AP Calculus AB

2020-2021 Summer Packet

Welcome to AP Calculus AB! This packet is intended to prepare you for the course by:

- Reviewing prerequisite algebra and pre-calculus skills.
- Covering Chapter 1 of the calculus textbook. (With the amount of curriculum to be covered before the AP Exam in May, we need to cover this review chapter before the school year.)

It is due on the first day of school and worth 20 homework points. During the first two weeks of school, there will be **Summer Packet assessments** covering this review content.

The packet is lengthy, so please start early. While many of the exercises cover basic algebra skills, you will encounter a few tough exercises. If you need assistance completing this packet, please email us and we can schedule a Zoom with you!

Have a wonderful and relaxing summer! We are looking forward to delving into the study and exploration of a branch of mathematics which has been referred to as "one of the supreme accomplishments of the human intellect." Calculus truly is a fascinating course—you will love it!

– Mr. Kemp and Mr. Fusco

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The following formulas and identities will help you complete this packet.

Additionally, students are expected to know ALL of these by memory for the course.

Linear forms: Slope-intercept: $y = mx + b$ Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$ Horizontal line: $y = b$ (slope = 0)

Vertical line: $x = a$ (slope is undefined)

Parallel → Equal slopes Perpendicular → Slopes are opposite reciprocals

Quadratic forms: $y = ax^2 + bx + c$ $y = a(x - h)^2 + k$ $y = a(x - p)(x - q)$

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin(2x) = 2\sin x \cos x$ $\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

Exponential Properties: $x^a \cdot x^b = x^{a+b}$ $(xy)^a = x^a y^a$ $x^0 = 1$ for all $x \neq 0$

$\frac{x^a}{x^b} = x^{a-b}$ $\left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$ $\sqrt[b]{x^n} = x^{n/b}$ $x^{-n} = \frac{1}{x^n}$

Logarithms: $y = \log_a x$ is equivalent to $a^y = x$

Logarithmic Properties: $\log_b mn = \log_b m + \log_b n$ $\log_b \left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \cdot \log_b m$ If $\log_b m = \log_b n$, then $m = n$ $\log_a n = \frac{\log_b n}{\log_b a}$

For #1-10, write an equation for each line in point-slope form.

1. Containing $(4,-1)$ with a slope of $\frac{1}{2}$

2. Crossing the x-axis at $x=-3$ and the y-axis at $y=6$

3. Containing the points $(-6,-1)$ and $(3,2)$

4. Write an equation of a line passing through $(5,-3)$ with an undefined slope.

5. Write an equation of a line passing through $(-4,2)$ with a slope of 0.

6. Write an equation in point-slope form passing through $(0,5)$ with a slope of $\frac{2}{3}$.

7. Write an equation of a line passing through $(2,8)$ that is parallel to $y=\frac{5}{6}x-1$.

8. Write an equation of a line passing through $(4,7)$ that is perpendicular to the y-axis.

9. Write an equation of a line with an x-intercept of $(2,0)$ and a y-intercept of $(0,3)$.

10. Write an equation of a line passing through $(6,-7)$ that is perpendicular to $y=-2x-5$.

For #11-18, solve each equation for x. Note that some equations with have a specific value, but most will have a solution for x in terms of other variables. For example: $x = \frac{a+b}{c}$ *would be a solution.*

11. $x^2 + 3x = 8x - 6$

12. $\frac{2x-5}{x+y} = 3 - y$

13. $3xy + 6x - xz = 12$

14. $A = ax + bx$

15. $cx = vx$

16. $r = t - x(z - y)$

17. $\frac{3+x}{5-x} = 6 + y$

18. $\frac{y+2}{4-x} = 4(2 - z)$

For #19-24, solve each quadratic by factoring.

19. $x^2 - 4x - 12 = 0$

20. $x^2 - 6x + 9 = 0$

21. $x^2 - 9x + 14 = 0$

22. $x^2 - 36 = 0$

23. $9x^2 - 1 = 0$

24. $4x^2 + 4x + 1 = 0$

For #25-29, evaluate the following knowing that $f(x) = 5 - \frac{2x}{3}$ *and* $g(x) = \frac{1}{2}x^2 + 3x$.

25. $f\left(\frac{1}{2}\right) =$

26. $g(-2) =$

27. $f(1) + g(0) =$

28. $f(0) \cdot g(0) =$

29. $\frac{g(-6)}{f(-6)} =$

Recall function composition notation $(f \circ g)(x)$ is the same thing as $f\big(g(x)\big)$.

For #30-39, use $f(x) = x^2 - 1$, $g(x) = 3x$, *and* $h(x) = 5 - x$ *to find each composite function.*

30. $(f \circ g)(x) =$

31. $(g \circ f)(x) =$

32. $(f \circ f)(4) =$

33. $(g \circ h)(-4) =$

34. $\big(f \circ (g \circ h)\big)(1) =$

35. $\big(g \circ (g \circ g)\big)(5) =$

36. $f\big(g(x-1)\big) =$

37. $g\big(f(x^3)\big) =$

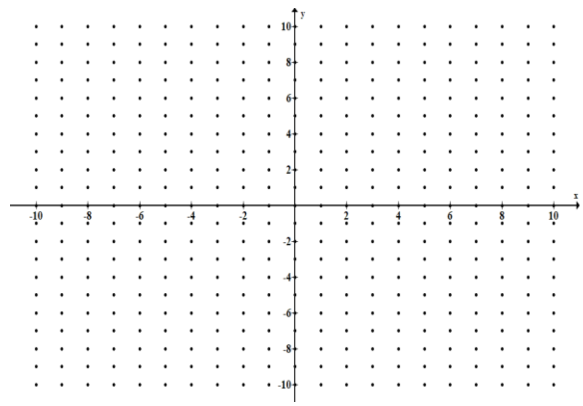
38. $\frac{f(x+h)-f(x)}{h} =$

39. The expression in the previous problem is very significant and important in Calculus. Think back to Pre-Calculus… what is the name of that expression?

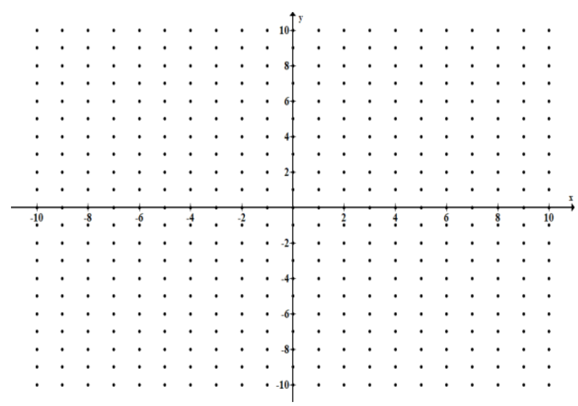
____

For #40-42, graph each piecewise function.

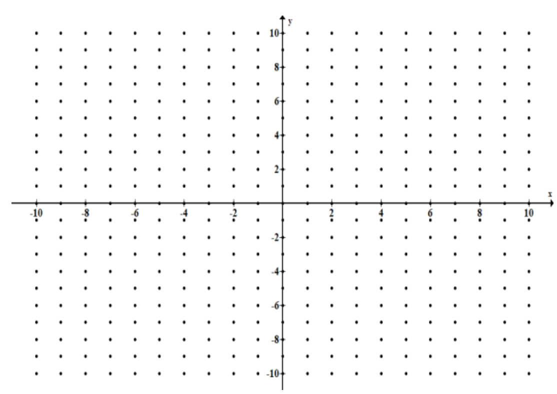
40. $f(x) = \begin{cases} x+3 & ; \ x < 0 \\ -2x+5 & ; \ x \geq 0 \end{cases}$



41. $g(x) = \begin{cases} \frac{1}{2}x & ; \ -4 \leq x \leq 2 \\ 2x-3 & ; \quad x > 2 \end{cases}$



42. $h(x) = \begin{cases} |x| & ; \ x \leq 1 \\ 2-|x-2| & ; \ x > 1 \end{cases}$



An ***exponential equation*** *is an equation in which the variable is in the exponent. To solve an exponential equation, you must use a logarithm to solve it.*

For #43-47, solve each exponential equation, rounding answers to the nearest thousandth. Note that some equations can be solved by writing each side as the same base instead of using a logarithm.

43. $5^x = \frac{1}{5}$

44. $6^x = 1296$

45. $6^{2x-7} = 216$

46. $5^{3x-1} = 49$

47. $10^{x+5} = 125$

For #48-51, simplify each expression without the use of a calculator.

48. $e^{\ln 4} =$

49. $e^{2\ln 3} =$

50. $\ln e^9 =$

51. $5\ln e^3 =$

For #52-57, solve each equation using natural logarithm. Round answers to the nearest thousandth.

52. $e^x = 34$

53. $3e^x = 120$

54. $e^x - 8 = 51$

55. $\ln x = 2.5$

56. $\ln(3x-2) = 2.8$

57. $\ln(e^x) = 5$

For #58-66, find each exact value of the expression using the Unit Circle. ***NO CALCULATOR!***

58. $\sin 120° =$ _________

59. $\cos\frac{11\pi}{6} =$ _________

60. $\tan 225° =$ _________

61. $\sin\left(-\frac{2\pi}{3}\right) =$ ________

62. $\sin 150° =$ _________

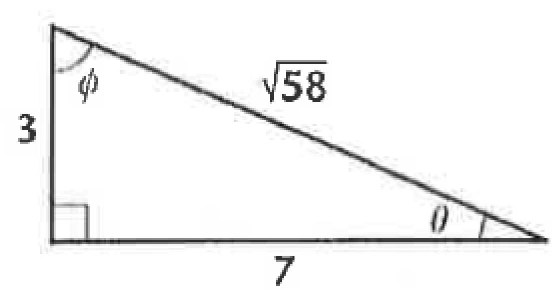
63. $\tan\frac{7\pi}{4} =$ _________

64. $\csc\left(\frac{\pi}{4}\right) =$ ________

65. $\sec(-210°) =$ ______

66. $\cot\left(\frac{5\pi}{4}\right) =$ __________

For #67-74, evaluate each trigonometric expression using the right triangle provided.



67. $\sin\theta =$ _________

68. $\cos\theta =$ _________

69. $\tan\phi =$ _________

70. $\csc\phi =$ _________

71. $\sec\theta =$ _________

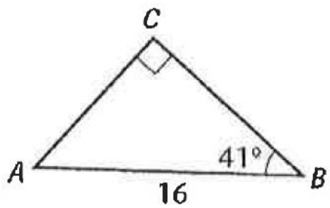
72. $\cot\theta =$ _________

73. $\sin\phi =$ _________

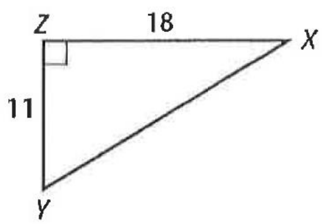
74. $\sec\phi =$ _________

For #75-76, solve each triangle, rounding all angles and sides to the nearest thousandth. Recall that "solve a triangle" means to find all missing sides and angles.

75.



76.



For #77-84, evaluate each inverse trigonometric function. ***NO CALCULATOR!***

77. $\sin^{-1}\left(\frac{1}{2}\right) =$ ________

78. $\sin^{-1}(-1) =$ ________

79. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ ________

80. $\tan^{-1}(\sqrt{3}) =$ ________

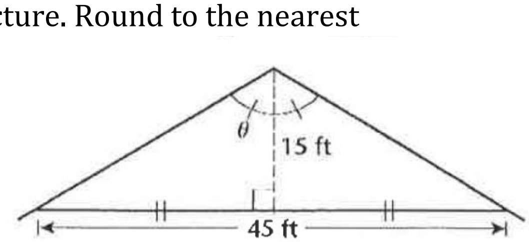
81. $\tan^{-1}(-1) =$ ________

82. $\tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) =$ ________

83. $\sin\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right) =$ _______

84. $\sin^{-1}(\cos(0)) =$ ________

85. Find the angle at the peak of the roof, as shown in the picture. Round to the nearest thousandth.



86. Explain how the graph of $f(x)$ and $f^{-1}(x)$ compare.

Recall that to find an inverse of a function, simply switch the x and y and solve for y. We use the notation $f^{-1}(x)$ to define the inverse of $f(x)$.

For #87-89, find the inverse of each function.

87. $g(x) = \frac{5}{x-2}$

88. $f(x) = \frac{x^2}{3}$

89. $y = \sqrt{4-x} + 1$

90. If the graph of $f(x)$ has the point $(2,7)$, then what is one point on the graph of $f^{-1}(x)$?

*For #91-94, convert the inequalities in to **interval notation**. For example, $x > 3$ becomes $(3, \infty)$.*

91. $1 < x \leq 10$

92. $x < 0$ or $x \geq 4$

93. $x \geq -2$

94. $x \geq 4$ and $x > 10$

For #95-100, find the domain and range of each function. Write the answer in interval notation. Confirm your answer by graphing the function in your graphing calculator.

95. $f(x) = \sqrt{x+5}$

D: ____________________ R: ____________________

96. $f(x) = x^2 - 5$

D: ____________________ R: ____________________

97. $f(x) = \frac{1}{x+7}$

D: ____________________ R: ____________________

98. $f(x) = \frac{5}{x^2+1}$

D: ______________________ R: ______________________

99. $f(x) = \sqrt{x^2+5}$

D: ______________________ R: ______________________

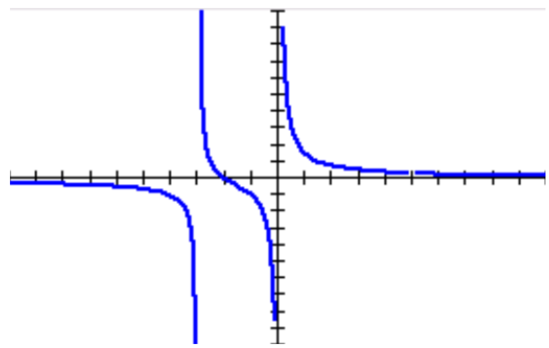
100. $g(x) = x^3 + 2x - 7$

D: ______________________ R: ______________________

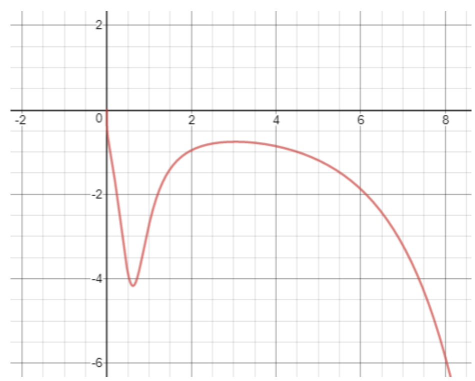
For #101-103, answer the question by referring to the function and its graph.

101. State the domain and range of $f(x) = \frac{2x^2-6x-20}{x^3-2x^2-15x}$

Hint: There is a hole.



102. Consider the function $f(x) = \frac{e^x}{\log x - x^3}$.

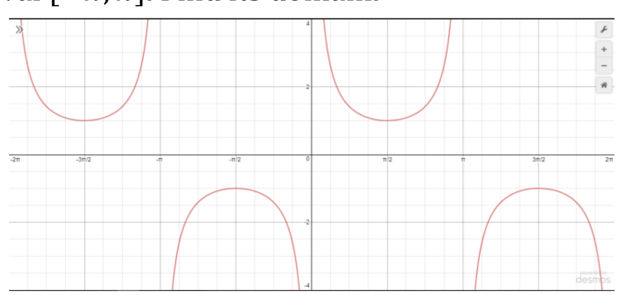


Find the maximum and minimum y-value of the function.

State the domain of $f(x)$.

State when the function is increasing and decreasing (write in interval notation).

103. Consider the function $f(x) = \csc x$ on the interval $[-\pi, \pi]$. Find its domain.



The difference quotient is defined to be $\frac{f(x+h)-f(x)}{h}$ *and is a core concept for the development of calculus. For #104-107, find the difference quotient of each function.*

104. $f(x) = 9x + 3$

105. $f(x) = 5 - 2x$

106. $f(x) = x^2 - 3x$

107. $f(x) = \frac{2}{x+1}$

108. Evaluate the following limits algebraically. If you cannot do it algebraically, view its graph.

$\lim_{x \to 1} e^{x^3 - x} =$

$$\lim_{x \to -3} \frac{x^2 - 9}{x^2 + 2x - 3} =$$

$$\lim_{x \to 5^+} \frac{x+5}{x-5} =$$

$$\lim_{x \to 0} \frac{\sin x}{3x} =$$

$$\lim_{h \to 0} \frac{(h-1)^3 + 1}{h} =$$

$$\lim_{x \to 0} \frac{\sqrt{4+x} - 2}{x} =$$

$$\lim_{x \to c} x =$$

$$\lim_{x \to a} c =$$

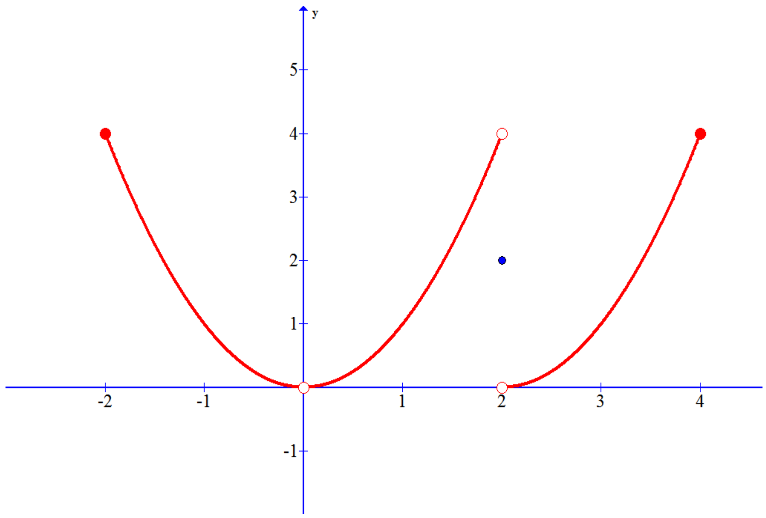
$$\lim_{h \to 0} \frac{h}{\sqrt{x+h} - \sqrt{x}} =$$

$$\lim_{x \to \infty} \frac{3x^3 + 5x^2 - 7x}{8x^3 - 13} =$$

$$\lim_{v \to 4^+} \frac{4-v}{|4-v|} =$$

$$\lim_{x \to \infty} \frac{\sqrt{x^2 - 9}}{2x - 6} =$$

109. Use the graph of the function to answer the following questions. Be as specific as possible



a. $f(2) = ?$ ________

b. $\lim_{x \to 2^+} f(x) = ?$ ________

c. $f(0) = ?$ ________

d. $\lim_{x \to 0} f(x) = ?$ ________

e. $f'(2) = ?$ ________

110. A particle moves along the y-axis. Its position at time t is given by $s(t) = 2t^3 - 4t + 1$.

a. What is the position of the particle at time $t = 2$?

b. At what time(s), if any, is the velocity of the particle zero? Explain how you found your answer(s).

c. What was the average velocity of the particle on the interval $[-1,2]$?

d. Find a formula for the **instantaneous velocity** of the particle at any time t.
Hint: derivative

111. Find the instantaneous rate of change at any point x (in other words, find the derivative) for the function given by $f(x) = 2x^2 - x$ using the definition of the derivative below.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

112. Find the derivative of the following—the ***Power Rule*** okay to use! Leave no negative exponents in your answer.

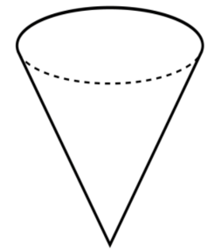
$f(x) = x^4 - 10x + 3 - \frac{3}{\sqrt{x}} + \frac{1}{x}$

113. Find the equation of the line tangent to the graph of $f(x) = 1 - \frac{x^3}{6}$ at $x = 2$.

114. A rectangular sheet of tin measures 20 inches by 12 inches. Suppose you cut a square out of each corner and fold up the sides to make an open-topped box. What size square should you cut out in order to maximize the volume of the box? Please show your work/justify your answer.

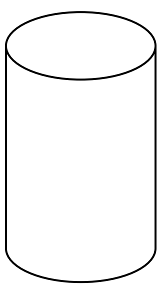
115. An inverted conical reservoir has a height of 10 inches and a base diameter of 12 inches. It is slowly being filled with water .Write an expression for the volume of the water in terms of its… (*Hint:* you might want to take advantage of similar triangles)

a. …radius



b. …height

116. You have been asked to design a cylindrical can that will hold 1000 cubic centimeters. What dimensions (height and radius) will use the least amount of material?



For #117-121, these problems were taken from problems that we will cover this coming year. Just assume we completed all of the calculus steps up to this point. Now, you need to solve each equation.

117. $\frac{\sqrt{\frac{1}{4}+x^2}}{2} = \frac{2-x}{3}$

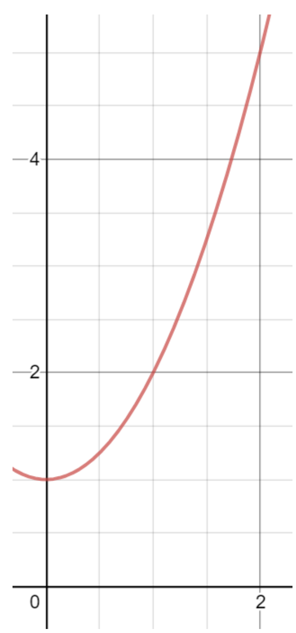
118. $2 - \frac{1800}{x^2} = 0$

119. $-6000x^{-2} + \frac{32}{3}\pi x = 0$

120. Solve for y: $\frac{-1}{y-1} = \frac{1}{\pi}\sin(\pi x) + 1$

121. Solve for y: $\ln y = \sin x + c$

122. Estimate the area under the curve $y = x^2 + 1$ on the interval [0,2] using Riemann Sums with four equally-spaced subdivisions with heights determined by:



a. Use **L**RAM (**left endpoint**)

b. Use **R**RAM (**right endpoint**)

c. Use **M**RAM (**midpoint**)