Brockton High School
Content: Math III
Week of: May 18 to May 24, 2020
Greetings Math III Students! We hope you are safe and well with your families!
This assignment is for the week, use your time wisely. You do not have to complete this in one sitting. Here is the lesson plan for this week:

Goals for this week

Learning Objectives:
Students will be able to:
1) Recognize the standard, factored, and vertex forms of quadratic functions.
2) Solve quadratic equations using factoring or the quadratic formula.
3) Transform one form of a quadratic function to another form.
4) Rewrite square roots of negative numbers and powers of i.
5) Identify, add, subtract, multiply, and divide complex numbers.
6) Identify and multiply complex conjugates.

(Standards A. REI, F. IF, F. IB)

Literacy Objectives:
Students will be able to:
1. to create, interpret and explain a table, chart or graph
2. to compute, interpret and explain numbers
3. to generate a response from what one has read, viewed or heard.

(https://www.bpsma.org/schools/brockton-high-school/about-us/mission-literacy-charts)

Carnegie Learning (use with Carnegie Resources provided below)

Carnegie Learning (Log-in through Clever)

Instructional Video Links: Please watch at least one of these videos to help guide you.
Module 2 Unit 3: Forms of Quadratic Functions

1. https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:quadratic-forms-features/v/rewriting-a-quadratic-function-to-find-roots-and-vertex

2. .https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:transformations/x2ec2f6f830c9fb89:trans-all-together/v/shifting-and-reflecting-functions
3. .https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:transform-quadratic-functions/v/example-translating-parabola

4. https://www.youtube.com/watch?v=VdVPg04t_6w
5. https://www.khanacademy.org/math/algebra-home/alg-quadratics/alg-features-of-quadratic-functions/v/quadratic-functions-2?modal=1

Module 2 Unit 4: Operations with Complex Numbers
1. https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:complex/x2ec2f6f830c9fb89:complex-num/v/complex-number-intro
2. https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:complex/x2ec2f6f830c9fb89:imaginary/v/imaginary-roots-of-negative-numbers
3.https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:complex/x2ec2f6f830c9fb89:complex-add-sub/v/adding-complex-numbers

Feel free to also search for your own tutorial videos to guide you. Your teacher will also provide you with a list of instructional videos for this week.

Printable Resources for Module 2 Week 2: : optional resources

The Root of a Problem pages M1-79 through M1 – 90

Key Terms

- **Standard form of a quadratic function:** The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where *a*, *b* and *c* are real numbers and a is not equal to 0.
- **Quadratic formula:**: A formula used to find the solutions of a quadratic equation. For a quadratic equation in the form, $ax^2 + bx + c = 0$, the solutions can be found by substituting the values for *a, b* and *c* in the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- **Vertex form of a quadratic function:** The vertex form of a quadratic equation is y = *a* $(x - h)^2$ + k, where *a, h* and *k* are real numbers and *a* is not equal to zero. The coordinates of the vertex are *(h, k*).
- **Imaginary unit i:** The imaginary unit ***i*** is equal to $\sqrt{-1}$
- **Complex number:** a number that can be written in the form a + bi, where a and b are real numbers and I is the imaginary unit. The number a is the real part and the number b is the imaginary part.
- **Imaginary number:** A complex number that can be written as a real number multiplied by the imaginary unit i.
- **Complex conjugate:** Pairs of complex numbers of the form *a* + *bi* and *a* – *bi*. The product of complex conjugates is a real number.

Practice Activities:

On-Line:

All students now have access to an on-line program called **Mathia(Carnegie Learning)**

- If you are already in Mathia, please continue to work in the program. **Please finish through Module 2 Unit 4 by the end of the week.**
- If you are new to Mathia: Please see the log-in information below and **begin working on Modules 1 and 2. Please finish through Module 2 Unit 4 by the end of the week.**

Extension Activities:

Complete the **REVIEW activity** on the Printable Resource form page M1 – 92 and send a copy to your teacher.

Log-in Information

1. Log-in to Clever
2. Under Math (in the left hand column) , click on Carnegie Learning (MATHia)
3. Username: 6-digit BHS school ID # @bpsma.org
   Password: Date of birth bps 1920
   Example: Student with ID #:123456
   Date of birth: January 1st, 2000
   Username: 123456@bpsma.org
   Password: 01012000bps1920

Additional Support

- Please email your math teacher with specific questions.
- Here is a list of math teachers' office hours:
- https://brocktonpublicschools-my.sharepoint.com/:x:/g/personal/danielcorbett_bpsma_org/EWk_ij9UwjpPtRAHBUkEpS4B3vue-_IG8VYz0AwG9ovJjQ?e=4%3arkcL1r&at=9&CT=1588698277992&OR=OWA-NT&CID=8d0078f2-9a27-460f-e632-64578875ee60

5

The Root of the Problem

Solving Quadratic Equations

Warm Up

Use the Distributive Property to determine each product.

1. $(x + 1)(x + 2)$

2. $(x + 4)(x - 5)$

3. $(2x - 3)(x - 4)$

4. $(x + 2)^2$

Learning Goals

- Factor quadratic trinomials to determine the roots of quadratic equations and to rewrite quadratic functions in forms that reveal different key characteristics.
- Complete the square to determine the roots of quadratic equations of the form $ax^2 + bx + c$.
- Use the Quadratic Formula to determine roots and zeros.
- Solve systems of two quadratic equations.

Key Term

- Quadratic Formula

You have analyzed the different structures of quadratic equations. How can the structure of a quadratic equation help you determine a solution strategy? Is there a single strategy that works to solve any quadratic equation?

GETTING STARTED

Grassroots

You know how to use the Properties of Equality to solve equations in the forms shown.

$$y = x^2 + d$$
$$y = (x - c)^2$$
$$y = a(x - c)^2$$
$$y = a(x - c)^2 + d$$

1. Use Properties of Equality to solve each equation. State the property used in each step of your solution.

a. $27 = x^2 - 9$

b. $(x + 3)^2 = 121$

c. $48 = 3(x - 1)^2$

d. $\frac{1}{2}(x + 5)^2 - 18 = 0$

2. Describe the strategy Oscar used to solve part (a) in Question 1.

Oscar

$$27 = x^2 - 9$$
$$27 - 27 = x^2 - 9 - 27$$
$$0 = x^2 - 36$$
$$0 = (x - 6)(x + 6)$$

$(x - 6) = 0$ and $(x + 6) = 0$

$x = 6$ and $x = -6$

ACTIVITY 5.1

Solving Quadratic Equations by Factoring

Let's consider strategies to solve quadratics in the form $y = ax^2 + bx + c$ using the factoring strategies you have learned.

Worked Example

You can use factoring to calculate the roots for the quadratic equation $x^2 - 4x = -3$.

$$x^2 - 4x = -3$$
$$x^2 - 4x + 3 = -3 + 3$$
$$x^2 - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$

$(x - 3) = 0$	and	$(x - 1) = 0$
$x - 3 + 3 = 0 + 3$	and	$x - 1 + 1 = 0 + 1$
$x = 3$	and	$x = 1$

Remember:

The Zero Product Property states that if the product of two or more factors is equal to zero, then at least one factor must be equal to zero.

1. Why is 3 added to both sides in the first step of the worked example?

Think about:

What is the connection between the worked example and determining the roots from factored form, $y = a(x - r_1)(x - r_2)$?

2. Determine each student's error and then solve each equation correctly.

Angela

$x^2 + 6x = 7$

$x(x + 6) = 7$

$x = 7$ and $x + 6 = 7$

$x = 1$

Dwight

$x^2 + 5x + 6 = 6$

$(x + 2)(x + 3) = 6$

$x + 2 = 6$ and $x + 3 = 6$

$x = 4$ and $x = 3$

3. **Use factoring to solve each quadratic equation, if possible.**

a. $x^2 - 8x + 12 = 0$

b. $x^2 + 8x = -7$

c. $\frac{2}{3}x^2 - \frac{5}{6}x = 0$

d. $f(x) = x^2 + 10x + 12$

Think about:

What efficiency strategies did you use to solve linear equations with fractional coefficients?

4. **Describe the different strategies and reasoning that Jim and Pam used to solve $4x^2 - 25 = 0$.**

Jim

$$4x^2 - 25 = 0$$
$$4x^2 = 25$$
$$x^2 = \frac{25}{4}$$
$$x = \pm\sqrt{\frac{25}{4}}$$
$$x = \pm\frac{5}{2}$$

Pam

$$4x^2 - 25 = 0$$
$$(2x - 5)(2x + 5) = 0$$
$2x - 5 = 0$ and $2x + 5 = 0$

$2x = 5 \qquad 2x = -5$

$x = \frac{5}{2}$ and $\quad x = -\frac{5}{2}$

ACTIVITY
5.2

Completing the Square to Determine Roots

You can use the completing the square method to determine the roots of a quadratic equation that cannot be factored.

Worked Example

Complete the square to determine the roots of the equation $x^2 + 10x + 12 = 0$.

Isolate $x^2 + 10x$.

$$x^2 + 10x + 12 - 12 = 0 - 12$$
$$x^2 + 10x = -12$$

Determine the constant term that would complete the square. Add this term to both sides of the equation.

$$x^2 + 10x + \_\_\_\_ = -12 + \_\_\_\_$$
$$x^2 + 10x + 25 = -12 + 25$$
$$x^2 + 10x + 25 = 13$$

Rewrite the left side as a perfect square.

$$(x + 5)^2 = 13$$

Take the square root of each side of the equation.

$$\sqrt{(x + 5)^2} = \pm\sqrt{13}$$
$$x + 5 = \pm\sqrt{13}$$

Set the factor of the perfect square trinomial equal to each square root of the constant. Then solve for x.

$$x + 5 = \sqrt{13} \quad \text{and} \quad x + 5 = -\sqrt{13}$$
$$x = -5 + \sqrt{13} \quad \text{and} \quad x = -5 - \sqrt{13}$$
$$x \approx -1.39 \quad \text{and} \quad x \approx -8.61$$

The roots are approximately -1.39 and -8.61.

Ask yourself:

How was equality of the equation maintained through the completing the square process?

1. **Complete the square to determine the roots of each equation.**

   a. $x^2 - 6x + 4 = 0$

   b. $x^2 - 12x + 6 = 0$

2. **A ball is thrown straight up from 4 feet above the ground with a velocity of 32 feet per second. The height of the ball over time can be modeled with the function $h(t) = -16t^2 + 32t + 4$. What is the maximum height of the ball?**

3. **Jessie is fencing in a rectangular plot outside of her back door so that she can let her dogs out to play. She has 60 feet of fencing and only needs to place it on three sides of the rectangular plot because the fourth side will be bound by her house. What dimensions should Jesse use for the plot so that the maximum area is enclosed? What is the maximum area? Draw a diagram to support your work.**

ACTIVITY
5.3

Using the Quadratic Formula

The **Quadratic Formula**, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, can be used to calculate the solutions to any quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c represent real numbers and $a \neq 0$.

Worked Example

You can use the Quadratic Formula to determine the zeros of the function $f(x) = -4x^2 - 40x - 99$.

Rewrite the function as an equation to be solved for x when $y = 0$.

$$-4x^2 - 40x - 99 = 0$$

Determine the values of a, b, and c.

$$a = -4,\ b = -40,\ c = -99$$

Substitute the values into the Quadratic Formula.

$$x = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(-4)(-99)}}{2(-4)}$$

Perform operations to rewrite the expression.

$$x = \frac{40 \pm \sqrt{1600 - 1584}}{-8}$$

$$x = \frac{40 \pm \sqrt{16}}{-8}$$

$$x = \frac{40 \pm 4}{-8}$$

$$x = \frac{40 + 4}{-8} \quad \text{and} \quad x = \frac{40 - 4}{-8}$$

$$x = \frac{44}{-8} \quad \text{and} \quad x = \frac{36}{-8}$$

$$x = -5.5 \quad \text{and} \quad x = -4.5$$

The zeros of the function $f(x) = -4x^2 - 40x - 99$ are $x = -5.5$ and $x = -4.5$.

The Seaside Serpents baseball team has a new promotional activity to encourage fans to attend games: launching free T-shirts! They can launch a T-shirt in the air with an initial velocity of 91 feet per second from $5\frac{1}{2}$ feet off the ground (the height of the team mascot).

A T-shirt's height can be modeled with the quadratic function $h(t) = -16t^2 + 91t + 5.5$, where t is the time in seconds and $h(t)$ is the height of the launched T-shirt in feet. They want to know how long it will take for a T-shirt to land back on the ground after being launched (if no fans grab it before then!).

Ask yourself:

What would a sketch showing the height of the T-shirt over time look like?

1. **Why does it make sense to use the Quadratic Formula to solve this problem?**

2. **Use the Quadratic Formula to determine how long it will take for a T-shirt to land back on the ground after being launched.**

Ask yourself:

Do you think an exact solution or approximate solution is more appropriate for this context?

3. **Meredith is solving the quadratic equation $x^2 - 7x - 8 = 3$. Her work is shown.**

   a. **Identify Meredith's error.**

   b. **Determine the solution to Meredith's quadratic equation.**

Meredith

$x^2 - 7x - 8 = 3$

$a = 1, b = -7, c = -8$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 + 32}}{2}$$

$$x = \frac{7 \pm \sqrt{81}}{2}$$

$$x = \frac{7 \pm 9}{2}$$

$$x = \frac{7 + 9}{2} \text{ or } x = \frac{7 - 9}{2}$$

$$x = \frac{16}{2} \text{ or } x = \frac{-2}{2} = -1$$

The roots are 8 and -1.

4. **Use the Quadratic Formula to determine the zeros for each function. Round the solutions to the nearest hundredth.**

   a. $f(x) = 2x^2 + 10x - 1.02$

   b. $h(x) = 3x^2 + 11x - 2$

ACTIVITY **5.4**

Solving Systems of Quadratic Equations

You previously solved a system of a linear equation and a quadratic equation using methods similar to solving a system of linear equations. You can use these same methods to solve a system of two quadratic equations.

1. Consider the system of two quadratic equations.

$$\begin{cases} y = x^2 + 3x - 5 \\ y = -x^2 + 10x - 1 \end{cases}$$

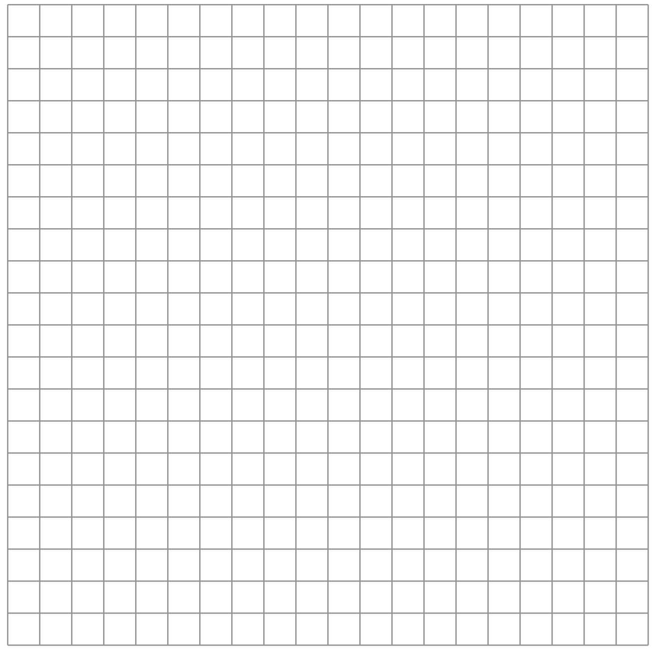
a. Set the expressions equal to each other.

b. Solve the resulting equation for *x*.

c. Calculate the corresponding values for *y*.

d. What is the solution to the system of equations?

e. **Graph each equation of the system and calculate the points of intersection.**

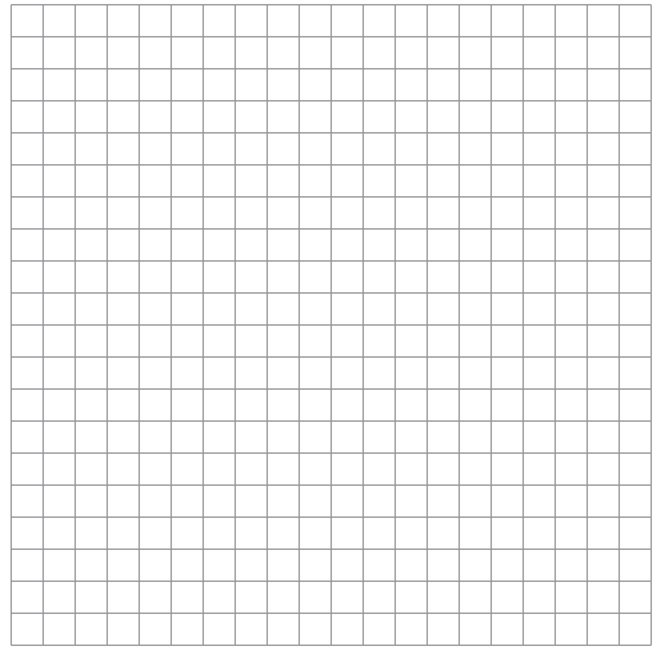


f. **What do you notice about the solutions you calculated algebraically and graphically?**

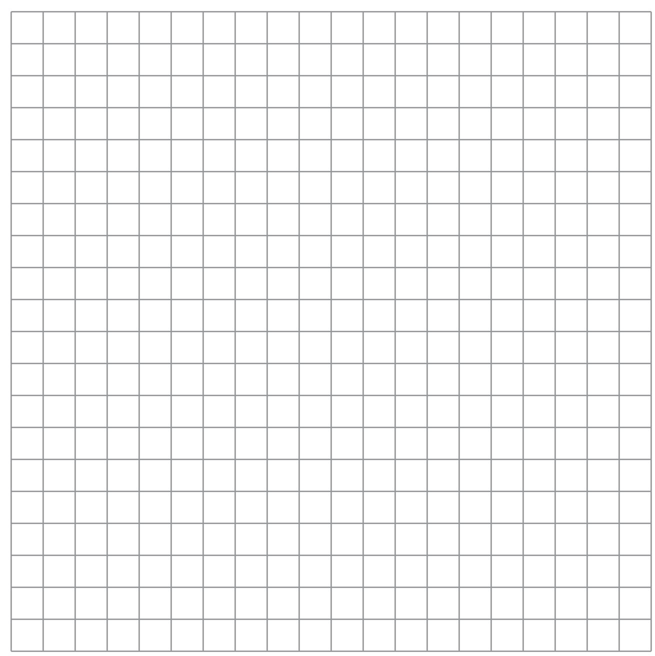
2. **Think about the graphs of two quadratic equations. Describe the different ways in which the two graphs can intersect and provide a sketch of each case.**

3. Solve each system of equations algebraically over the set of real numbers. Then verify the solution graphically.

a. $\begin{cases} y = x^2 + 2x + 1 \\ y = 2x^2 - x - 3 \end{cases}$



b. $\begin{cases} y = 2x^2 - 7x + 6 \\ y = -2x^2 + 5x - 3 \end{cases}$



NOTES

TALK the TALK

Show Me the Ways

1. **Determine the real roots of the quadratic equation $y = 2x^2 + 4x - 6$ using each method.**

Factoring

Completing the Square

$y = 2x^2 + 4x - 6$

Using the Quadratic Formula

Graphing

Assignment

Write

Describe the solution to a system of equations.

Remember

You can solve quadratic equations using factoring, completing the square, the Quadratic Formula, and graphing.

Practice

1. Solve each equation.
   a. $0 = x^2 - 7x - 18$
   b. $x^2 + 10x = 39$
   c. $0 = x^2 - 10x + 12$
   d. $2x^2 + 4x = 0$
   e. $3x^2 - 22x + 7 = 0$
2. Determine the roots of each equation. Check your solutions.
   a. $y = x^2 + 9x + 3$
   b. $y = 3x^2 + 24x - 6$
3. Kian is driving 48 miles per hour and is speeding up to merge onto the highway. He gradually accelerates at a rate of 7 miles per hour for several seconds. The formula $s = ut + \frac{1}{2}at^2$ can be used to calculate the distance, s, an object travels in t seconds. In this formula, u represents the initial velocity, and a represents a constant acceleration.
   a. Substitute the initial velocity and constant acceleration into the formula to write an equation to represent the distance Kian travels.
   b. Use the Quadratic Formula to determine the roots of the equation. What do the roots represent in the context of the problem situation? Explain your reasoning.
4. Determine the solution to each system of equations
   a. $\begin{cases} y = x^2 - 4x - 8 \\ y = -x^2 - 10x + 12 \end{cases}$
   b. $\begin{cases} y = 2x^2 + 3x - 2 \\ y = -2x^2 + 7x - 3 \end{cases}$

Stretch

The function g is defined by $g(x) = x^2 - 3x - 10$. If $g(x + 3) = x^2 + bx - c$, what are the values of b and c? Show your work and justify your answer.

Review

1. Consider the function $f(x) = (x + \frac{1}{2})(x - \frac{3}{4})$.
   a. Identify the form of the function as factored, general, or vertex.
   b. Identify the zeros and axis of symmetry of the function.
2. Write a quadratic equation for the parabola that passes through the point $(-2, 12)$ with roots $(-5, 0)$ and $(-3, 0)$.
3. Solve each equation for the unknown value.
   a. $|2x + 3| = 25$
   b. $9 = |-3x - 1| - 7$