. • When both expressions of an inequality are multiplied or divided by a negative number, the inequality symbol reverses (e.g., $-3x < 15$ is equivalent to $x > -5$). • Solutions to inequalities can be represented using a number line. • In an inequality, there can be more than one value for the variable that makes the inequality true. There can be many solutions. (i.e., $x + 4 > -3$ then the solution is $x > -7$. This means that x can be any number greater than -7. A few solutions might be -6.5, -3, 0, 4, 25, etc.) • Properties of real numbers and properties of inequality can be used to solve inequalities, justify solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard). – Commutative property of addition: $a + b = b + a$. – Commutative property of multiplication: $a \cdot b = b \cdot a$. – Subtraction and division are not commutative. – The additive identity is zero (0) because any number added to zero is the number. The multiplicative identity is one (1) because any number multiplied by one is the number. There are no identity elements for subtraction and division.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Apply properties of real numbers and the multiplication and division properties of inequality to solve one-step inequalities in one variable, and the addition, subtraction, multiplication, and division properties of inequality to solve two-step inequalities in one variable. Coefficients and numeric terms will be rational. • Represent solutions to inequalities algebraically and graphically using a number line. • Write verbal expressions and sentences as algebraic expressions and inequalities. • Write algebraic expressions and inequalities as verbal expressions and sentences. • Solve practical problems that require the solution of a one- or two-step inequality. • Identify a numerical value(s) that is part of the solution set of a given inequality.

7.13 The student will solve one- and two-step linear inequalities in one variable, including practical problems, involving addition, subtraction, multiplication, and division, and graph the solution on a number line.

Understanding the Standard	Essential Knowledge and Skills
– Identity property of addition (additive identity property): $a + 0 = a\ and\ 0 + a = a$. – Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a\ and\ 1 \cdot a = a$. – Inverses are numbers that combine with other numbers and result in identity elements (e.g., 5 + (–5) = 0; $\frac{1}{5} \cdot 5 = 1$). – Inverse property of addition (additive inverse property): $a + (-a) = 0\ and\ (-a) + a = 0$. – Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1\ and\ \frac{1}{a} \cdot a = 1$. – Zero has no multiplicative inverse. – Multiplicative property of zero: $a \cdot 0 = 0\ and\ 0 \cdot a = 0$. – Division by zero is not a possible mathematical operation. It is undefined. – Substitution property: If $a = b$, then *b* can be substituted for *a* in any expression, equation, or inequality. – Addition property of inequality: If $a < b$, then $a + c < b + c$; if $a > b$, then $a + c > b + c$. – Subtraction property of inequality: If $a < b$, then $a - c < b - c$; if $a > b$, then $a - c > b - c$.. – Multiplication property of inequality: If $a < b\ and\ c > 0$, then $a \cdot c < b \cdot c$; if $a > b\ and\ c > 0$, then $a \cdot c > b \cdot c$. – Multiplication property of inequality (multiplication by a negative number): If $a < b\ and\ c < 0$, then $a \cdot c > b \cdot c$; if $a > b\ and\ c < 0$, then $a \cdot c < b \cdot c$. – Division property of inequality: If $a < b\ and\ c > 0$, then $\frac{a}{c} < \frac{b}{c}$; if $a > b\ and\ c > 0$, then $\frac{a}{c} > \frac{b}{c}$. – Division property of inequality (division by a negative number): If $a < b\ and\ c < 0$, then $\frac{a}{c} > \frac{b}{c}$; if $a > b\ and\ c < 0$, then $\frac{a}{c} < \frac{b}{c}$.	