AP Calculus BC

My Dear and Stellar Honors Pre-Calculus Student,

Next September, we hope to hit the ground running beginning on the first day! Therefore, it is our distinct pleasure to present you this packet of summer fun. This packet is intended to prepare you for the course by:

- Reviewing prerequisite algebra and pre-calculus skills.
- Covering Chapter 1 of the calculus textbook. (With the amount of curriculum to be covered before the AP Exam in May, we need to cover this review chapter before the school year.)

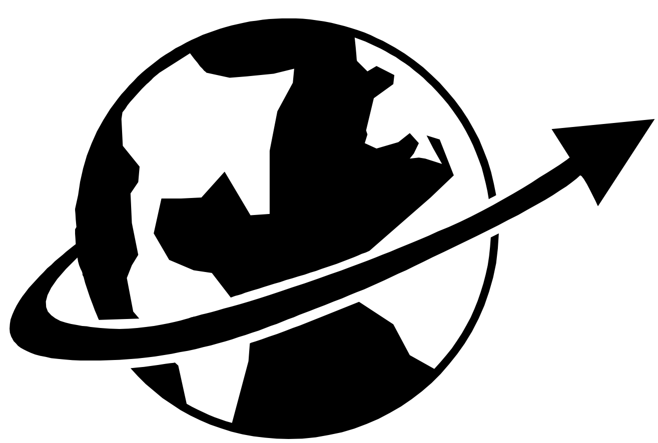
It is due on the first day of school and worth 20 points in the homework category. At the end of the first week of school, there will be the **Chapter 1 Test** assessing you on this review content.

You might need to look up an occasional formula or definition—taking shortcuts with this packet won't benefit you in the Fall! **Questions marked with an asterisk (*) should be solvable without having to rely on a graphing calculator.** Use your resources (friends, internet, etc.) if necessary. If there is anything in this packet you don't know how to do, make sure you learn it before the first day of school!

It is also suggested that you watch the original *Star Wars* trilogy and Monty Python's *Search For the Holy Grail* sometime this summer so that your education can be considered somewhat complete. You'll miss a lot of references if you come to class without having watched them.

Have a relaxing and enjoyable summer!

– Mr. Haugh and Mr. Kemp



Show all work in the space provided. Put final answers on the lines on the right side, if applicable.

*1. Find the slope of the line 3x + 2y = 4

*2. Given the points (1, 2) & (-3, 0)

a. Find the <u>exact</u> distance between the points

b. Find the equation of the line that passes through the two points
(in *both* slope-intercept AND point-slope form)

c. Find the equation of the line <u>normal</u> (perpendicular) to the segment joining the two points at the segment's midpoint.

y =

3. Let $f(x) = 3x^2 - 6x - 4$, and let $g(x) = \frac{x}{2} + 1$. Use a graphing calculator to determine the following:

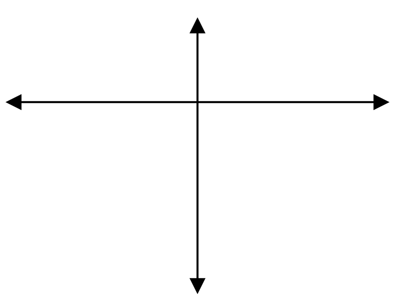
a. The roots of the parabola

b. The vertex of the parabola

c. The point(s) of intersection of the two graphs

*4. Find the exact, fully reduced solutions to the equation $2x^2 - 4x + 1 = 0$

*5. Sketch a graph of $y = x^2 + 5x - 6$. Find and label roots, vertex, and y-intercept. What is the equation of the parabola <u>in vertex form</u>?



*6. Use whatever means available (except a calculator) to find the exact roots of the function $y = x^3 + 2x^2 - 7x + 4$. (Try Synthetic Division &/or Factoring?)

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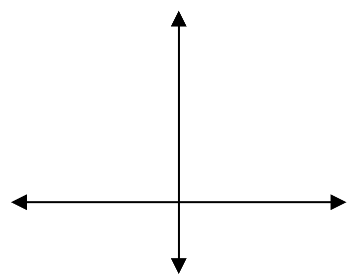
*7. Find the solutions to the following: | x + 2 | - 1 < 4. ____________________

*8. Sketch a graph of y = | x – 1 |



*9. State the domain and range of the function $f(x) = x^2 + 4$.

D: __________

R: __________

*10. Let $f(x) = x^2$, and let $g(x) = \sqrt{x+1}$, and let $h(x) = x + 1$. Does *h(x) = f(g(x))*, or does *h(x) = g(f(x))*, (or neither?) What is *h(f(g(x)))?*

____

h(f(g(x))) = _____________

*11. Let $f(x) = \dfrac{1}{2x+1}$. Find $f^{-1}(x)$. _____________

*12. Simplify as far as possible; leave no radicals in your answer (rational powers are okay):

$$\frac{4x^4 - 8x^2}{4x^2\sqrt[5]{(x^2-2)^3}}$$

____

*13. Solve each of the following:

a. $\log_2 x = 5$ b. $\ln x = 1$ c. $\ln e^2 = x$ d. $3^x = 3\sqrt{3}$

__________ __________ __________ __________

14. Write the degree equivalent under each of the following radian measures

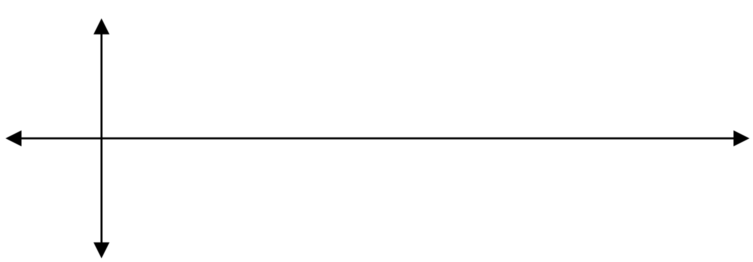
11π/6 π/2 4π/3 7π/6 5π/4

_____ _____ _____ _____ _____

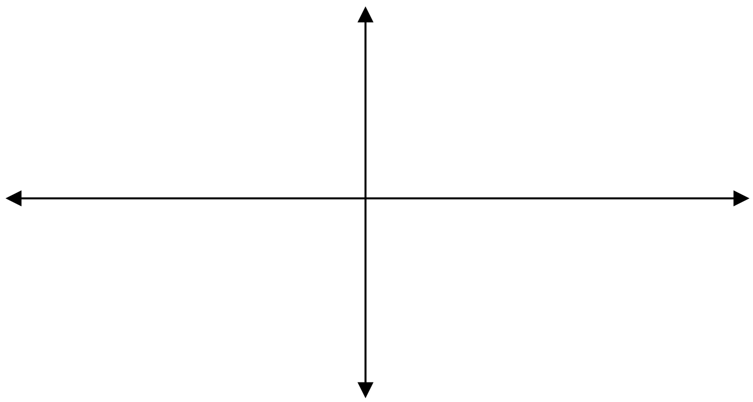
15. Graph two full periods of each of the following. Label the axes, especially roots (x-intercepts) and extrema!

a. y = 2 sin 4x

b. y = 1 – cos πx



(15 cont'd) c. y = tan x



16. Solve for x: $5^{x+1} = 250$

*17. Evaluate: a. $\sin^{-1}\left(\frac{1}{2}\right)$ b. arccos (-1) c. $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

18. Find all solutions between 0 and 2π for each of the following:

a. $\sin x = \frac{1}{2}$

b. $\cos x = \frac{-\sqrt{3}}{2}$

c. tan x = -1

d. 2 cos 2x = 1

(18 cont'd)

e. $\cot x = \dfrac{1}{\sqrt{3}}$

f. $\sec \pi x = 2$

g. $2 \sin x \cos x = -1$

h. $\cos^2 x - \sin^2 x = \dfrac{1}{2}$

19. Determine if the following functions are even, odd, or neither; then describe their symmetry (if any).

a. $y = \dfrac{|x|}{\sqrt{x^2 - 1}}$

b. $y = \dfrac{x^3 - x}{x^4}$

E, O or N? _____

Symmetry? _____________

E, O, or N? _____

Symmetry? _____________

*20. Evaluate each of the following:

a. $\lim\limits_{x \to \infty} \dfrac{x}{x^2 + 1}$

b. $\lim\limits_{x \to \infty} \dfrac{3x^3 + 2x}{1 - 5x^3}$

c. $\lim\limits_{x \to \infty} \dfrac{\sqrt{2x^2 - 10}}{2x}$

_____________ _____________ _____________

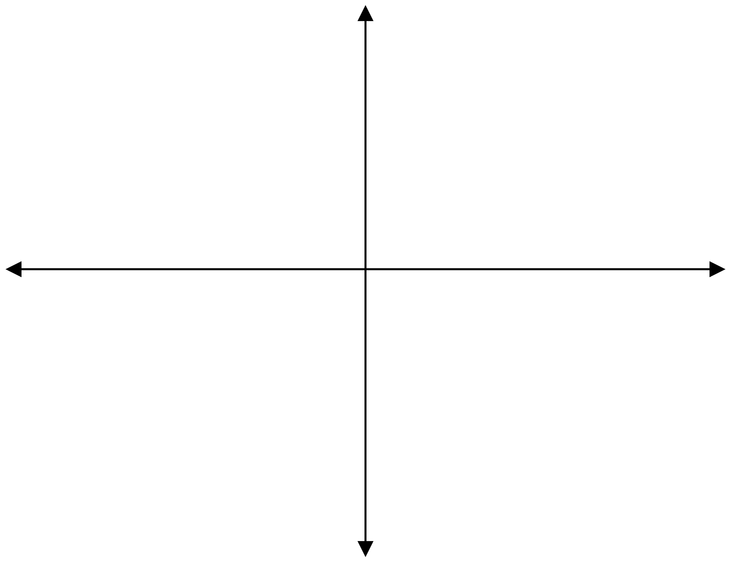
*21. Evaluate each of the following:

a. $\lim\limits_{x \to 2} \dfrac{x^2 - 3}{2}$

b. $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2}$

22. Graph the following rational function. Indicate any and all roots, vertical asymptotes (infinite discontinuities), horizontal asymptotes, open circles (removable discontinuities), and other points you find. Take it as far as possible before resorting to using a calculator (in other words, show your work, including limit statements!)

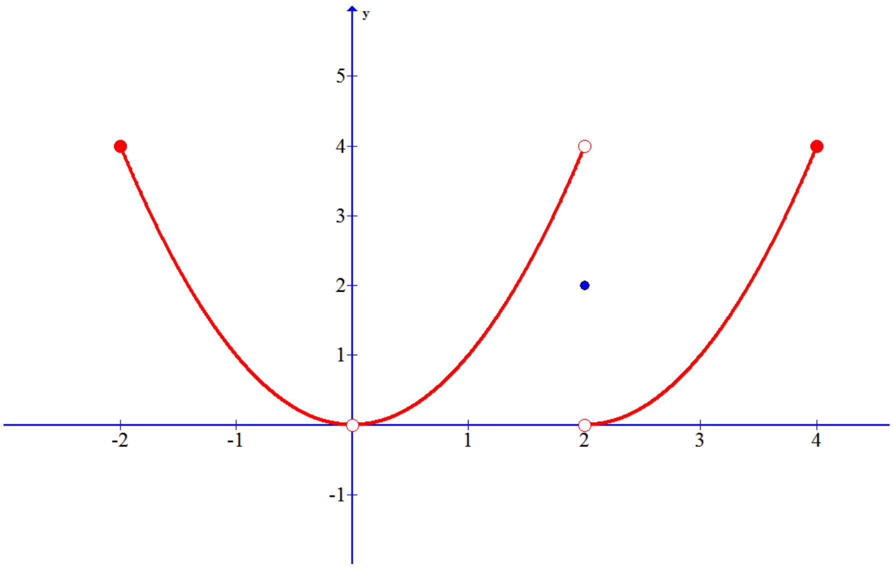
$$f(x) = \frac{x^3 + 3x^2}{x^4 - 4x^2}$$



*23. Evaluate: a. $\sum\limits_{i=1}^{6} (i^2 - 4)$

b. $\sum\limits_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$

* 24. Use the graph of the function to answer the following questions. Be as specific as possible



a. $f(2) = ?$ ____________

b. $\lim_{x \to 2^+} f(x) = ?$ ____________

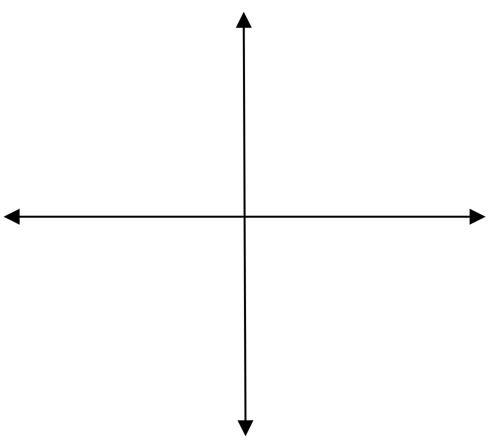
c. $f(0) = ?$ ____________

d. $\lim_{x \to 0} f(x) = ?$ ____________

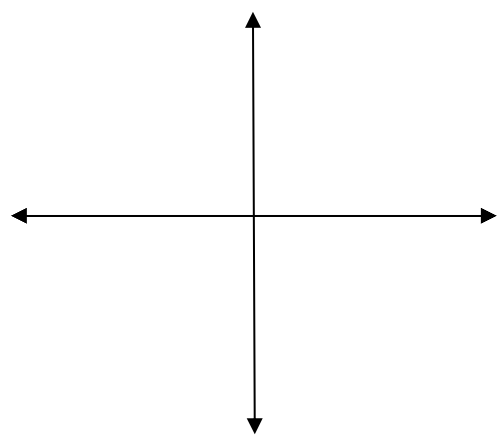
e. $f'(2) = ?$ ____________

25. Sketch a graph of the following polar equations

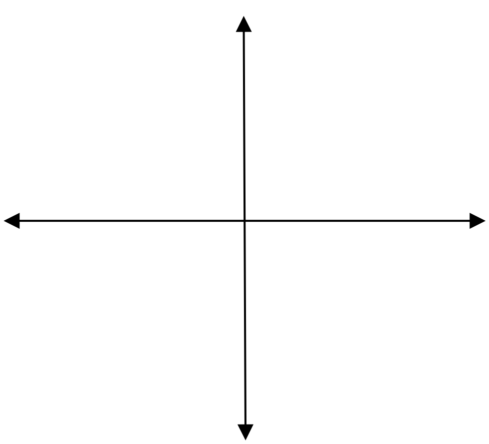
a. $r = 2 \sin 2\theta$



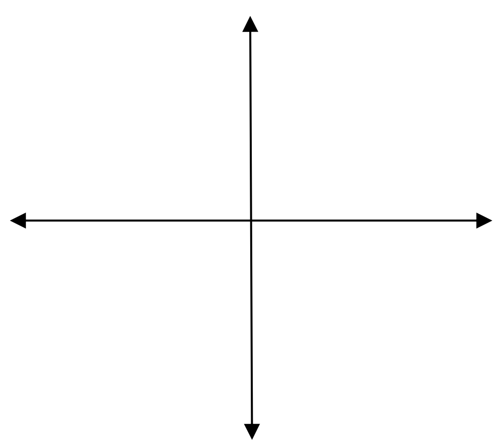
* b. $1 - \cos \theta$



c. $r = \theta + \sin(2\theta), 0 \leq \theta \leq \pi$



* d. $r = 1 - 2 \sin \theta$



26. A particle moves along the y-axis. Its position at any time t can be found via $s(t) = 2t^3 - 4t + 1$.

a. What is the position of the particle at time t = 2? ______________

b. At what time(s) – if any – is the velocity of the particle zero? Explain how you found your answer(s).

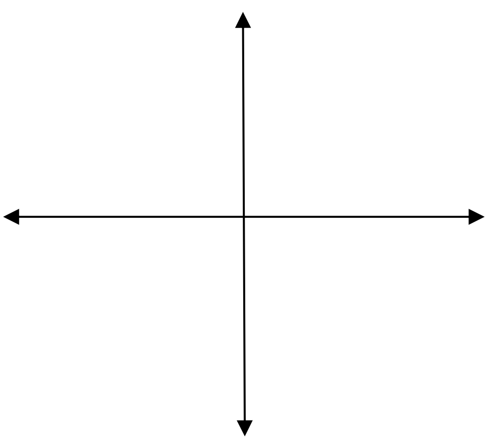
c. What was the average velocity of the particle on the interval [-1, 2]? ______________

d. Find a formula for the instantaneous velocity of the particle at any time t. _____________________

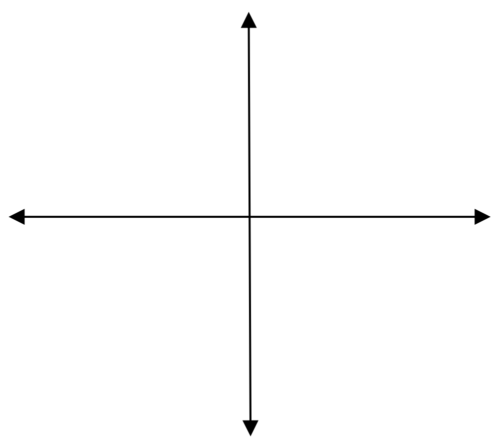
e. What was the average acceleration of the particle on the interval [-1, 2]? ______________

27. Make a sketch of the parametric curves. Then eliminate the parameter to find an equation that relates x and y directly.

* a. $x(t) = 2t + 3,\ y(t) = 4t - 3$

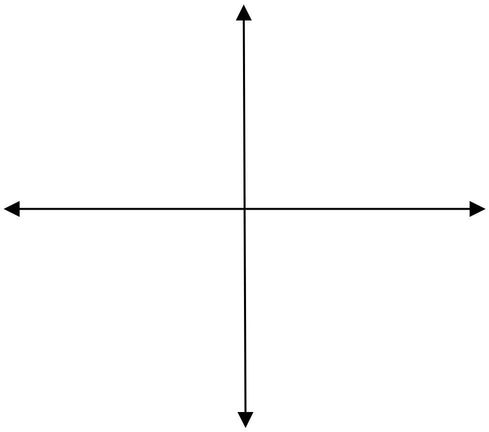


* b. $x(t) = \sqrt{t-2}\,,\ y(t) = \frac{t+5}{4}$

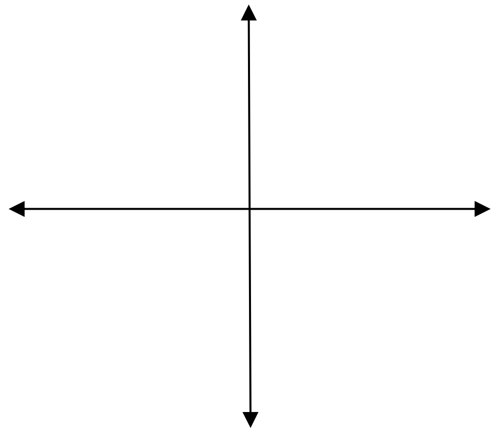


28. Make a sketch of the parametric curves. Then eliminate the parameter to find an equation that relates x and y directly.

* a. $x(t) = \sin t\,,\ y(t) = \cos t$

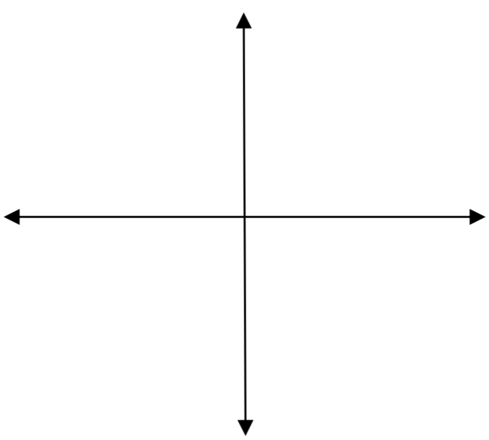


b. $x(t) = \tan t\,,\ y(t) = \sec t,\ \ 0 \le t \le \frac{\pi}{4}$



* 29. Eliminate the parameter to find an equation that relates x and y directly, and <u>then</u> sketch the curve.

$$x(t) = e^{t} + e^{-t}\,, \qquad y(t) = e^{t} - e^{-t}$$



* 30. Find the instantaneous rate of change at any point x (in other words, find the derivative) for the function given by $f(x)=2x^2-x$ using the definition of the derivative below. (No power rule or other shortcuts)

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

* 31. Find the derivative of the following (Power Rule okay to use!!) Leave no negative exponents in your answer.

$$f(x) = x^4 - 10x + 3 - \frac{3}{\sqrt{x}} + \frac{1}{x}$$

____

* 32. Find the equation of the line tangent to the graph of $f(x) = 1 - \frac{x^3}{6}$ at $x = 2$.

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* 33. Given a continuous function f such that $f(0) = -1$ and $f(4) = 6$, which of the following must be true?

a. $f(c) = 0$ for some value of c such that $0 \le c \le 4$

b. $f'(c)$ exists for all values of c, $0 \le c \le 4$

c. Somewhere on the interval $0 \le c \le 4$, $f'(c) = 0$. _______________

34. A rectangular sheet of tin measures 20 inches by 12 inches. Suppose you cut a square out of each corner and fold up the sides to make an open-topped box. What size square should you cut out in order to maximize the volume of the box? Please show your work/justify your answer.

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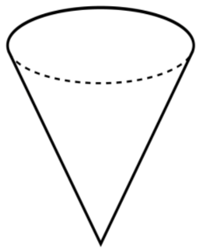
* 35. An inverted conical reservoir has a height of 10 inches and a base diameter of 12 inches. It is slowly being filled with water .Write an expression for the volume of the water in terms of its…

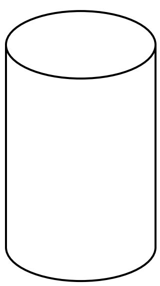
a. …radius



b. …height

(hint: you might want to take advantage of similar triangles)

36. You have been asked to design a cylindrical can that will hold 1000 cubic centimeters. What dimensions (height and radius) will use the least amount of material?



37. Suppose a particle moves along the x-axis with a velocity given by $v(t) = e^{-t}$. Estimate the total distance traveled by the particle on the interval [0, 2] using Riemann Sums with four equally spaced subdivisions.

a. Use LRAM

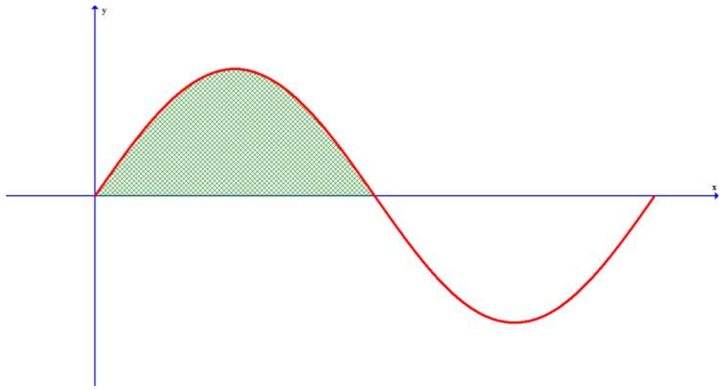
b. Use RRAM

c. Use MRAM

38. Using the same info from problem #37, estimate the TDT by the particle with the Trapezoidal Rule, again with four equally spaced subdivisions.

39. Use your GC to find the actual TDT accurate to three decimal places

* 40. It is a fact that $\int_0^{\pi} \sin x \, dx = 2$. Use this information to help you determine the values of each of the following.



a. $\int_{\pi}^{2\pi} \sin x \, dx$ __________

b. $\int_0^{2\pi} \sin x \, dx$ __________

c. $\int_0^{\pi/2} \sin x \, dx$ __________

d. $\int_0^{\pi} (2 + \sin x) \, dx$ __________

41. Evaluate on your GC: 41. __________

$$4 \int_0^1 \frac{1}{1 + x^2} \, dx$$

* 42. Write each of the following in sigma notation:

a. $\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} + \cdots + \frac{99}{100}$ 42a. __________

b. $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots$ 42b. __________

* 43. Find the sum of the series from #42b 43. __________

* 44. Use partial fractions to write the following as the sum of two separate fractions:

$$\frac{x - 1}{x^2 - 6x - 27}$$

* 45. Make use of Partial Fractions to evaluate the following:

$$\sum_{i=1}^{99} \frac{1}{i(i + 1)}$$