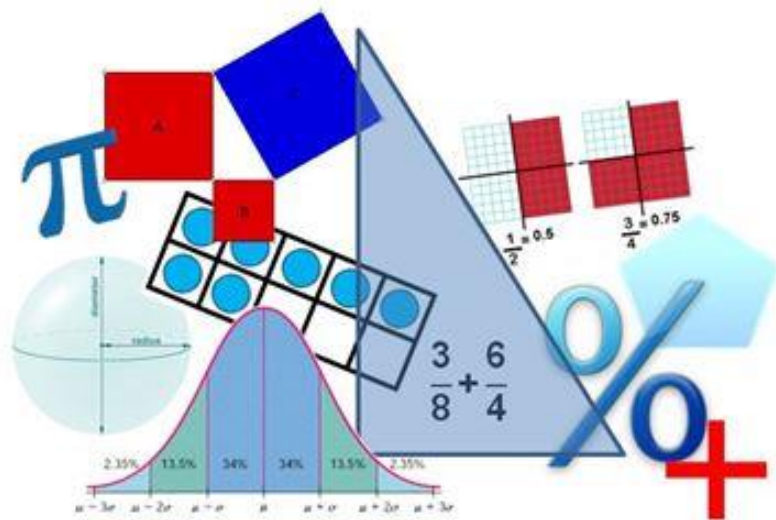# Mathematics

2016 Standards of Learning

Algebra, Functions, and Data Analysis
Curriculum Framework

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Virginia 2016 *Mathematics Standards of Learning Curriculum Framework*
Introduction

The 2016 *Mathematics Standards of Learning Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

Essential Knowledge and Skills
This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

Mathematical Process Goals for Students

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

Mathematical Problem Solving

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

Mathematical Communication

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

Mathematical Reasoning

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

Mathematical Connections

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

Instructional Technology

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, "… the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations." State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (*).

Computational Fluency

Mathematics instruction must develop students' conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades. Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of second grade and those for multiplication and division by the end of grade four. Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

Algebra Readiness

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. "Algebra readiness" describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

Equity

> **"**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement."
> – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students' prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

AFDA.1   **The student will investigate and analyze linear, quadratic, exponential, and logarithmic function families and their characteristics. Key concepts include**
  a) **domain and range;**
  b) **intervals on which a function is increasing or decreasing;**
  c) **absolute maxima and minima;**
  d) **zeros;**
  e) **intercepts;**
  f) **values of a function for elements in its domain;**
  g) **connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;**
  h) **end behavior; and**
  i) **vertical and horizontal asymptotes.**

Understanding the Standard	Essential Knowledge and Skills
• A relation is a function if and only if each element in the domain is paired with a unique element of the range. • Functions are used to model practical phenomena. • Functions describe the relationship between two variables where each input is paired to a unique output. • Function families consist of a parent function and all transformations of the parent function. • The domain of a function is the set of all possible values of the independent variable. • The range of a function is the set of all possible values of the dependent variable. • For each x in the domain of f, x is a member of the input of the function f, $f(x)$ is a member of the output of f, and the ordered pair $(x, f(x))$ is a member of f. • The domain of a function may be restricted algebraically, graphically, or by the practical situation modeled by the function.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically. Domains may be limited by problem context or in graphical representations. (a, d, e) • Identify intervals on which the function is increasing or decreasing. (b) • Identify the location and value of the absolute maximum and absolute minimum of a function over the domain of the function graphically or by using a graphing utility. (c) • For any x value in the domain of f, determine $f(x)$. (f) • Represent relations and functions using verbal descriptions, tables, equations, and graphs. Given one representation, represent the relation in another form. (g) • Detect patterns in data and represent arithmetic and geometric patterns algebraically. (g) • Describe the end behavior of a function. (h)

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Understanding the Standard	Essential Knowledge and Skills
- A function can be described on an interval as increasing, decreasing, or constant over a specified interval or over the entire domain of the function. - A function, *f(x)*, is increasing over an interval if the values of *f(x)* consistently increase over the interval as the *x* values increase. - A function, *f(x)*, is decreasing over an interval if the values of *f(x)* consistently decrease over the interval as the *x* values increase. - A function, *f(x)*, is constant over an interval if the values of *f(x)* remain constant over the interval as the *x* values increase. - A function, *f*, has a maximum at *x = a* if *f(a)* is the largest value of *f* over its domain. - A function, *f*, has a minimum in some interval at *x = a* if *f(a)* is the smallest value of *f* over its domain. - Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation.	- Determine the equations of the horizontal asymptote of an exponential function and the vertical asymptote of a logarithmic function. (i) - Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a, b, c, d, e, f, g, h ,i)

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e) intercepts;
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h) end behavior; and
i) vertical and horizontal asymptotes.

Understanding the Standard	Essential Knowledge and Skills
– Examples may include: | Equation/ Inequality	Set Notation	Interval Notation | |---|---|---| | $x = 3$ | {3} | | | $x = 3$ or $x = 5$ | {3, 5} | | | $0 \leq x \leq 3$ | $\{x \| 0 \leq x \leq 3\}$ | [0, 3) | | $y \geq 3$ | $\{y: y \geq 3\}$ | [3, ∞) | | Empty (null) set Ø | { } | | • A value x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$. • The x-intercept is the point at which the graph of a relation or function intersects with the x-axis. It can be expressed as a value or a coordinate. • The y-intercept is the point at which the graph of a relation or function intersects with the y-axis. It can be expressed as a value or a coordinate.	

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Understanding the Standard	Essential Knowledge and Skills
• Given a polynomial function $f(x)$, the following statements are equivalent for any real number, k, such that $f(k) = 0$: – k is a zero of the polynomial function $f(x)$ located at $(k, 0)$; – k is a solution or root of the polynomial equation $f(x) = 0$; – the point $(k, 0)$ is an x-intercept for the graph of polynomial $f(x) = 0$; and – $(x - k)$ is a factor of polynomial $f(x)$. • Connections between multiple representations (graphs, tables, and equations) of a function can be made. • In an arithmetic pattern, the common difference is the value that is added to obtain the next value in the pattern. • In geometric number patterns, the common ratio is a multiplier used to obtain the next value in the pattern. • End behavior describes a function's values as x approaches positive or negative infinity.	

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Understanding the Standard	Essential Knowledge and Skills
• Asymptotes can be used to describe local behavior and end behavior of graphs. They are lines or other curves that approximate the graphical behavior of a function.	

AFDA.2   **The student will use knowledge of transformations to write an equation, given the graph of a linear, quadratic, exponential, and logarithmic function.**

Understanding the Standard	Essential Knowledge and Skills
• Knowledge of transformational graphing using parent functions can be used to verify a mathematical model from a scatterplot that approximates the data. • The graph of a parent function is an anchor graph from which other graphs are derived using transformations. • Transformations of graphs include: – Translations (horizontal and vertical shifting of a graph); – Reflections (over the *x*- and *y*-axis); and – Dilations (stretching and compressing of graphs). • The equation of a line can be determined by two points on the line or by the slope and a point on the line.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Write an equation of a line when given the graph of a line. • Recognize graphs of parent functions for linear, quadratic, exponential and logarithmic functions. • Write the equation of a linear, quadratic, exponential, or logarithmic function in vertex form, given the graph of the parent function and transformation information. • Describe the transformation from the parent function given the equation written in vertex form or the graph of the function. • Given the equation of a function, recognize the parent function and transformation to graph the given function. • Recognize the vertex of a parabola given a quadratic equation in vertex form or graphed. • Describe the parent function represented by a scatterplot.

AFDA.3 **The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve practical problems using models of linear, quadratic, and exponential functions.**

Understanding the Standard	Essential Knowledge and Skills

• Data and scatterplots may indicate patterns that can be modeled with a function.
 	
• The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate.
 	
• Knowledge of transformational graphing using parent functions can be used to verify a mathematical model from a scatterplot that approximates the data.
 	
• Graphing utilities can be used to collect, organize, represent, and generate an equation of a curve of best fit for a set of data.
 	
• Data that fit linear ($y = mx + b$), quadratic ($y = ax^2 + bx + c$), and exponential ($y = ab^x$) models arise from practical situations.
 	
• A correlation coefficient measures the degree of association between two variables that are related linearly.
 	
• Rounding that occurs during intermediate steps of problem solving may reduce the accuracy of the final answer.
 	
• Evaluation of the reasonableness of a mathematical model of a practical situation involves asking questions including:
 	
   – "Is there another curve (linear, quadratic, or exponential) that better fits the data?"
   – "Does the curve of best fit make sense?"
   – "Could the curve of best fit be used to make reasonable predictions?"
	The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
	
	• Determine an equation for the curve of best fit, given a set of no more than 20 data points in a table, on a graph, or practical situation.
	
	• Make predictions, using data, scatterplots, or the equation of the curve of best fit.
	
	• Solve practical problems involving an equation of the curve of best fit.
	
	• Evaluate the reasonableness of a mathematical model of a practical situation.

AFDA.4   **The student will use multiple representations of functions for analysis, interpretation, and prediction.**

Understanding The Standard	Essential Knowledge And Skills
• The most appropriate representation of a function depends on the questions to be answered and/or the analysis to be done. • Given data may be represented as discrete points or as a continuous graph with respect to the practical context. • Practical data may best be represented as a table, a graph, or a formula.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Given an equation, graph a linear, quadratic, exponential or logarithmic function. • Make predictions given a table of values, a graph, or an algebraic formula. • Describe relationships between data represented in a table, in a scatterplot, and as elements of a function. • Determine the appropriate representation of data derived from real-world situations. • Analyze and interpret the data in context of the practical situation. • Use a graphing utility to graph, analyze, interpret, and make predictions.

AFDA.5   **The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.**

Understanding the Standard	Essential Knowledge and Skills
• Linear programming models an optimization process. • A linear programming model consists of a system of constraints and an objective quantity that can be maximized or minimized. • Any maximum or minimum value will occur at the vertices of a feasible region.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model practical problems with systems of linear inequalities. • Solve systems of no more than four linear inequalities with pencil and paper and using a graphing utility. • Solve systems of no more than four equations algebraically and graphically. • Identify the feasible region of a system of linear inequalities. • Identify the coordinates of the vertices of a feasible region. • Determine and describe the maximum or minimum value for the function defined over a feasible region.

AFDA.6   **The student will calculate probabilities. Key concepts include**
- a) **conditional probability;**
- b) **dependent and independent events;**
- c) **mutually exclusive events;**
- d) **counting techniques (permutations and combinations); and**
- e) **Law of Large Numbers.**

Understanding the Standard	Essential Knowledge and Skills
- The Fundamental Counting Principle states that if one decision can be made *n* ways and another can be made *m* ways, then the two decisions can be made *nm* ways. - A sample space is the set of all possible mutually exclusive outcomes of a random experiment. - An event is a subset of the sample space. *P(E)* is a way to represent the probability that the event *E* occurs. - Mutually exclusive events are events that cannot both occur simultaneously. - Mutually exclusive events are calculated using the addition or multiplication rules. - If *A* and *B* are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$. - The complement of event *A* consists of all outcomes in which event *A* does not occur. *P(B|A)* is the probability that *B* will occur given that *A* has already occurred. *P(B|A)* is called the conditional probability of *B* given *A*. - Exploration of the representation of conditional statements using Venn diagrams may assist in deepening student understanding.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** - Analyze, interpret and make predictions based on theoretical probability within practical context. (a, b, c, e) - Determine conditional probabilities for dependent, independent, and mutually exclusive events. (a, b, c) - Represent and calculate probabilities using Venn diagrams and probability trees. (a) - Define and give contextual examples of complementary, dependent, independent, and mutually exclusive events. (b, c) - Given two or more events in a problem setting, determine whether the events are complementary, dependent, independent, and/or mutually exclusive. (b, c) - Compare and contrast permutations and combinations, including those occurring in practical situations. (d) - Calculate the number of permutations of *n* objects taken *r* at a time, without repetition. (d) - Calculate the number of combinations of *n* objects taken *r* at a time, without repetition. (d)

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   e) **Law of Large Numbers.**

Understanding the Standard	Essential Knowledge and Skills
$$P(B\|A) = \frac{P(A \cap B)}{P(A)}$$ $$\Rightarrow P(A \cap B) = P(A)P(B\|A)$$ • Two events, *A* and *B*, are independent if the occurrence of one does not affect the probability of the occurrence of the other. If *A* and *B* are not independent, then they are said to be dependent. • If *A* and *B* are independent events, then $P(A \cap B) = P(A)P(B)$ $P(A \cap B) = P(A)P(B)$. • A permutation is the number of possible ways to arrange a group of objects without repetition and when order matters (e.g., the outcome 1, 2, 3 is different from the outcome 3, 2, 1 when order matters; therefore, both arrangements would be included in the possible outcomes).	

AFDA.6   **The student will calculate probabilities. Key concepts include**
   a)   **conditional probability;**
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   d)   **counting techniques (permutations and combinations); and**
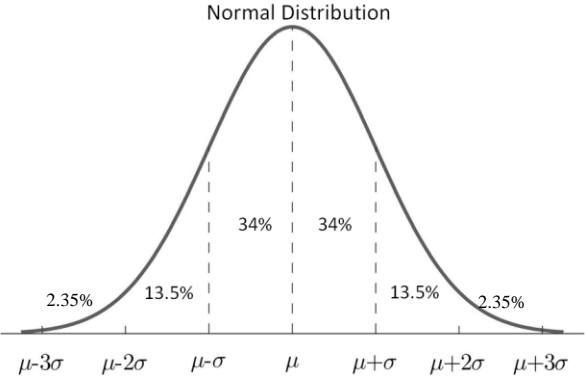   e)   **Law of Large Numbers.**

Understanding the Standard	Essential Knowledge and Skills
• A combination is the number of possible ways to select or arrange objects when there is no repetition and order does not matter (e.g., the outcome 1, 2, 3 is the same as the outcome 3, 2, 1 when order does not matter; therefore, both arrangements would not be included in the possible outcomes). • The Law of Large Numbers states that as a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.	

AFDA.7   **The student will**
    a)   **identify and describe properties of a normal distribution;**
    b)   **interpret and compare *z*-scores for normally distributed data; and**
    c)   **apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.**

Understanding the Standard	Essential Knowledge and Skills
• The focus of this standard is on the interpretation of descriptive statistics, *z*-scores, probabilities, and their relationship to the normal curve in the context of a data set.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**
• Descriptive statistics include measures of center (mean, median, and mode) and dispersion or spread (variance and standard deviation).	• Identify the properties of a normal distribution. (a)
• Variance (σ^2) and standard deviation (σ) measure the spread of data about the mean in a data set.	• Describe how the standard deviation and the mean affect the graph of the normal distribution. (a)
• Standard deviation is expressed in the original units of measurement of the data.	• Given standard deviation and mean, calculate and interpret the *z*-score for a data point. (b)
• The greater the value of the standard deviation, the further the data tends to be dispersed from the mean.	• Compare two sets of normally distributed data using a standard normal distribution and *z*-scores, given mean and standard deviation. (b)
• In order to develop an understanding of standard deviation as a measure of dispersion (spread), students should have experiences analyzing the formulas for and the relationship between variance and standard deviation.	• Represent probability as area under the curve of a standard normal distribution. (c)
• The normal distribution curve is the family of symmetrical, bell-shaped curves defined by the mean and the standard deviation of a data set. The arithmetic mean (μ) is located on the line of symmetry of the curve and is approximately equivalent to the median and mode of the data set.	• Use a graphing utility or a table of Standard Normal Probabilities to determine probabilities associated with areas under the standard normal curve. (c)
• The normal curve is a probability distribution and the total area under the curve is 1.	• Use a graphing utility to investigate, represent, and determine relationships between a normally distributed data set and its descriptive statistics. (a, b, c)

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Understanding the Standard	Essential Knowledge and Skills
• For a normal distribution, approximately 68 percent of the data fall within one standard deviation of the mean, approximately 95 percent of the data fall within two standard deviations of the mean, and approximately 99.7 percent of the data fall within three standard deviations of the mean. This is often referred to as the Empirical Rule or the 68-95-99.7 rule. Normal Distribution NOTE: This chart illustrates percentages that correspond to subdivisions in one standard deviation increments. Percentages for other subdivisions require the table of Standard Normal Probabilities or a graphing utility. • The mean and standard deviation of a normal distribution affect the location and shape of the curve. The vertical line of symmetry of the normal distribution falls at the mean. The greater the standard deviation, the wider ("flatter" or "less peaked") the distribution of the data.	

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   b) **interpret and compare *z*-scores for normally distributed data; and**
   c) **apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.**

Understanding the Standard	Essential Knowledge and Skills
• A *z*-score derived from a particular data value tells how many standard deviations that data value falls above or below the mean of the data set. It is positive if the data value lies above the mean and negative if the data value lies below the mean. • A standard normal distribution is the set of all *z*-scores. The mean of the data in a standard normal distribution is 0 and the standard deviation is 1. This allows for the comparison of unlike normal data. • The table of Standard Normal Probabilities and graphing utilities may be used to determine normal distribution probabilities. • Given a *z*-score (*z*), the table of Standard Normal Probabilities (*z*-table) shows the area under the curve to the left of *z*. This area represents the proportion of observations with a *z*-score less than the one specified. Table rows show the *z*-score's whole number and tenths place. Table columns show the hundredths place. • Graphing utilities can be used to represent a normally distributed data set and explore relationships between the data set and its descriptive statistics.	

AFDA.8   **The student will design and conduct an experiment/survey. Key concepts include**
   a) **sample size;**
   b) **sampling technique;**
   c) **controlling sources of bias and experimental error;**
   d) **data collection; and**
   e) **data analysis and reporting.**

Understanding the Standard	Essential Knowledge and Skills
• The value of a sample statistic may vary from sample to sample, even if the simple random samples are taken repeatedly from the population of interest. • Poor data collection can lead to misleading and meaningless conclusions. • Considerations such as sample size, randomness, and bias affect experimental design. • The purpose of sampling is to provide sufficient information so that population characteristics may be inferred. • Inherent bias diminishes as sample size increases. • Experiments must be carefully designed in order to detect a cause-and-effect relationship between variables. • Principles of experimental design include comparison with a control group, randomization, and blindness. • The precision, accuracy and reliability of data collection can be analyzed and described.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and describe sampling techniques, such as simple random sampling, stratified sampling, and cluster sampling. (a, b) • Determine which sampling technique is best, given a particular context. (b) • Identify biased sampling methods. (c) • Given a plan for a survey, identify possible sources of bias, and describe ways to reduce bias. (c) • Plan and conduct an experiment or survey. The experimental design should address control, randomization, and minimization of experimental error. (a, b, c, d) • Compare and contrast controlled experiments and observational studies and the conclusions one may draw from each. (e) • Write a report describing the experiment/survey and the resulting data and analysis. (e)