Brockton High School Content: Math II Week of: May 18 to May 24, 2020



Greetings Math II Students! We hope you are safe and well with your families! This assignment is for the week, use your time wisely. **You do not have to complete this in one sitting**. Here is the lesson plan for this week:

Goals for this week: Module 3 on Mathia

Learning Objectives:

Students will be able to:

1. Investigate proportionality using trigonometric ratios, arc length of circles, and volume of 3D shapes.

(Standards G.C.1, G.C.5, G.SRT.6, G.SRT.8)

Literacy Objectives:

Students will be able to:

- 1. to compute, interpret and explain numbers
- 2. to read, break down, and solve a word problem
- 3. to explain and/or interpret relationships of space and time

(https://www.bpsma.org/schools/brockton-high-school/about-us/mission-literacy-charts)

Carnegie Learning (use wi	th Carnegie Resources provided below)			
Instructional Video Links: Please watch at least one of these videos to help guide you. Introduction to Trigonometry Finding sides using trig ratios Finding angles using trig ratios Finding arc length of a circle Volume of cylinders, Cones, and Spheres Volume and Surface Area of Pyramids	 <u>og-in through Clever</u>) <u>Printable Resources:</u> Skills Practice: Module 2 topic 2 Trigonometry Module 2 topic 3 Building Volume and Surface Area formulas for Pyramids, cones, and Spheres\ Module 2 Topic 3 All circles great and small See Below 			
 Practice Activities: <u>On-Line:</u> All students now have access to an on-line program called Mathia! Mathia- If you are already in Mathia, please continue to work in the program. If you are new to Mathia: Please see the log-in information below. 	 Key Terms: Sine: The ratio of the opposite side over the hypotenuse Cosine: The ratio of the adjacent side over the hypotenuse Tangent: The ratio of the opposite side over the adjacent side Unit circle: A circle whose radius is 1 unit Arc length: A portion of a circles circumference (circumference) (measure of arc 360) Circumference: The distance around a circle(2πr) 			

Radian: Unit used to measure angles (one radian=
_ <u>180</u> π degrees
 Sector: A portion of a circle bounded by two radii and one of the arcs they intercept
 Volume: The number of cubic units that will completely fill the interior of a solid
 Surface area: The sum of all the areas of the polyhedron's faces
• Lateral Face: A face of a polyhedron that is not part of the base
Height: The perpendicular distance between the two bases
 Vertex of a solid: A point where multiple edges meet
Radius: The distance from the center of a circle to a point on the circle

Extension Activities:

Stretch

A gumball machine is shown in the figure. The diameter of the sphere with gumballs on the top of the machine is 12 inches. The cylinder with gumballs under the sphere is 6 inches in diameter and 7 inches tall. If a gumball is 1 inch in diameter, approximately how many gumballs will fit in this machine? Use 3.14 for π and assume that due to the spacing of the gumballs, only 75% of the total capacity for gumballs will be used.



Log-in Information

- 1. Log-in to Clever
- 2. Under Math, click on MATHia
- Username: 6-digit BHS school ID # @bpsma.org Password: Date of birth bps 1920 Example: Student (Michael) with ID #:123456 Date of birth: January 1st, 2000 Username: <u>123456@bpsma.org</u> Password: 01012000bps1920

Additional Support

<u>Email:</u>

• Please email your math teacher with specific questions.

Office Hours:

<u>Click here for teacher Office hours!</u>

Carnegie Learning Resource

The Tangent Ratio

TRIGONOMETRIC RATIOS

Definition:

In a right triangle, the tangent of an angle is equal to the ratio of the length of the side opposite the angle to the length of the side adjacent to the angle.

Example:

In triangle *ABC*, the length of the side opposite angle *A* is 6 inches, and the length of the adjacent to angle *A* is 8 inches. Therefore, the tangent of angle *A*, or tan *A*, is

 $\frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A} = \frac{6 \text{ in.}}{8 \text{ in.}} = \frac{3}{4}.$



Definition:

In a right triangle, the sine of an angle is equal to the ratio of the length of the side opposite the angle to the length of the hypotenuse.

Example:

In triangle *ABC*, the length of the side opposite angle *A* is 6 inches, and the length of the hypotenuse is 10 inches. Therefore, the sine of angle *A* or sin *A*, is

 $\frac{\text{length of side opposite to } \angle A}{\text{length of hypotenuse}} = \frac{6 \text{ in.}}{10 \text{ in.}} = \frac{3}{5}.$

The Cosine Ratio

Example:

In triangle *ABC*, the length of the side adjacent to angle *A* is 8 inches, and the length of the e^{-ent} to hypotenuse is 10 inches. Therefore, the cosine of angle *A*, or cos *A*, is

 $\frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{8 \text{ in.}}{10 \text{ in.}} = \frac{4}{5}.$



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A helpful acronym for remembering the definitions of the trigonometric ratios is:



These trigonometric ratios can also be used to find missing side and angle measures in a right triangle.



Skills Practice

Name.

Topic 2

TRIGONOMETRY

Date _____

I. Calculating Trigonometric Ratios

Calculate the tangent of the indicated angle in each triangle. Write your answers in simplest form.



Calculate the sine of the indicated angle in each triangle. Write your answers in simplest form.



Calculate the cosine of the indicated angle in each triangle. Write your answers in simplest form.





3.







5.





II. Determining Unknown Lengths Using Trigonometric Ratios

Calculate the missing length of each triangle. Round your answers to the nearest hundredth.



B. Solve each problem. Round your answers to the nearest hundredth.

 A boat travels in the following path. How far north did it travel?



 During a group hike, a park ranger makes the following path. How far west did they travel?



III. Determining Unknown Angle Measures Using Trigonometric Ratios

Calculate the measure of angle X for each triangle. Round your answers to the nearest hundredth.



Solve each problem. Round your answers to the nearest hundredth.

 A moving truck is equipped with a ramp that extends from the back of the truck to the ground. When the ramp is fully extended, it touches the ground 12 feet from the back of the truck. The height of the ramp is 2.5 feet. Calculate the measure of the angle formed by the ramp and the ground.



3. A lifeguard is sitting on an observation chair at a pool. The lifeguard's eye level is 6.2 feet from the ground. The chair is 15.4 feet from a swimmer. Calculate the measure of the angle formed when the lifeguard looks down at the swimmer.



 A park has a skateboard ramp with a length of 14.2 feet and a length along the ground of 12.9 feet. The height is 5.9 feet. Calculate the measure of the angle formed by the ramp and the ground.



 A surveyor is looking up at the top of a building that is 140 meters tall. His eye level is 1.4 meters above the ground, and he is standing 190 meters from the building. Calculate the measure of the angle from his eyes to the top of the building.



Table of Trigonometric Ratios

Angle	Sine	Cosine	Tangent	Angle	Sine	Cosine	Tangent
1°	.0175	.9998	.0175	46°	.7193	.6947	1.0355
2°	.0349	.9994	.0349	47°	.7314	.6820	1.0724
3°	.0523	.9986	.0524	48°	.7431	.6691	1.1106
4°	.0698	.9976	.0699	49°	.7547	.6561	1.1504
5°	.0872	.9962	.0875	50°	.7660	.6428	1.1918
6 °	.1045	.9945	.1051	51°	.7771	.6293	1.2349
7 °	.1219	.9925	.1228	52°	.7880	.6157	1.2799
8°	.1392	.9903	.1405	53°	.7986	.6018	1.3270
9.	.1564	.9877	.1584	54°	.8090	.5878	1.3764
10°	.1736	.9848	.1763	55	.8192	.5736	1.4281
11°	.1908	.9816	.1944	56°	.8290	.5592	1.4826
12°	.2079	.9781	.2126	57°	.8387	.5446	1.5399
13°	.2250	.9744	.2309	58°	.8480	.5299	1.6003
14	.2419	.9703	.2493	59°	.8572	.5150	1.6643
15"	.2588	.9659	.2679	60~	.8660	.5000	1.7321
16°	.2756	.9613	.2867	61°	.8746	.4848	1.8040
17°	.2924	.9563	.3057	62°	.8829	.4695	1.8807
18°	.3090	.9511	.3249	63°	.8910	.4540	1.9626
19°	.3256	.9455	.3443	64°	.8988	.4384	2.0503
20°	.3420	.9397	.3640	65°	.9063	.4226	2.1445
21°	.3584	.9336	.3839	66°	.9135	.4067	2.2460
22°	.3746	.9272	.4040	67°	.9205	.3907	2.3559
23°	.3907	.9205	.4245	68°	.9272	.3746	2.4751
24°	.4067	.9135	.4452	69°	.9336	.3584	2.6051
25°	.4226	.9063	.4663	70°	.9397	.3420	2.7475
26°	.4384	.8988	.4877	71 °	.9455	.3256	2.9042
27°	.4540	.8910	.5095	72°	.9511	.3090	3.0777
28°	.4695	.8829	.5317	73°	.9563	.2924	3.2709
29°	.4848	.8746	.5543	74°	.9613	.2756	3.4874
30°	.5000	.8660	.5774	75°	.9659	.2588	3.7321
31°	.5150	.8572	.6009	76°	.9703	.2419	4.0108
32°	.5299	.8480	.6249	77°	.9744	.2250	4.3315
33°	.5446	.8387	.6494	78°	.9781	.2079	4.7046
34°	.5592	.8290	.6745	79°	.9816	.1908	5.1446
35"	.5736	.8192	.7002	80°	.9848	.1736	5.6713
36°	.5878	.8090	.7265	81°	.9877	.1564	6.3138
37°	.6018	.7986	.7536	82°	.9903	.1392	7.1154
38°	.6157	.7880	.7813	83°	.9925	.1219	8.1443
39°	.6293	.7771	.8098	84°	.9945	.1045	9.5144
40°	.6428	.7660	.8391	85°	.9962	.0872	11.4301
41°	.6561	.7547	.8693	86°	.9976	.0698	14.3007
42°	.6691	.7431	.9004	87°	.9986	.0523	19.0811
43°	.6820	.7314	.9325	88°	.9994	.0349	28.6363
44°	.6947	.7193	.9657	89°	.9998	.0175	57.2900
45°	.7071	.7071	1.0000				

activity **1.2**

Determining Arc Length



Consider the large gear represented by circle *O*, containing a central angle, $\angle AOB$, whose measure is equal to 60°; a minor arc, \widehat{AB} ; and a major arc, \widehat{ACB} , as shown. Consider the small gear represented by circle *E*, containing a central angle, $\angle FEG$, whose measure is equal to 60°; a minor arc, \widehat{FG} ; and a major arc, \widehat{FDG} , as shown.

1. Compare the two gears.



- a. Is the length of the radii in the large gear proportional to the length of the radii in the small gear? Explain your reasoning.
- b. Determine the degree measure of the minor arc in each circle.
- c. What is the ratio of the degree measure of the minor arc to the degree measure of the entire circle for each of the two gears?
- 2. Explain why Casey is incorrect.

Casey

The two minor arcs, \widehat{AB} and \widehat{FG} , on the gears have the same measure, which is 60°. So, the two arcs are the same length.

In a circle, the degree measure of an arc is equal to the measure of the central angle. **Arc length** is a linear measurement that represents a portion of the circumference of a circle. To determine the arc length of the minor arc, you need to know the circumference of the circle, which requires knowing the radius of the circle.

The length of an arc is different from the degree measure of the arc.

- 3. If the length of the radius of the large gear, or *OB*, is equal to 4 centimeters, determine the circumference of circle *O*.
- 4. Use the circumference of circle *O* determined in Question 3 and the ratio determined in Question 1, part (c) to solve for the length of the minor arc.
- 5. If the length of the radius of the small gear, or \overline{EF} , is equal to 2 centimeters, determine the circumference of circle *E*.
 - 6. Use the circumference of circle *E* determined in Question 5 and the ratio determined in Question 1, part (c) to solve for the length of the minor arc.

To determine arc length, s, you multiplied the circumference of the circle by a fraction representing the portion of the circumference determined by the central angle measure, m.

arc length =
$$\frac{\text{central angle measure}}{360^{\circ}}$$
 (circumference)
 $s = \frac{m}{360^{\circ}} (2\pi r)$

There is a proportional relationship between the measure of an arc length and the circumference of the circle.

> $\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle measure}}{360^{\circ}}$ $\frac{s}{2\pi r} = \frac{m}{360^\circ}$

Worked Example

You can apply the formula $s = \frac{m}{360^{\circ}} (2\pi r)$ to determine the length of an arc for a circle with a radius of 10 inches and central angle of 80°.



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7. Determine the arc length of an entire circle with each given radius.

a. 1 inch

b. r units

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 $s = \frac{m}{360^\circ} \cdot 2\pi r$ $s = \frac{m}{180^{\circ}} \cdot \pi r$ $\frac{s}{r} = \frac{\pi}{180^\circ} \cdot m$

- The relationship also implies that the ratio of the arc length to the radius, $\frac{s}{r}$, is directly proportional to m, the measure of the central angle.
- 8. What is the constant of proportionality in this proportional relationship?

Arc length is directly

proportional to the circumference.

9. Use the formula that compares the ratio $\frac{s}{r}$ to calculate the length of the minor arc *AB* in each circle. Express your answer in terms of π .





d. Compare the worked example and part (a), as well as parts (b) and (c). In each pair, the central angle is the same but the radius has been doubled. What effect does doubling the radius have on the length of the arc? Justify why this relationship exists.



LESSON 1: All Circles Great and Small • M2-219

Consider the diagram of tree rings shown. Let the radius of the small tree ring (small circle) be *r* and the radius of the larger tree ring (large circle) be 10*r*.



- 11. Compare the tree rings.
 - a. How does the length of the minor arc in the small tree ring compare to the length of the minor arc in the large tree ring?

b. If the length of the minor arc in the small tree ring is equal to 3 inches, what is the length of the minor arc in the large tree ring?

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c. If m∠A = 20° and the length of the minor arc of the small tree ring is 3 inches, determine the circumference of the large tree ring.

Measuring Angles and Arcs Using Radians

ACTIVITY

1.4

So far you have described the measures of angles and arcs using degrees and determined the arc length intercepted by a central angle as a linear measurement.

Let's investigate another unit of angular measure called a *radian*. One **radian** is defined as the measure of a central angle whose arc length is the same as the radius of the circle. The radian measure, θ , of a central angle is the ratio of the length of the intercepted arc to the radius of a circle.

The length of the intercepted arc of the central angle is proportional to the radius. The radian measure of the angle is the constant of proportionality.

 $\theta = \frac{s}{r}$

 $s = \theta r$

\$2

1. Use the fact that all circles are similar to show that the length of the arc intercepted by an angle is proportional to the radius. Identify the constant of proportionality.



- 2. What is the radian measure of an entire circle?
- 3. Write the equivalent radian measure for each degree measure.

180°





The central angle of

360

one radian of a unit circle produces an arc length of 1.

90°

270°

The degree measure and radian measure are related by the equations $360^\circ = 2\pi$ radians and $180^\circ = \pi$ radians.

The formulas you can use to convert from radians to degrees and degrees to radians are shown.



- 4. Write an equivalent equation to show each relationship.
 - a. 1° and radians

b. 1 radian and degrees

The decimal approximation is not used to convert between radians and degrees.

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5. Determine the unknown arc length. Explain your reasoning.



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6. The measure of a central angle is 120°. The length of the radius is 20 centimeters. Determine the arc length using each equation.

a. $s = \frac{\text{measure of angle}}{360^\circ} \cdot 2\pi r$ b. $s = \theta r$

c. Compare your answers in part (a) and part (b).

- 7. If $\theta = \frac{\pi}{2}$ and r = 4, solve for the length of the intercepted arc.
- 8. If *r* = 2 and the intercepted arc length is 5, what is the measure of the central angle?
 - 9. Describe the similarities and differences between radians and degrees.

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10. Which unit do you prefer to use? Explain your reasoning.



Short Stack

You have stacked congruent two-dimensional figures to create solid figures. Now let's investigate how stacking similar figures can create solid figures also.

- 1. Imagine you stack similar circular pancakes on top of each other so that each layer of the stack is composed of a slightly smaller pancake than the previous layer.
 - a. Identify the name of the solid formed by this stack of pancakes.
 - b. Relate the dimensions of a single pancake to the dimensions of the solid.
- 2. Imagine you stack similar square pancakes on top of each other so that each layer of the stack is composed of a slightly smaller pancake than the previous layer.
 - a. Identify the name of the solid formed by this stack of pancakes.
 - b. Relate the dimensions of a single pancake to the dimensions of the solid.

4.1

Building the Pyramid Volume Formula



Let's think about how to derive the formula for the volume of a pyramid. The diagrams show two pyramids—one with two layers and one with three layers.

1. Use centimeter cubes to construct these pyramids. Then answer the questions and explain your reasoning.



- a. Describe the two-dimensional shape of the top surface of each layer of cubes.
- b. Explain why the two-dimensional surfaces of the cube layers represent similar figures.

- c. What is the volume of each pyramid?
- d. For each of the two pyramids, determine the volume of a prism that has the same base.

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4.2 Building the Cone Volume Formula



You have learned that the formula for the volume of a cone, like the formula for the volume of a pyramid, is $V = \frac{1}{3}Bh$. You can use Cavalieri's Principle to understand why the volume formulas for the two different solids are the same.

Consider the cone and pyramid shown. Their bases have equal area.



1. If the length of the radius of the base of the cone is *r*, what is the area of its base?

2. If the side length of the base of the square pyramid is *s*, what is the area of its base?

3. You are given that the areas of the bases are equal. Set the areas equal to each other and rewrite the equation in terms of the side length of the base of the square pyramid.

Cavalieri's Principle states that if a plane cross-section parallel to the given plane has the same area in both solids, then the volumes of the solids are equal.

Remember:

ACTIVITY 4.4

Surface Area

In the previous activity, you explored the different volume created by rolling a piece of paper in two different ways. The cylinders that you created had the same lateral surface area. The lateral surface area of a threedimensional figure is the sum of the areas of its lateral faces. The total surface area of a three-dimensional figure is the sum of the areas of its bases and lateral faces.

- 1. Consider the right cylinder shown.
 - a. Sketch the bases and lateral faces of the cylinder. Include the dimensions.
 - b. Determine the area of each face.
 - c. Use your sketch to write the formulas for the total surface area and lateral surface area of the cylinder. Explain your reasoning.
- of the lateral face of a cylinder is equal to the circumference of the base.
- Recall that the width

A lateral face of a three-dimensional

figure is a face that is

Remember:

h

not a base.

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b. Apply the formula to determine the total surface area of the paint roller.

a. Apply the formula to determine the lateral surface area of the

2. A cylindrical paint roller has a diameter of 2.5 inches and a

length of 10 inches.

paint roller.

3. Consider the right rectangular prism shown. Its bases are shaded.



- a. Sketch the bases and lateral faces of the prism. Include the dimensions of each.
- b. Determine the area of each face.

For right rectangular prisms, you can call any pairs of opposite faces "bases."

c. Use your sketch to write the formulas for the total surface area and lateral surface area of the right rectangular prism. Explain your reasoning.



4. David says that the lateral surface area of a right rectangular prism can change, depending on what the bases of the prism are. He calculates 3 different lateral surface areas, *L*, for the prism shown.

 $L_1 = 36 \text{ ft}^2$ $L_2 = 60 \text{ ft}^2$ $L_3 = 48 \text{ ft}^2$ Is David correct? Explain your reasoning.



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4.5 Spheres

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Recall that a circle is the set of all points in two dimensions that are equidistant from the center of the circle. A sphere can be thought of as a three-dimensional circle.



The volume of a sphere is given by the equation $V = \frac{4}{3}\pi r^3$, where *r* represents the radius of the sphere.

- 1. Determine the volume of each sphere with the given radius length.
 - a. 2.2 in.

c. $\frac{1}{2}$ yd

b. 4 cm

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d. 4.4 in.

A **sphere** is the set of all points in three dimensions that are equidistant from a given point called the center.

The **radius of a sphere** is a line segment drawn from the center of the sphere to a point on the sphere.

The **diameter of a sphere** is a line segment drawn between two points through the center.

A great circle of a sphere is a cross section of a sphere when a plane passes through the center of the sphere.

A **hemisphere** is half of a sphere bounded by a great circle.

Assignment

Write

Describe a similarity and a difference between each term.

- 1. lateral surface area and total surface area
- 2. radius of a sphere and diameter of a sphere

Remember

The volume formula for a cone is $V = \frac{1}{3}\pi r^2 h$, where *r* is the radius of the base and *h* is the height of the cone. The volume formula for a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height of the pyramid.

The lateral surface area of a three-dimensional figure is the sum of the areas of its lateral faces. The total surface area of a three-dimensional figure is the sum of the areas of its bases and lateral faces. The volume formula for a sphere is $V = \frac{4}{3}\pi r^3$, where *r* is the radius of the sphere. The formula for the total surface area of a sphere is $SA = 4\pi r^2$, where *r* is the radius of the sphere.

Practice

- The Luxor Hotel in Las Vegas is a replica of the Great Pyramid at Giza. The Luxor's base is a square with a side length of 600 feet. Each lateral face has a slant height of 460 feet, and the hotel is 350 feet tall.
 - a. What is the volume of the Luxor Hotel?
 - b. What is the lateral surface area of the Luxor Hotel?
- 2. Your municipality is replacing the storage tanks in the community. Which plan provides the greater total capacity?

Plan 1: Install one cylindrical tank that is 150 feet tall and has a radius of 50 feet.

- Plan 2: Install two cylindrical tanks that are 75 feet tall. One cylindrical tank has a radius of 30 feet, and one tank has a radius of 25 feet.
- Use 3.14 for π . Round your answers to the nearest tenth, if necessary.
- 3. A traffic cone has a radius of 9 inches and a height of 30 inches. What is the volume of this traffic cone? What is its lateral and total surface area if the slant height of the traffic cone is approximately 31 inches?
- 4. Today's deal at the ice cream shop is a mini cone with one scoop of ice cream.
 - a. A mini ice cream cone has a diameter of 3.5 centimeters and a height of 6 centimeters. How much ice cream fits in the cone?
 - b. One scoop of ice cream has the same diameter as the cone, 3.5 centimeters. What's the volume of 1 scoop of ice cream?
- 5. A soccer ball has a circumference of 70 centimeters at its widest point. What is the volume and total surface area of the soccer ball?